KIER DISCUSSION PAPER SERIES

KYOTO INSTITUTE
OF
ECONOMIC RESEARCH

Discussion Paper No. 1079

"Comparative risk and ambiguity aversion: an experimental approach"

Takashi Hayashi and Ryoko Wada

May 2022

KYOTO UNIVERSITY

KYOTO, JAPAN

Comparative risk and ambiguity aversion: an experimental approach *

Takashi Hayashi† and Ryoko Wada‡

May 30, 2022

Abstract

This paper experimentally studies comparative properties of risk aversion and ambiguity aversion in the way that the role of heterogeneity is allowed for. We examine correlation between the degrees of risk aversion and the degree of ambiguity aversion, how the latter changes across geometric properties of objective sets of possible probability distributions and how ambiguous information is generated.

JEL Classification Code: C91, D81

Keywords: Ambiguity aversion, risk aversion, comparative definitions, choice experiments

*The experiments in this paper are funded by Keiai University. We gratefully appreciate their supports.

†University of Glasgow, Takashi.Hayashi@glasgow.ac.uk

‡Keiai University, rwada@u-keiai.ac.jp

1 Introduction

Significance of ambiguity and ambiguity aversion as compared to risk and risk aversion has been well-documented, after Ellsberg [14], in both theoretical[1] and empirical/experimental literature.[2]

The current paper aims at getting a better empirical account of heterogeneity in the degrees of risk aversion and ambiguity aversion, while majority of the existing studies are concerned with model selection questions. We investigate comparative properties of risk and ambiguity aversion across subjects, and comparative properties of ambiguity aversion across information sources, which differ in the ranges of objectively possible probability distributions and in how ambiguous information is generated.

For this purpose, we present ambiguous information as objective but imprecise probabilistic information, following Hayashi and Wada [21], Cohen, Tallon and Vergnaud [9]. Such information is given in the form of a set, called a probability possibiity set, such that the decision maker knows only that the true probability distribution lies it it but does not know anything about which one in it is true or which one in it is more likely to be true.

We measure probability equivalent of a bet under ambiguity as well as its certainty equivalent. When we restrict attention to choice between binary bets, one is ambiguous and the other is with known probability, and vary the value of known probability of winning in the latter, the probability equivalent of a bet under ambiguity is the value at which the subject is indifferent between the two bets.

We vary probability possibility sets and measure how the distribution of degrees

[1] See Schmeidler [27], Gilboa and Schmeidler [20], Epstein [16], Ghirardato and Marinacci [19], Ghirardato, Maccheroni and Marinacci [18] among many.

[2] Earlier contributions to the experimental literature are Becker and Brownson [3], Slovic and Tversky [28], Yates and Zukowski [29], Einhorn and Hogarth [13], Fox and Tversky [15], Chow and Sarin [8], Halevy [22], Ahn, Choi, Gale and Kariv [1], Bossaerts, Ghirardato, Guarnaschelli and Zame [5].

of ambiguity aversion respond to such variation. In particular we are interested in whether and how the degrees of ambiguity aversion are sensitive to geometric properties of probability possibility sets.

We vary how we generate ambiguous information as well. One of the key issues in lab experiment on choice under ambiguity is how we generate objectively ambiguous information so that there is no room for manipulation and speculation. A traditional way of presenting an Ellsberg urn/box is simply telling nothing about the proportion of colors in it, but this is problematic when we carry out a real choice experiment, since we have to determine the actual proportion of colors in the end: this has to lead the subjects to speculate about manipulation by the experimenter, as pointed out by Hayashi and Wada [21], Hey and Pace [23], Dominiak and Duersch [12], Oechssler and Roomets [25].

We run the experiments in two ways. One is the traditional method in which we simply tell nothing. The other follows Hayashi and Wada [21], where we generate ambiguous information in the way that there is no room for manipulation by the experimenter and nevertheless the problem does not reduce to calculating a probability distribution over probability distributions.

The paper proceeds as follows. In Section 2 we extend the definition of comparative risk aversion and ambiguity aversion, since the existing theoretical definition allows us to compare degrees of ambiguity aversion only between subjects with identical risk preferences. In Section 3 we explain our experimental method. We present the experimental results in Section 4, and Section 5 concludes.

2 Comparative risk and ambiguity aversion: theory

2.1 Definitions of comparative risk and ambiguity aversion

Let X be the set of pure outcomes and $\Delta(X)$ be the set of (simple) lotteries over X. Let Ω be the set of states of the world, which is assumed to be finite for simplicity. Let $\mathcal{H}$ be the set of lottery acts, which are mappings from Ω into $\Delta(X)$. Note that the set of lotteries $\Delta(X)$ is regarded as a subset of $\mathcal{H}$ consisting of constant mappings. Let $\succsim_1$ and $\succsim_2$ denote two generic preference relations over $\mathcal{H}$. When restricted to $\Delta(X)$ they are understood as risk preferences.

We follow the definition of comparative risk aversion according to Arrow [2] and Pratt [26] .

Definition 1 $\succsim_1$ is more risk-averse than $\succsim_2$ if it holds

$$l \succsim_1 \delta(x) \quad \Longrightarrow \quad l \succsim_2 \delta(x)$$

and

$$l \succ_1 \delta(x) \quad \Longrightarrow \quad l \succ_2 \delta(x)$$

for all $x \in X$ and $l \in \Delta(X)$, where $\delta(x)$ denote the lottery degenerated at x.

Note that this definition implies the two preferences coincide over deterministic outcomes, although it is not restrictive when the outcome space is one-dimensional.

For the degree of ambiguity aversion, here is the existing definition due to Ghirardato and Marinacci [19].

Definition 2 $\succsim_1$ is more ambiguity-averse than $\succsim_2$ if it holds

$$h \succsim_1 l \quad \Longrightarrow \quad h \succsim_2 l$$

and

$$h \succ_1 l \quad \Longrightarrow \quad h \succ_2 l$$

for all $l \in \Delta(X)$ and $h \in \mathcal{H}$.

Note again that this definition implies the two preferences coincide over lotteries. It is a restrictive condition, as one can compare ambiguity attitudes only between preferences with the identical risk attiude.

Thus we also consider a weaker definition of comparative ambiguity aversion as below (see Dimmock, Kouwenberg, Wakker [10], Wang [30]). Let $(x; E, y; E^c)$ denote a bet in which x is given if event E occurs and y is given otherwise. Let $(x; \lambda, y; 1-\lambda)$ denote a lottery which yields x with probability λ and y with probability $1-\lambda$.

Definition 3 $\succsim_1$ is weakly more ambiguity averse than $\succsim_2$ if it holds

$$(x; E, y; E^c) \succsim_1 (x; \lambda, y; 1-\lambda) \quad \Longrightarrow \quad (x; E, y; E^c) \succsim_2 (x; \lambda, y; 1-\lambda)$$

and

$$(x; E, y; E^c) \succ_1 (x; \lambda, y; 1-\lambda) \quad \Longrightarrow \quad (x; E, y; E^c) \succ_2 (x; \lambda, y; 1-\lambda)$$

for all $x, y \in X$ and $\lambda \in (0,1)$ and $E \subset \Omega$.

Also, one can consider a richer domain of choice which consist of pairs of "information" and act, where "information" takes the form of a set of probability distributions over states such that the decision maker knows the true distribution lies in it but does not know which one in it is true or which one in it is more likely to be true. We call such set a *probability possibility set*. Let $\mathcal{P}$ be the set of probability possibility sets, which are compact and convex subsets of $\Delta(\Omega)$.

