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“Pay-as-you-go pension systems supported by the old rich”

Kotono Tanigawa and Tomoya Sakagami

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# Pay-as-you-go pension systems supported by the old rich

Kotono Tanigawa

Graduate School of Economics, Kumamoto Gakuen University, JAPAN

Tomoya Sakagami\*

Faculty of Economics, Kumamoto Gakuen University, JAPAN

\*Corresponding author. E-mail: tomoya@kumagaku.ac.jp

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## Abstract

In this paper, we present a pension policy that supplements the pay-as-you-go pension system with payments by old generations with a high assets income. This supplement is intended to reduce intergenerational inequity. To analyze the effect of this pension policy on both capital stock in the economy and the utilities of the rich and the poor, we build an Over-Lapping Generations model with different incomes when young. This model finds that the stable steady-state capital stock level increases as the old rich generation contributes to the pension system. We also find by numerical simulations that there is a Pareto efficient premium level between high-income and low-income people.

**Key words:** pay-as-you-go pension system, intergenerational inequity, overlapping generations model

## 1. Introduction

There are various, clear, problems with the Japanese social security system. The most serious problem of the pay-as-you-go pension system is the one of intergenerational inequity due to the declining birthrate and aging population. That is, the per capita pension burden of younger generations increases as the birthrate declines and the population ages.

In this paper, we present a new pension policy which reduces the intergenerational inequity in pension burden, which is an increasing consequence of the present pay-as-you-go pension system. Specifically, in addition to younger generations, elder generations who have a high asset income will also contribute to the insurance premiums. The burden on younger generations, and the intergenerational inequity, will therefore be reduced.

The purpose of this paper is to analyze the impact of this new pension policy on capital stock and welfare. In order to do so, we build an Over-Lapping Generations model (OLG model) with different incomes. In the model, those with a high income in their youth have a high asset income in their elder years. Using this model, we analyze the impact of the introduction of this pension policy on the dynamics and steady-state of the capital stock. As a result, we find that the stable steady-state capital stock level increases as the rich old generations contribute to the pension system. We also find by simulation analysis that there is a Pareto efficient premium level between high-income and low-income people.

The standard model for analyzing the effect of the pension system on the dynamics of capital stock is Diamond (1965), which incorporates capital accumulation into the OLG model. The studies on the properties of this model and its extensions have been discussed in numerous publications, including de la Croix and Michel (2002), Acemoglu (2009), and Yakita (2017). In these models, a dynamic analysis of the economy under the pay-as-you-go pension system is carried out assuming an economy consisting of two generations, the young and the old generations. These papers point out that the pay-as-you-go pension system impedes capital accumulation. However, since many models dealing with the pay-as-you-go pension system assume homogeneous individuals within the same generation, income heterogeneity such as high-income and low-income is not considered.

In contrast, there are many OLG models that deal with income inequality and education. Among them, Glomm and Ravikumar (1992), Dahan and Tsiddon (1998), Sakagami and Matsuo (2021) are focusing on the education system and income inequality. The model they use is a two-generation OLG model consisting of a child generation which makes educational choices and a parent generation which performs production activities. In particular, in Sakagami and Matsuo (2021), high-income and low-income households have appeared, and those who choose education at a young age become high-income in

the next period with a given probability. Based on these assumptions, they analyze the effects of class-size policy on the private and public education systems.

In this paper, we incorporate such income inequality into the pay-as-you-go pension model so that the high-income old also contribute pension premiums. In our model, starting with an exogenous existence of high-income and low-income households in youth makes an endogenous difference in savings, so that the formerly young, high income generation has a high asset income when old. We then model this old generation paying insurance premiums together with a young generation of the same period.

We analyze the dynamic behavior of capital stock and show the existence of a stable steady-state. We also show that the level of capital stock in the stable steady-state is higher than the current pay-as-you-go system in which the old do not contribute insurance premiums. If the high-income old also pay pension premiums, the high-income young increase their savings in preparation for future premiums. As a result, the capital stock also increases. This finding corrects the previous conclusion<sup>1</sup> that pay-as-you-go pension systems impede capital accumulation. It is difficult to determine mathematically the level of steady-state capital stock in this model, so we have numerically simulated levels of capital stock and lifetime utility based on consumption in young and old age.