Then one can consider preference over $\mathcal{P} \times \mathcal{H}$, where $(P, f) \succsim (Q, g)$ says the decision maker weakly prefers taking bet f under P over taking bet g under Q. In experiment, such P and Q are provided by the experimenter as the Ellsberg boxes. This allows us to make a distiction between the set of objecievely possible probability distributions and the subjective set of priors as in Gilboa and Schmeidler [20].

In such an environment, ambiguity aversion is revealed as an attitude to avoid betting under imprecise information. The following definition is due to Gajdos, Hayashi, Tallon and Vergnaud [17].

Definition 4 On the domain of pairs of probability possibility sets and acts, $\succsim_1$ is more imprecision-averse than $\succsim_2$ if it holds

$$(P,(x;E,y;E^c)) \succsim_1 (\{p\},(x;E,y;E^c)) \implies (P,(x;E,y;E^c)) \succsim_2 (\{p\},(x;E,y;E^c))$$

and

$$(P,(x;E,y;E^c)) \succ_1 (\{p\},(x;E,y;E^c)) \implies (P,(x;E,y;E^c)) \succ_2 (\{p\},(x;E,y;E^c))$$

for all $x, y \in X$, $E \subset \Omega$ and $p \in \Delta(X)$ and $P \in \mathcal{P}$.

2.2 Models

While our measurement methods are model-free, introducing models helps us to get a better understanding of the measurement observations.

The most prominent model of risk preference is expected utility theory preference, which allows representation in the form

$$U(l) = E_l[u(x)]$$

for $l \in \Delta(X)$, where $u : X \to \mathbb{R}$ is called von-Neumann/Morgenstern (vNM) index and E_l denotes expectation with regard to l.

The following proposition is standard.

Proposition 1 Let u_1 and u_2 be vNM indices that form expected utility representations of $\succsim_1$ and $\succsim_2$, respectively. Then, $\succsim_1$ is more risk-averse than $\succsim_2$ if and only if there is a monotone and concave transformation ϕ such that $u_1 = \phi \circ u_2$.

The most prominent model of ambiguity averse preferences is maxmin expected utility preference due to Gilboa and Schmeidler [20], which is represented in the form

$$U(h) = \min_{p \in P} E_p[u \circ h]$$

for $h \in \mathcal{H}$, where $P \subset \Delta(\Omega)$ is the subjective set of beliefs and E_p denotes expectation with regard ot each possible $p \in P$, and $u \circ h : \Omega \to \mathbb{R}$ is a composite mapping which maps each $\omega \in \Omega$ into $E_{h(\omega)}[u(x)]$.

One can show the follwing propsition from the standard uniqueness argument.

Proposition 2 Let (u_1, P_1) and (u_2, P_2) be pairs of vNM indices and sets of priors that form maxmin expected utility representations of $\succsim_1$ and $\succsim_2$, respectively. Then, $\succsim_1$ is more mbiguity-averse than $\succsim_2$ if and only if u_1 and u_2 are cardinally equivalent and $P_1 \supset P_2$.

Also, one can show

Proposition 3 Let (u_1, P_1) and (u_2, P_2) be pairs of vNM indices and sets of priors that form maxmin expected utility representations of $\succsim_1$ and $\succsim_2$, respectively. Then, $\succsim_1$ is weakly more ambiguity-averse than $\succsim_2$ if and only if u_1 and u_2 are ordinally equivalent and $\min_{p \in P_1} p(E) \leq \min_{p \in P_2} p(E)$.

Note that when $|\Omega| = 2$ the above two propositions are identical.

One may consider a more flexible class, so-called α-maxmin preference which is represented in the form

$$U(h) = (1 - \alpha) \min_{p \in P} E_p[u \circ h] + \alpha \max_{p \in P} E_p[u \circ h]$$

As pointed out Ghirardato, Maccheroni and Marinacci [18], however, α-maxmin preference lacks of uniqueness of representation, in the sense that one preference can be represented by two (α, P) and (β, Q) where α and β are different and β and Q are different. Thus the two definition of comparative ambiguity aversion does not have an implication to parametric comparison in this class.

Such lack of uniqueness motivates us to consider a richer domain in which probability possibiity sets are treated as objects. One such model is a natural extension of the α-model, which has the form

$$U(P, h) = (1 - \alpha) \min_{p \in P} E_p[u \circ h] + \alpha \max_{p \in P} E_p[u \circ h]$$

Note that when $\alpha = 0$ the decision maker is completely pessimistic in the sense tha she takes the worst-case from the entire probability possibility set, and when $\alpha = 1$ she is completely optimistic in the analougous way.

Thw following observation is immediate.

Proposition 4 Let (u_1, α_1) and (u_2, α_2) be pairs which form α-maximin representations of $\succsim_1$ and $\succsim_2$ respectively. Then $\succsim_1$ is more imprecision averse than $\succsim_2$ if and only if u_1 and u_2 are ordinally equilavent and $\alpha_1 \leq \alpha_2$.

One important property of the α-maximin model is insensitive to geometric properties of probability possibility sets, as only the best case and the worst case should matter.

Another prominent model is called the *contraction model*, in which the decision maker forms her subjective set of beliefs by shrinking the given probability possibility set toward its "center" at certain degree (Gajdos, Hayashi, Tallon and Vergnaud [17]). Denote the "center"[3] of set P by $s(P)$, then the preference is represented in the form

$$U(P, h) = (1 - \varepsilon)E_{s(P)}[u \circ h] + \varepsilon \min_{p \in P} E_p[u \circ h] = \min_{p \in (1-\varepsilon)\{s(P)\}+\varepsilon P} E_p[u \circ h]$$

Note that when $\varepsilon = 0$ the decision maker shrinks the given probability possibility set into a single point $s(P)$, and when $\varepsilon = 1$ she does not shrink the set at all and takes the worse case in the entire probability possibility set.

It is now intuitive to observe the following.

Proposition 5 Let (u_1, ε_1) and (u_2, ε_2) be pairs which form contraction representations of $\succsim_1$ and $\succsim_2$ respectively. Then $\succsim_1$ is more imprecision averse than $\succsim_2$ if and only if u_1 and u_2 are ordinally equilavent and $\varepsilon_1 \leq \varepsilon_2$.

3 Experimental design

3.1 Risky/unambiguous box

For the measurement of probability-equivalent, we introduce a risky/unambiguous box, which we call Box A throughout. There are two colors, Red and Blue. Let

[3] Gajdos, Hayashi, Tallon and Vergnaud [17] characterize the "center" as Steiner point, which is the weighted average of vertices in which the weight on each vertex is proportional to its outer angle. In the current setting it suffices to know that the Steiner point of a probability simplex is the point of uniform distribution.

(p_R, p_B) denote a generic probability distribution over two colors. In particular, Box H refers to the case that $(p_R, p_B) = (0.5, 0.5)$, which will be used in the measurement of certainty equivalent.