We found that pension premium contributions from the high-income old relieves the pension burden of the young through transfer from the old. It also has the “intra-generational” transfers effect from the high-income old to the low-income old. Therefore, when the premiums for the high-income old are increased, the lifetime utility of low-income rises. On the other hand, we also show that the lifetime utility of high-income does not decrease monotonically in relation to the increase in insurance premiums. In particular, when insurance premiums are small, the lifetime utility of high-income increases because the consumption of high-income young increases when the old support the premium, while the consumption of high-income old decreases. Therefore, the increase in consumption of the young outweighs the decrease in consumption when old, so the lifetime utility of high-income households increases.

The paper is organized as follows. Section 2 develops the model and derives the optimal savings function for high-income and low-income. Section 3 formulates the profit maximization behavior of a firm. In section 4, we find the steady-state capital stock and analyse the effect of the new contributions of insurance premiums on the steady-state and welfare. In section 5, we simulate the model to find the Pareto efficient premium level between high-income and low-income people.

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<sup>1</sup> See Acemoglu(2009) and Yakita(2017) for example.

## 2. The model

We set up a two-period OLG model in which young and old generations coexist in each period. In this model, we assume that there are high-income and low-income workers when young (see Figure 1). The generation born in the period  $t$  is called generation  $t$ . We consider the lifetime budget constraint of generation  $t$  separately for high-income and low-income workers.

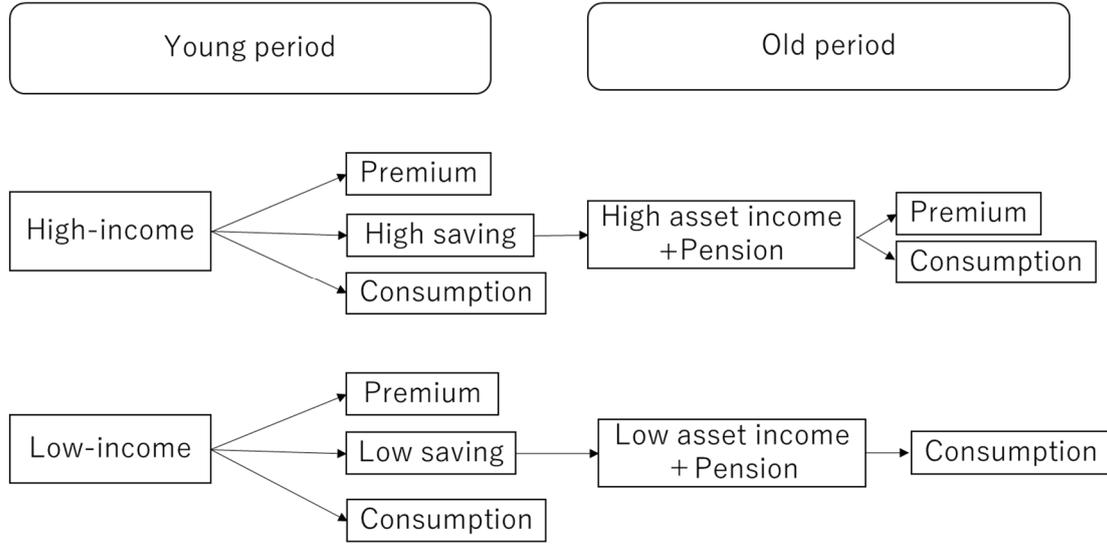


Figure 1: Household's income and expenditure in each period

First, let us consider the budget constraint for high-income households. They earn high wages in their young period. It is distributed over savings, consumption, and payments of pension insurance premiums. In this model, not only the young generation, but also the high income old also contribute to insurance premiums. That is, the contributions paid in the period  $t$  are paid by all the generation  $t$ , who are the young, and a part of the generation  $t-1$ , who are the old.

In the  $t + 1$  period when the generation  $t$  becomes old, the old who were high-income earners in their youth (period  $t$ ) get a higher asset income (details below). They spend all their asset income and pensions on consumption and contributions payments. All contributions are redistributed as a pension to the whole old generation.

The budget constraints of generation  $t$  of high-income workers can be expressed in the young (period  $t$ ) and the old (period  $t + 1$ ) as follows:

$$c_t^{yh} + s_t^h + \left( d - \frac{H_t^o}{H_t^y + L_t^y} d^o \right) = w_t^h, \quad (1)$$

$$c_{t+1}^{oh} + d^o = (1 + r_{t+1})s_t^h + \frac{(H_{t+1}^y + L_{t+1}^y) \left( d - \frac{H_{t+1}^o}{H_{t+1}^y + L_{t+1}^y} d^o \right) + H_{t+1}^o d^o}{H_{t+1}^o + L_{t+1}^o} \quad (2)$$

Here,  $c_t^{yh}$  in (1) is the consumption of the high-income young in the period  $t$ , and  $s_t^h$  is the savings of the high-income young in the period  $t$ .  $d$  and  $d^o$  are positive constants set by the government.  $d$  is the per capita contribution for young if only the young pay the contribution, and  $d^o$  is the per capita contribution by the old who had a high-income when young.  $H_t^o$  is the population of the old with a high asset income in period  $t$ ,  $H_t^y$  is the population of the high-income young in period  $t$ ,  $L_t^y$  is the population of the low-income young in period  $t$ , and  $w_t^h$  is the labor income (wage) of the young who receive a high income in period  $t$ . The third item on the left side of (1) is the per capita contribution of the whole young generation in period  $t$ . This is because the contribution for the old in period  $t$ ,  $H_t^o d^o$ , divided by the population  $H_t^y + L_t^y$  of all the young in the same period, is  $\frac{H_t^o}{H_t^y + L_t^y} d^o$ . The value obtained by subtracting this amount from  $d$  is the amount of contribution by the whole young generation in this model.

$c_{t+1}^{oh}$  in (2) is the consumption of old, high-income workers (type  $h$ ) in period  $t$ ,  $r_{t+1}$  is the interest rate in the period  $t + 1$ ,  $H_{t+1}^y$  is the high-income young population in period  $t + 1$ ,  $L_{t+1}^y$  is the low-income young population in the period  $t + 1$ , and  $H_{t+1}^o$  is the old population in period  $t + 1$  who had high incomes when they were young. The second item on the right-hand side of (2) expresses the amount of pension received by the high-income old. The numerator is the sum of  $(H_{t+1}^y + L_{t+1}^y) \left( d - \frac{H_{t+1}^o}{H_{t+1}^y + L_{t+1}^y} d^o \right)$ , which is the pension premium paid per capita by the young generation multiplied by the young population, and  $H_{t+1}^o d^o$  is the total premium paid by the high-income old in the period  $t + 1$ . We divide this contribution by the old population,  $H_{t+1}^o + L_{t+1}^o$  in the period  $t + 1$ , to get the amount of pension received per old person.

Next, we assume that the premiums paid by young low-income workers are the same as those paid by young high-income workers. Low-income old do not pay premiums. Therefore, the budget constraints of low-income people can be expressed in terms of the young and the old as shown in (3) and (4), respectively:

$$c_t^{yl} + s_t^l + \left( d - \frac{H_t^o}{H_t^y + L_t^y} d^o \right) = w_t^l \quad (3)$$

$$c_{t+1}^{ol} = (1 + r_{t+1})s_t^l + \frac{(H_{t+1}^y + L_{t+1}^y) \left( d - \frac{H_{t+1}^o}{H_{t+1}^y + L_{t+1}^y} d^o \right) + H_{t+1}^o d^o}{H_{t+1}^o + L_{t+1}^o} \quad (4)$$

Here,  $c_t^{yl}$  in (3) is the consumption of the low-income young in period  $t$ ,  $s_t^l$  is the savings of the low-income young in period  $t$ , and  $w_t^l$  is the labor income (wage) of the young who receive a low-income in period  $t$ . The insurance premium in the third item on the left-hand side of equation (3) is the same as in (1).  $c_{t+1}^{ol}$  in equation (4) is the consumption of the low-income old in the  $t + 1$  period. The amount of pension received in the second term on the right-hand side of equation (4) is the same as in equation (2).