3.2 Ambiguous boxes

We consider two ambiguous boxes, Box B and Box C, which consist of either two colors (Red and Blue) or three colors (Red, Blue and Yellow). Let (p_R, p_B) denote a generic probability distribution over two colors, and (p_R, p_B, p_Y) denote a generic probability distribution over three colors. Then the three boxes are formalized as follows. Then the two probability-possibility sets are given by

$$\begin{aligned} B &= \{(p_R, p_B) : p_R, p_B \geq 0,\ p_R + p_B = 1\} \\ C &= \{(p_R, p_B, p_Y) : p_R, p_B, p_Y \geq 0,\ p_R + p_B + p_Y = 1\} \end{aligned}$$

3.3 How do we generate a "real" Ellsberg box?

In most Ellsberg experiments the subjects are simply told nothing about the proportion of colors in the ambiguous boxes. This creates a room for manipulation, because in real-choice experiments the experimenter has to create a "real" Ellsberg box, that is, has to determine the proportion of colors in the end. If it is simply arbitrarily chosen by the experimenter, it leaves the subjects a room for supeculation, such as "because the experimenter seems simple-minded he/she would choose it symmetrically" or "because the experimenter would like to minimize the reward payment he/she would minimize the number of balls for the prize," etc, as pointed out by Hayashi and Wada [21], Hey and Pace [23], Dominiak and Duersch [12], Oechssler and Roomets [25]. Moreover, if we simply randomize the proportion according to some transparent distribution, such as uniform distribution, it is no longer a setting of ambiguity but a setting of two-stage risk.

To deal with this issue, we generate Ellsbergs boxes in two ways and compare. One is done by following the approach by Hayashi and Wada [21]. They provided a lab-experiment method to create Ellsberg boxes which is sufficiently complicated so that it is virtually impossible for any subject to calculate the final probability distribution and yet is simple so that there is no room for manipulation by the experimenter. We follow the same procedure here.

For each probability possibility set, we play a "snake and ladder" game, which is sufficiently complicated so that calculating final probability distribution about which point in the set will realize is virtually impossible to calculate but is simple enough so that everybody can understand the rule. We call this the *DICE* treatment. In the DICE treatment, Box B and C are generated through the procedures as illustrated in Appendix.

The other is to follow the traditional method taken in almost all of the Ellsberg experiments, where we simply tell nothing about the proportion of colors or about how it is being chosen. Thus we call it the *TN* (Telling Nothing) experiment.

3.4 Probability equivalent vs. certainty equivalent

For each of Box H, B, C and D, we measure certainty equivalent of a bet on Red in which one receives 2000 JPY/Point if Red is drawn and nothing otherwise, using the multiple price list (MPL) method.

For each of Box B, C and D, we measure *probability equivalent* of a bet on Red in the ambiguous box in an analogous way: Bring box A containing Red or Blue balls, in which the proportion of Red is known but variable. Start with the proportion of Red being zero and gradually raise it. Let the subject choose between the box with known proportion of Red and the given ambiguous box, where the subject wins 2000 JPY/Point if Red is drawn from the chosen box. Find the value of the proportion of Red such that the subject is indifferent between beting on Red in the box with known probabilities and betting on Red in the box with unknown probabilities. The instruction are summarized in the appendix.

Let p_B denote the probability equivalent of a bet on Red in Box B and p_C denote

Table 1: Experiments conducted

Date	University	numbers(female)	Treatment
2016/10/17	Keio	58(7)	DICE
2017/10/2	Keio	61(7)	DICE
2018/5/28	Keio	23(3)	TN
2018/7/18,7/19	Keiai	43(0)	TN
2019/7/23,25,30,31	Keiai	36 (5)	TN
2020/1/6	Tohoku Gakuin	59 (15)	TN
sum		278(37)	DICE(119)TN(159)

the probability equivalent of a bet on Red in Box C. Then, the α-maxmin model predicts

$$p_B = p_C = \alpha,$$

meaning that ambiguity attitude is insensitive to geometric properties of probability possibility sets. On the other hand, the contraction model predicts

$$p_B = \frac{1-\varepsilon}{2}, \quad p_C = \frac{1-\varepsilon}{3},$$

implying

$$p_C = \frac{2}{3} p_B,$$

meaning that ambiguity attitude is sensitive to geometric properties of probability possibility sets in a proportional way.

3.5 Conduct of the experiment

The experiments were conducted at Keio University, Keiai University and Tohoku Gakuin University, in which all the subjects were students (see Table 1). Each subject was paid 2000 JPY if he or she won in the problem that was randomly chosen from all the problems. The participation reward was 1000 JPY.

4 Results

4.1 Comparative risk/ambiguity aversion across subjects

Certainty equivalent in Box H and probability equivalent in Box B are plotted in Figure 1-3, for each of the DICE treatment group, TN treatment group and the group of all subjects. Also, the statistical analysis is summarized in Table 1. They show that correlation between risk aversion and ambiguity aversion is significant overall but the degree of risk aversion explains little about the degree of ambiguity aversion. Note that the correlation is weak in the DICE treatment.

Certainty equivalent in Box H and probability equivalent in Box C are plotted in Figure 4-6, for each of the DICE treatment group, TN treatment group and the group of all subjects. Again there is almost no correlation in each case. The statistical analysis is summarized in Table 2. Again, they show that correlation between risk aversion and ambiguity aversion is significant overall but the degree of risk aversion explains little about the degree of ambiguity aversion. Note that the correlation is weak in the DICE treatment.

This result contrasts to Cohen, Tallon and Vergnaud [9], who observe no correlation between risk aversion and ambiguity aversion, although it is partially consistent in the sense that the degree of risk aversion explains little of the degree of ambiguity aversion. It is also consistent with the result by Lauriola and Levin [24], Butler, Guiso Jappelli [6], which report positive correlation between risk aversion and ambiguity aversion in different settings.

4.2 Comparative ambiguity aversion across probability possibility sets

Probability equivalent in B and probability equivalent in C are plotted in Figure 7-9, for each of the DICE treatment group, TN treatment group and the group of all subjects. There is a correlation, where it is strong in the TN treatment but weak in

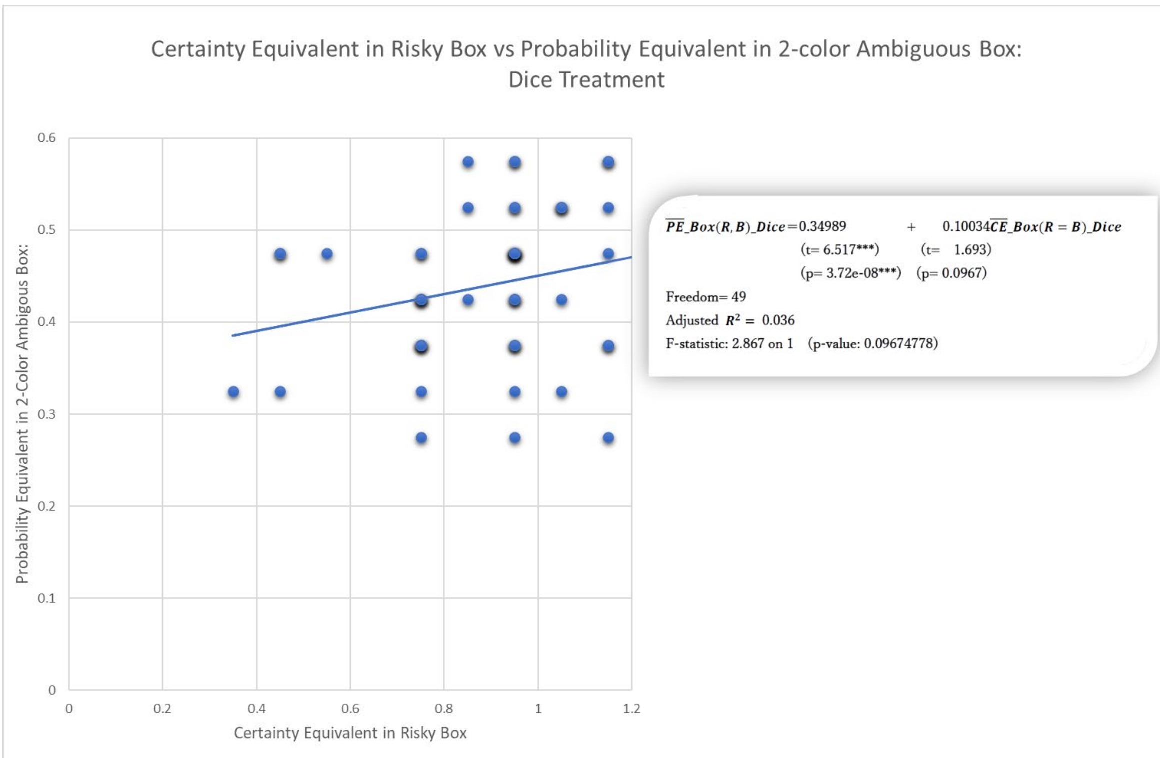

Figure 1: Certainty Equivalent in Risky Box and Probability Equivalent in 2-color Ambiguous Box:DICE Treatment

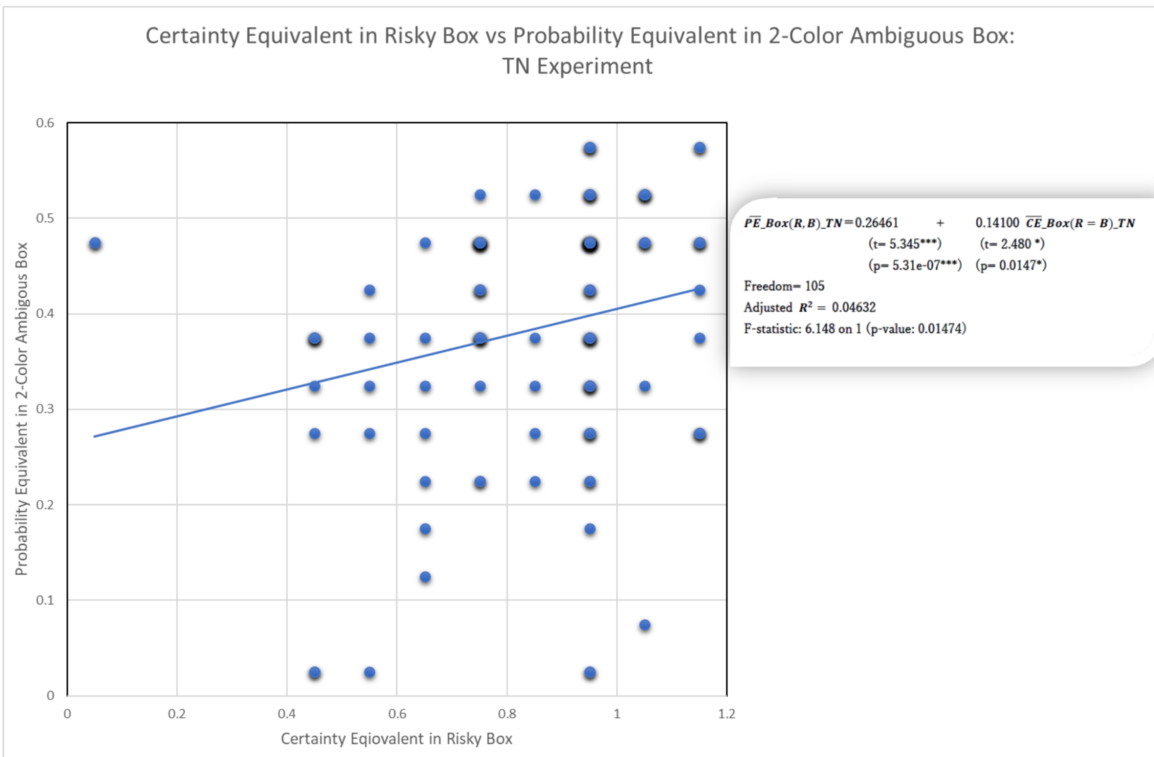

Figure 2: Certainty Equivalent in Risky Box and Probability Equivalent in 2-color Ambiguous Box:TN Treatment

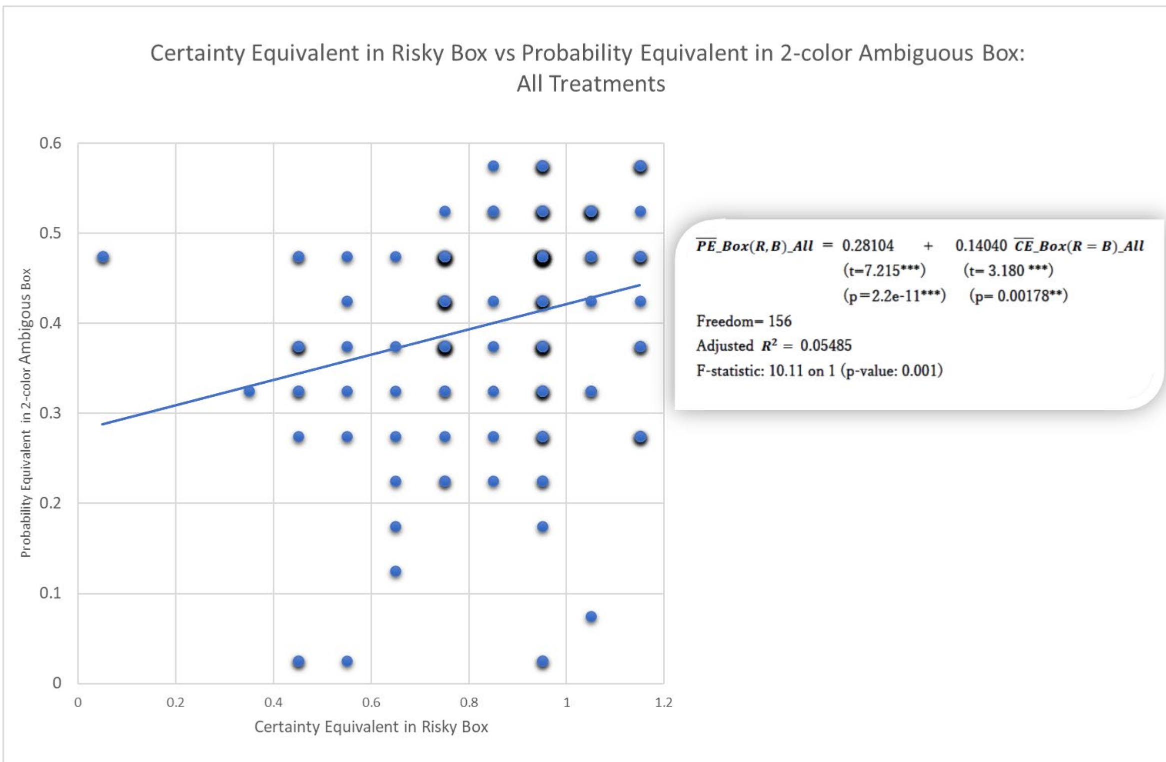

Figure 3: Certainty Equivalent in Risky Box and Probability Equivalent in 2-color Ambiguous Box:All Treatments