In the following, we assume the constraints  $d - \frac{H_t^o}{H_t^y + L_t^y} d^o \leq \min(w_t^l, w_t^h)$  and  $d^o \leq r_{t+1} s_t^h$ , which means that pension premiums do not exceed income or asset income. In the subsequent analysis, the population of each generation is normalized to 1, and we assume that the population does not grow. Therefore, the total population of young and old people in each period is always 2. Furthermore, we assume that the population of low-income and high-income (and thus the population ratio) are also constant. That is,

$$H_t^y = H_{t+1}^y = H_t^o = H_{t+1}^o = H(\text{const.}), \quad L_t^y = L_{t+1}^y = L_t^o = L_{t+1}^o = L(\text{const.}).$$

We assume  $H < L$  in the following discussion. Setting the ratio of high-income people to the generational population as  $\frac{H}{H+L} = v$ , we find  $v < \frac{1}{2}$  and  $\frac{L}{H+L} = 1 - v > \frac{1}{2}$ .

From these assumptions, we can verify that the pension received by the old is  $d$ . Because  $H_{t+1}^y + L_{t+1}^y = H_{t+1}^o + L_{t+1}^o = 1$ , the amount of pension received by the old in the second term on the right-hand side of (2) or (4) is

$$\frac{(H_{t+1}^y + L_{t+1}^y) \left( d - \frac{H_{t+1}^o}{H_{t+1}^y + L_{t+1}^y} d^o \right) + H_{t+1}^o d^o}{H_{t+1}^o + L_{t+1}^o} = d.$$

Then, the budget constraints (1) to (4) can be rewritten as follows:

$$c_t^{yh} + s_t^h + (d - v d^o) = w_t^h, \quad (5)$$

$$c_{t+1}^{oh} + d^o = (1 + r_{t+1})s_t^h + d, \quad (6)$$

$$c_t^{yl} + s_t^l + (d - v d^o) = w_t^l, \quad (7)$$

$$c_{t+1}^{ol} = (1 + r_{t+1})s_t^l + d. \quad (8)$$

Households act to maximize the utility they get from their consumption under the following log-utility function:

$$\max_{c_t^{yh}, c_{t+1}^{oh}} U(c_t^{yh}) + \frac{1}{1+\rho} U(c_{t+1}^{oh}) \equiv \ln c_t^{yh} + \frac{1}{1+\rho} \ln c_{t+1}^{oh} \quad (9)$$

$$\max_{c_t^{yl}, c_{t+1}^{ol}} U(c_t^{yl}) + \frac{1}{1+\rho} U(c_{t+1}^{ol}) \equiv \ln c_t^{yl} + \frac{1}{1+\rho} \ln c_{t+1}^{ol} \quad (10)$$

(9) and (10) show objective functions of high-income people and low-income people, respectively.  $\frac{1}{1+\rho}$  is a discount factor, and we assume  $\rho > 0$ . Substituting (5) and (6) into (9), and (7) and (8) into (10) respectively, we can rewrite these expressions as follows:

$$\max_{s_t^h} \ln[w_t^h - s_t^h - (d - vd^o)] + \frac{1}{1+\rho} \ln[(1+r_{t+1})s_t^h + d - d^o] \quad (11)$$

$$\max_{s_t^l} \ln[w_t^l - s_t^l - (d - vd^o)] + \frac{1}{1+\rho} \ln[(1+r_{t+1})s_t^l + d]. \quad (12)$$

Deriving the first-order conditions of (11) and (12), and rewriting them for  $s_t^h$  and  $s_t^l$ , we obtain the optimum saving functions. First, we derive the saving function for high-income. The first-order condition of (11) is

$$\frac{-1}{w_t^h - s_t^h - (d - vd^o)} + \frac{1}{1+\rho} \frac{1+r_{t+1}}{(1+r_{t+1})s_t^h + d - d^o} = 0.$$