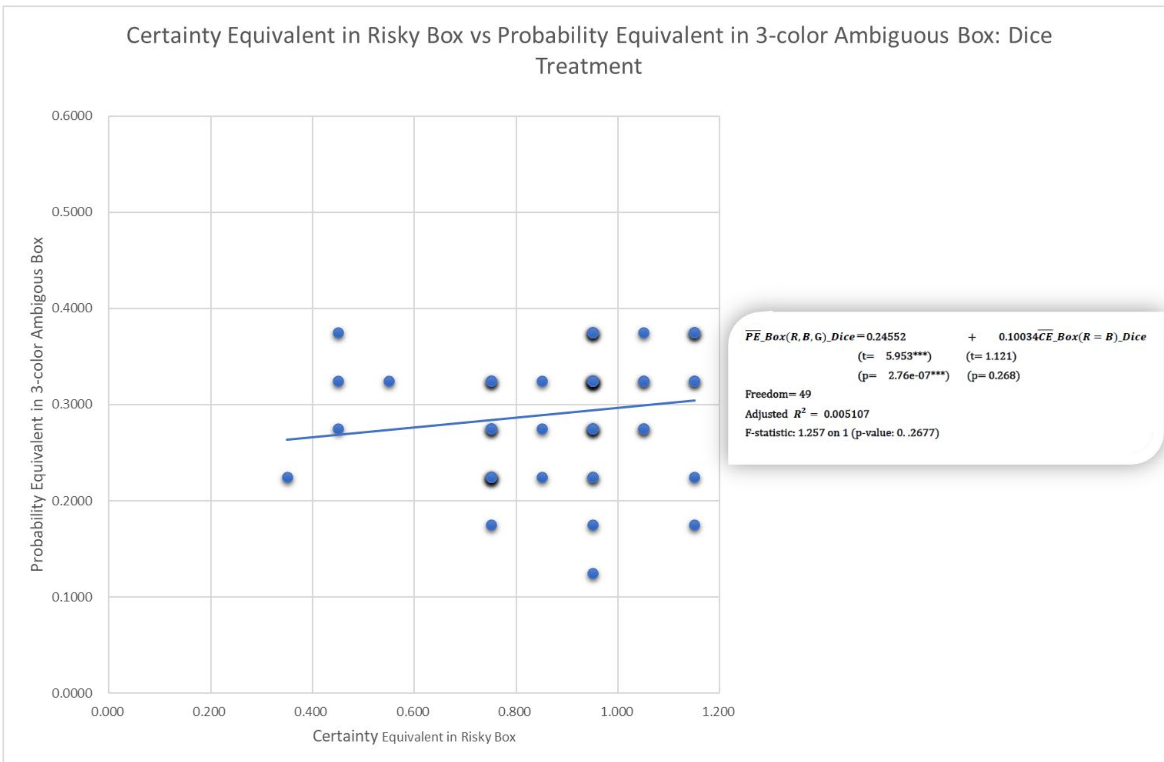

Figure 4: Certainty Equivalent in Risky Box and Probability Equivalent in 3-color Ambiguous Box:DICE Treatment

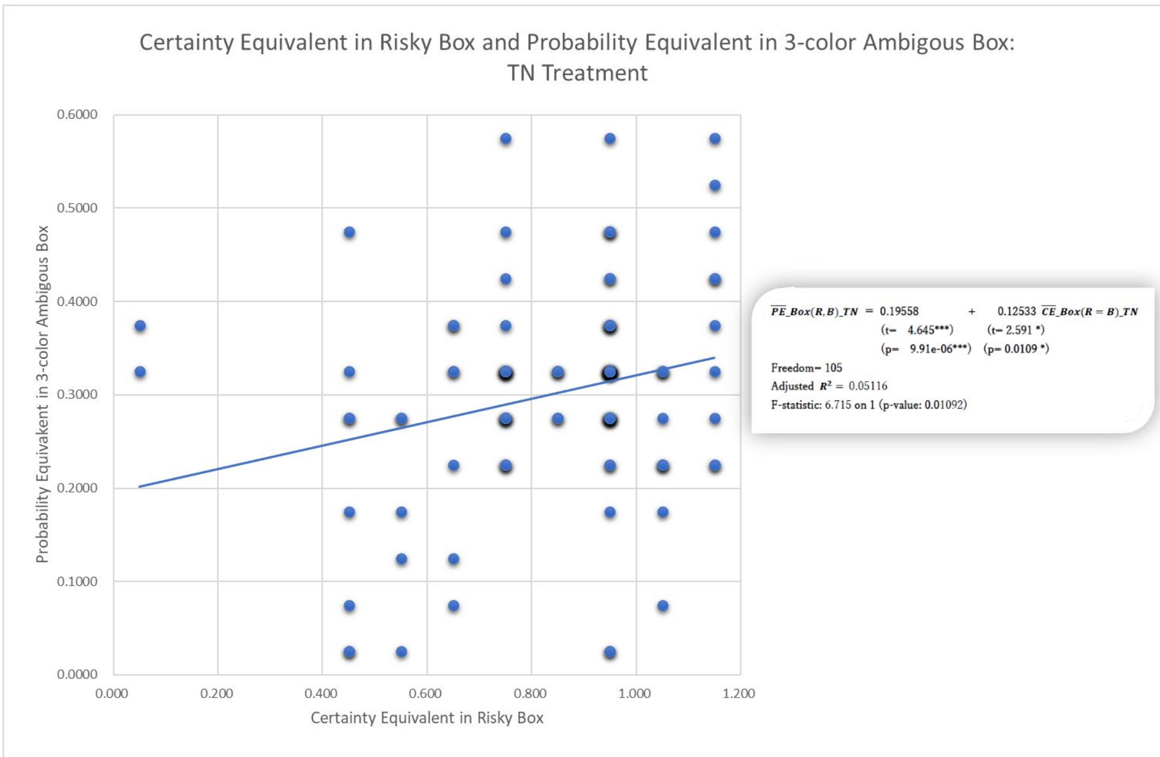

Figure 5: Certainty Equivalent in Risky Box and Probability Equivalent in 3-color Ambiguous Box:TN Treatment

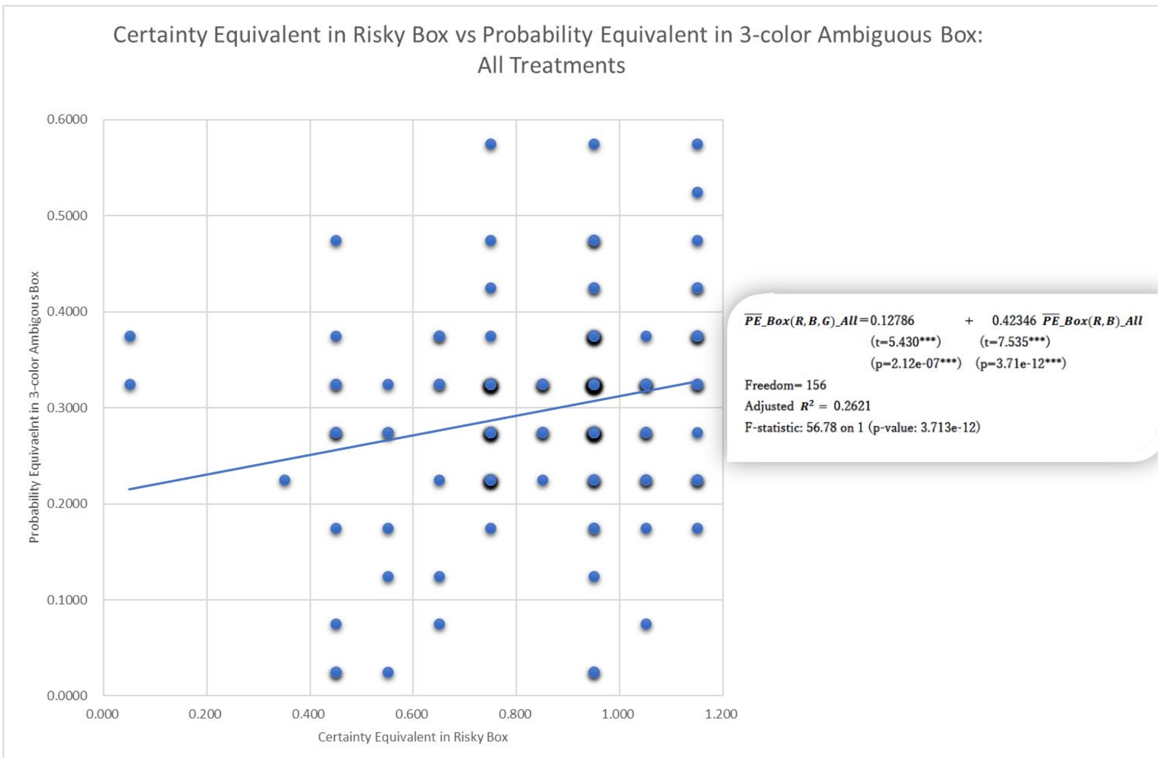

Figure 6: Certainty Equivalent in Risky Box and Probability Equivalent in 3-color Ambiguous Box:All Treatments

Table 2: Change in ambiguity aversion between 2-color and 3-color

	ALL	DICE	TN
mean of Ambigous Box(R,B)	0.4013	0.4387	0.3834
mean of Ambigous Box(R,B,G)	0.2978	0.2907	0.3012
$H_0 : Box(R,B,G) = Box(R,B)$	t = 11.822**	t = 11.47**	t = 7.6055**
paired t.test	p=2.2e-16	p =1.307e-15	p = 1.212e-11
$H_0 : Box(R,B,G) = \frac{2}{3}Box(B,G)$	t = 3.8295**	t = 0.83564	t = 4.1818**
	p = 0.0001852	p = 0.4073	p = 5.973e-05

**level of significance for 99% confidence interval

the DICE treatment.

Recall that the two must be distributed on the 45-degree line under the α-maxmin model in which the degree of ambiguity aversion is insensitive to geometric properties of probability possibility sets, while the ε-contraction model predicts that the slope is 2/3 and the intercept is 0. When we take it as a ground assumption that the slope is 0, the test statistics show that the actual distribution is violating both in the TN treatment but it is significantly violating only the α-maxmin model in the DICE treatment (see Table 2).

4.3 Comparative ambiguity aversion across information generating processes

Lastly we compare between the observed distributions of degree of ambiguity aversion between the DICE treatment and the TN treatment. Table 3 show that mean of probability equivalent is significantly higher in the DICE treatment than in the TN expetiment in Box B with two colars. We observe the same pattern in Box D with three colors but this difference is not significant. We cannot reject the null hypothesis that variance of probability equivalent in the DICE treatment is the same as that in the TN treatment, in each of Box B and Box C.

This is consistent with the result by Oechssler and Roomets [25] that less ambigu-

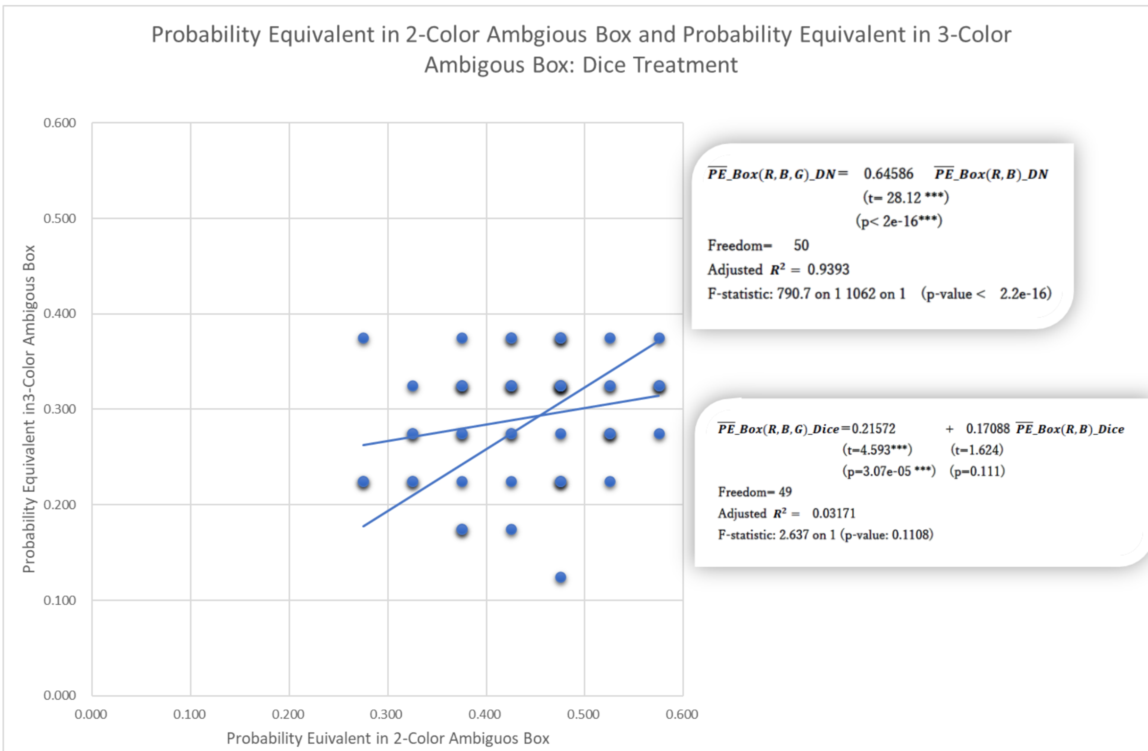

Figure 7: Probability Equivalent in 2-color Ambiguous Box and Probability Equivalent in 3-color Ambiguous Box:DICE Treatment

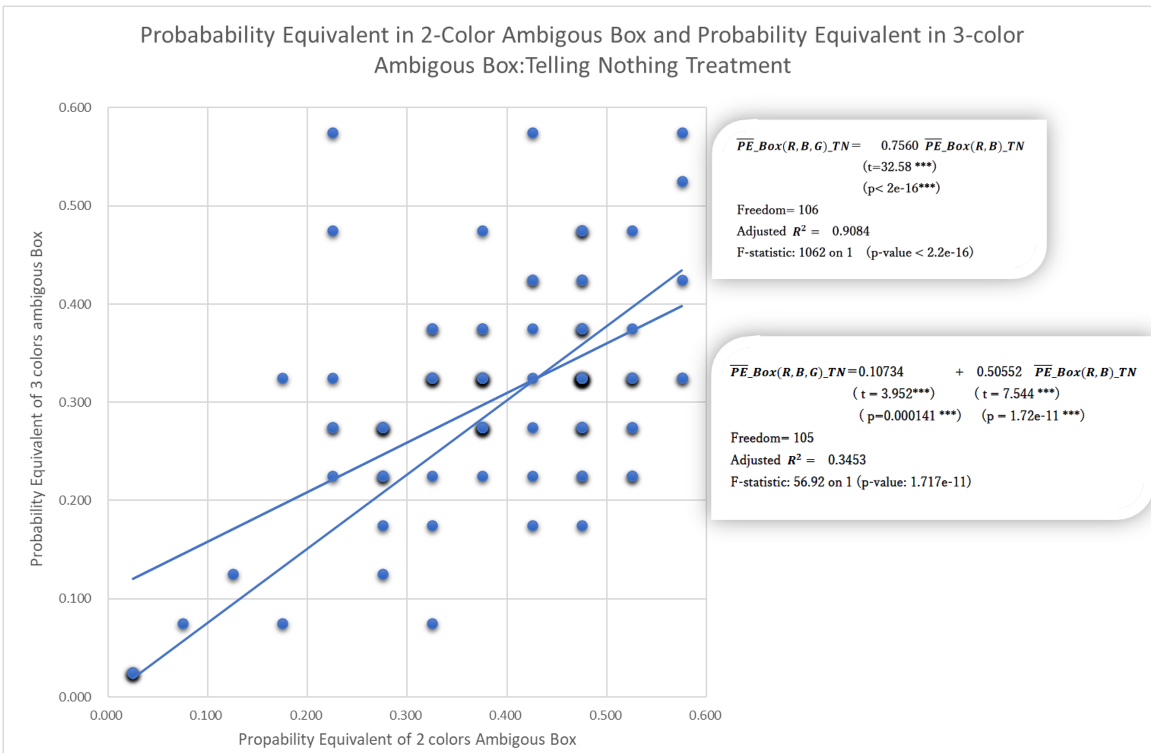

Figure 8: Probability Equivalent in 2-color Ambiguous Box and Probability Equivalent in 3-color Ambiguous Box:TN Treatment

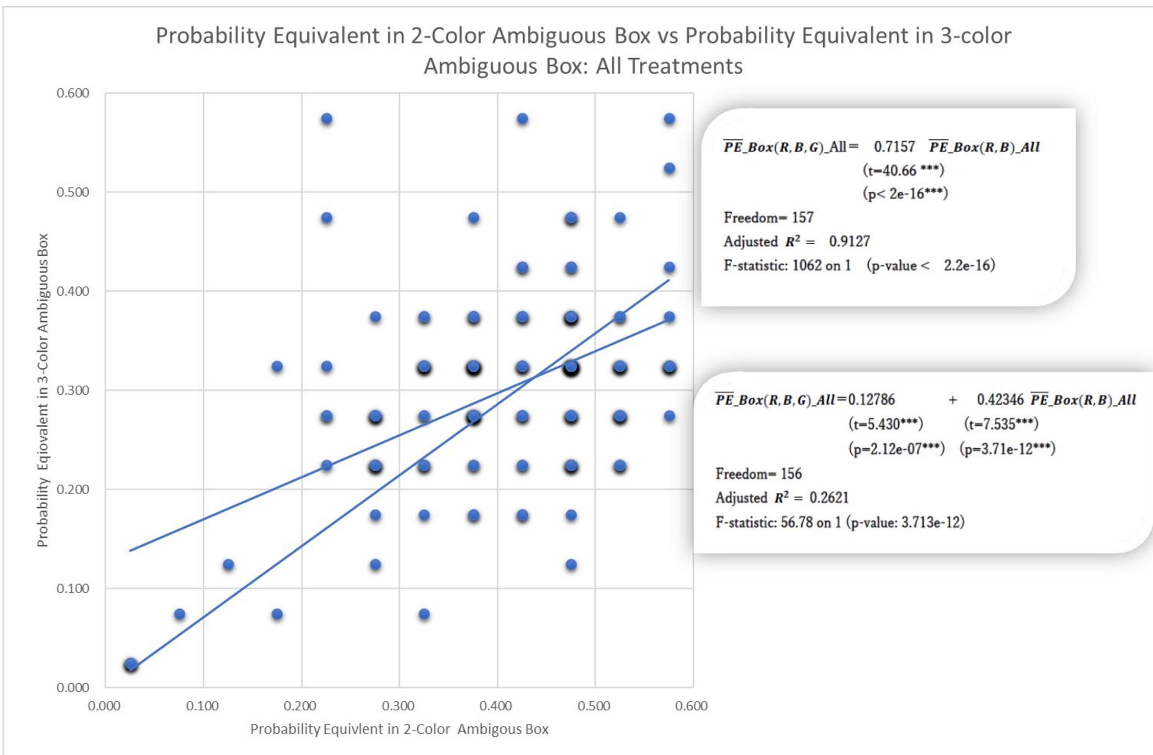

Figure 9: Probability Equivalent in 2-color Ambiguous Box and Probability Equivalent in 3-color Ambiguous Box:All Treatment

Table 3: Change in ambiguity aversion between DICE and TN

Wilcoxon Rank Sum Test	Box(R=B)	Box(R,B)	Box(R,B,G)
mean of $\bar{Box}_{Dice}$	842.5	0.4387	0.2907
mean of $\bar{Box}_{TN}$	885.3	0.3834	0.3012
W	W = 3009	W= 3315	W= 2501.5
p-value	p =0.2805	p = 0.02672	p = 0.3863
H_0:$\bar{Box}_{Dice} = \bar{Box}_{TN}$	not rejected	rejected	not rejected

ity aversion is observed when the information-generating process is made mechanical.

Also, as we see in Figure 1-3 and 4-6, correlation between risk aversion and ambiguity aversion is weaker in the DICE treatment and stronger in the TN treatment. This suggests that there are different kinds of "ambiguity" aversion for different ways of generating "ambiguous" information.

5 Conclusion

We have experimentally investigated comparative and distributional properties of risk aversion and ambiguity aversion, in which ambiguous information is presented as sets of objective but imprecise probabilistic information and the degree of ambiguity aversion is measured though observing the probability equivalent of a bet.

We observed (i) correlation between risk aversion and ambiguity aversion is significant but the degree of risk aversion explains little of ambiguity aversion, and also the significance of correlation differs across the ways how we generate ambiguous information; (ii) the degrees of ambiguity aversion are mostly significantly correlated across probability possibility sets, and they are sensitive to geometric properties of probability possibility sets; (iii) the degree of ambiguity aversion is reduced when ambiguous information is generated in a non-manipulable way.

References

[1] Ahn, D., Choi, S., Gale, D., & Kariv, S. (2014). Estimating ambiguity aversion in a portfolio choice experiment. Quantitative Economics, 5(2), 195-223.

[2] Arrow, Kenneth J. ”Aspects of the theory of risk-bearing (Yrjo Jahnsson Lectures).” (1965).

[3] Becker, S. W., & Brownson, F. O. (1964). What price ambiguity? Or the role of ambiguity in decision-making. Journal of political economy, 72(1), 62-73.

[4] Bossaerts, P., Plott, C., and Zame, W. R. (2007). Prices and portfolio choices in financial markets: Theory, econometrics, experiments. Econometrica, 75(4), 993-1038.

[5] Bossaerts, P., Ghirardato, P., Guarnaschelli, S., & Zame, W. R. (2010). Ambiguity in asset markets: Theory and experiment. The Review of Financial Studies, 23(4), 1325-1359.

[6] Butler, J. V., Guiso, L., & Jappelli, T. (2014). The role of intuition and reasoning in driving aversion to risk and ambiguity. Theory and decision, 77(4), 455-484.

[7] Baillon, A., Halevy, Y., and Li, C. (2014). Experimental elicitation of ambiguity attitude using the random incentive system. University of British Columbia working paper.

[8] Chua Chow, Clare, and Rakesh K. Sarin. ”Known, unknown, and unknowable uncertainties.” Theory and Decision 52.2 (2002): 127-138.

[9] Cohen, M., Tallon, J. M., & Vergnaud, J. C. (2011). An experimental investigation of imprecision attitude and its relation with risk attitude and impatience. Theory and Decision, 71(1), 81-109.

[10] Dimmock, S. G., Kouwenberg, R., & Wakker, P. P. (2016). Ambiguity attitudes in a large representative sample. Management Science, 62(5), 1363-1380.

[11] Dominiak, A., Duersch, P., and Lefort, J. P. (2012). A dynamic Ellsberg urn experiment. Games and Economic Behavior, 75(2), 625-638.

[12] Dominiak, A., and Duersch, P. (2015). Benevolent and Malevolent Ellsberg Games (No. 592). Discussion Paper Series.

[13] Einhorn, H. J., & Hogarth, R. M. (1986). Decision making under ambiguity. Journal of business, S225-S250.

[14] Ellsberg, D. (1961). Risk, ambiguity, and the Savage axioms. The quarterly journal of economics, 643-669.

[15] Fox, C. R., & Tversky, A. (1995). Ambiguity aversion and comparative ignorance. The quarterly journal of economics, 110(3), 585-603.

[16] Epstein, L. G. (1999). A definition of uncertainty aversion. The Review of Economic Studies, 66(3), 579-608.

[17] Gajdos, T., Hayashi, T., Tallon, J. M., and Vergnaud, J. C. (2008). Attitude toward imprecise information. Journal of Economic Theory, 140(1), 27-65.

[18] Ghirardato, Paolo, Fabio Maccheroni, and Massimo Marinacci. "Differentiating ambiguity and ambiguity attitude." Journal of Economic Theory 118.2 (2004): 133-173.

[19] Ghirardato, P., & Marinacci, M. (2002). Ambiguity made precise: A comparative foundation. Journal of Economic Theory, 102(2), 251-289.

[20] Gilboa, I., & Schmeidler, D. (1989). Maxmin expected utility with non-unique prior. Journal of mathematical economics, 18(2), 141-153.

[21] Hayashi, T., and Wada, R. (2010). Choice with imprecise information: an experimental approach. Theory and Decision, 69(3), 355-373.

[22] Halevy, Y. (2007). Ellsberg revisited: An experimental study. Econometrica, 75(2), 503-536.

[23] Hey, John D., and Noemi Pace. "The explanatory and predictive power of non two-stage-probability theories of decision making under ambiguity." Journal of Risk and Uncertainty 49.1 (2014): 1-29.

[24] Lauriola, M., & Levin, I. P. (2001). Relating individual differences in attitude toward ambiguity to risky choices. Journal of Behavioral Decision Making, 14(2), 107-122.

[25] Oechssler, J., & Roomets, A. (2015). A test of mechanical ambiguity. Journal of Economic Behavior & Organization, 119, 153-162.

[26] Pratt, John W. "Risk Aversion in the Small and in the Large." Econometrica 32.1/2 (1964): 122-136.

[27] Schmeidler, D. (1989). Subjective probability and expected utility without additivity. Econometrica: Journal of the Econometric Society, 571-587.

[28] Slovic, P., & Tversky, A. (1974). Who accepts Savage's axiom?. Behavioral science, 19(6), 368-373.

[29] Yates, J. F., & Zukowski, L. G. (1976). Characterization of ambiguity in decision making. Behavioral science, 21(1), 19-25.

[30] Wang, Fan (2019). Comparative ambiguity attitudes, mimeo.

Appendix: How to generate imprecise information in the dice experiment.

- How to Make an Imprecise Box B (R+B=180):
  - Step 1: The experimenter rolls a six-sided die and a 20-sided die, and the sum of the numbers of each die is "c."
  - Step 2: If c is odd, the experimenter rolls an additional die with 12 faces, and adds the number to c; we call this d. If c is even, c = d.
  - Step 3: The experimenter rolls a 10-face die. The number of the die "e" is multiplied by "d," and we obtain f.
    - If f = e×f＞180, we extract 180 from f, and if f≦180, the experimenter applies the number f.
  - Step 4: The experimenter rolls a six-face die again, and if an even face appears, f becomes the number of red balls. If an odd face appears, f becomes the number of blue balls.
- How to Make Box C (R+B+Y=180)
  - Step 1: The experimenter provides a triangle, as in Fig.2. One point "X(x,y,z)" on the triangle is selected to determine the distribution of balls between red, blue, and green. The experimenter rolls two 10-sided dice simultaneously, and obtains a sum of numbers of faces "a." From the top point A(180,0,0) of the triangle in Fig. _, "x" moves "10a" toward the top point B(0,180,0), and therefore, reaches point (180－10a, 10a, 0).
  - Step 2: The experimenter rolls a 10-sided dice again, and obtains a sum of numbers of faces "b." If a+b <35, the experimenter rolls a 10-sided dice, and obtains a face "c." If a+b=36, the experimenter does not roll a dice; c=0. The point x moves from a point (180－10a, 10a, 0) toward the inside of the triangles as much as the "10b+c."
    - If a = 18, x is tentatively at B(0,180,0), and goes back as much as 10b toward the top point A(180, 0,0).
    - If a+b≦18, x stops at (180-10a-(10b+c), 10a, (10b+c)).
    - If b＞18 and a+b≦36, after reaching the bottom of the triangle, x turns up parallel to the right-hand side, and stops at X(10a +(10 b+c) − 180, 180 –(10 b+c), 180 −10 a).
  - Step 3: The experimenter rolls a six-sided die, and decides which number corresponds to the three colors between red, blue, and yellow.
    - If a face is 1 → X(x,y,z) = (red, blue, yellow) 2→ X(x,y,z) = (red, yellow,blue) 3→ X(x,y,z) = (blue, red, yellow) 4→ X(x,y,z) = (blue, yellow, red) 5→ X(x,y,z) = (yellow, red, blue) 6→ X(x,y,z) = (yellow, blue, red)

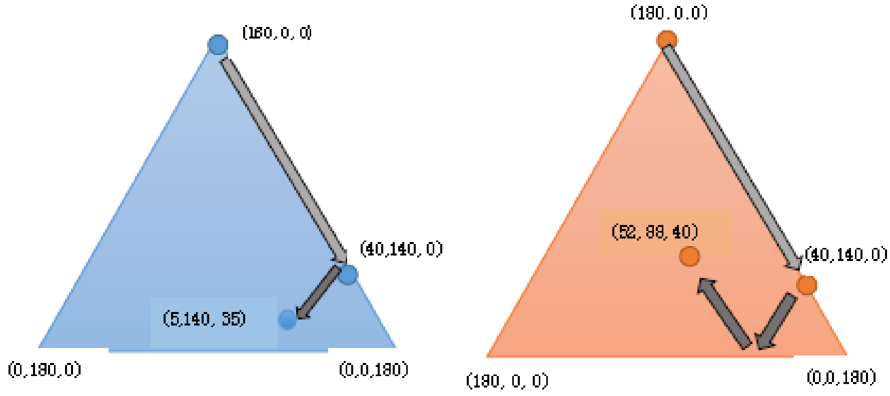

Fig. 2. Two examples of Box C;

The left triangle shows the case with a =14, b=3, therefore, (5,140,35) and in the step 3, the face of die in the step3 =4, X(5.140.35)=(blue,yellow, red)

The right triangle shows the case with a=14,b=9, therefore,(52,88,40) and in the step 3, the face of die in the step3=2, X(52,88,40)=(red,yellow, blue)