Solving for  $s_t^h$ , we obtain the following saving function for high-income:

$$s_t^h = \frac{w_t^h - (d - vd^o)}{2+\rho} - \frac{(1+\rho)(d - d^o)}{(2+\rho)(1+r_{t+1})}. \quad (13)$$

Similarly, we obtain the saving function of low-income as follows:

$$s_t^l = \frac{w_t^l - (d - vd^o)}{2+\rho} - \frac{(1+\rho)d}{(2+\rho)(1+r_{t+1})}. \quad (14)$$

### 3. Firm's behavior

In this section, we describe firms' behavior. The production function of a representative firm is the following Cobb-Douglas function:

$$Y_t = F(K_t, H, L) \equiv AK_t^\alpha H^\beta L^\gamma, \quad (15)$$

where  $K_t$  stands for the total capital stock of period  $t$ . In the following discussion, we assume  $\alpha + \beta + \gamma = 1$ ,  $0 < \alpha < 1$ ,  $0 < \beta < 1$ ,  $0 < \gamma < 1$  and  $\beta > \gamma$ .

Each firm faces a perfectly competitive market. The firm's profit maximization problem is described as follows:

$$\max_{K_t, H, L} F(K_t, H, L) - r_t K_t - w_t^h H - w_t^l L.$$

Then the firm's optimal conditions are given by

$$r_t = F_K(K_t, H, L) = \alpha A K_t^{\alpha-1} H^\beta L^\gamma \quad (16)$$

$$w_t^h = F_H(K_t, H, L) = \beta A K_t^\alpha H^{\beta-1} L^\gamma \quad (17)$$

$$w_t^l = F_L(K_t, H, L) = \gamma A K_t^\alpha H^\beta L^{\gamma-1} \quad (18)$$

We assume that the production function satisfies the linear homogeneity. Then the following exhaustion theorem is satisfied:

$$Y_t = r_t K_t + w_t^h H + w_t^l L. \quad (19)$$

From (17) and (18), we have

$$\frac{w_t^h}{w_t^l} = \frac{\beta A K_t^\alpha H^{\beta-1} L^\gamma}{\gamma A K_t^\alpha H^\beta L^{\gamma-1}} = \frac{\beta}{\gamma} \frac{L}{H} > 1.$$

The inequality can be shown using  $\frac{\beta}{\gamma} > 1$  from the assumption of  $\beta > \gamma$  and  $\frac{L}{H} > 1$  from the assumption of  $H < L$ . Then we can verify  $w_t^h > w_t^l$ . Also, regarding the second term of each of the savings functions (13) and (14),  $\frac{(1+\rho)(d-d^0)}{(2+\rho)(1+r_{t+1})} < \frac{(1+\rho)d}{(2+\rho)(1+r_{t+1})}$  holds, and regarding the first term  $\frac{w_t^h - (d-vd^0)}{2+\rho} > \frac{w_t^l - (d-vd^0)}{2+\rho}$  holds from  $w_t^h > w_t^l$ . Therefore, we obtain  $s_t^h > s_t^l$ .

Finally, from the goods market and the fund market equilibrium conditions, we have<sup>2</sup>

$$K_{t+1} = s_t^h H + s_t^l L. \quad (20)$$

#### 4. Dynamics and steady-state of capital stock

In this section, we derive the dynamic of capital stock using the fund market equilibrium condition (20) and the savings functions (13) and (14). First, dividing (20) by the generational population ( $1 = H + L$ ) and setting  $\frac{K_{t+1}}{H+L} = k_{t+1}$  gives;

$$k_{t+1} = s_t^h v + s_t^l (1 - v). \quad (20)'$$

Substituting the savings functions (13) and (14) for this equation, we have

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<sup>2</sup> See Appendix for the derivation of (20).

$$k_{t+1} = \left[ \frac{w_t^h - d + vd^o}{2+\rho} - \frac{(1+\rho)(d-d^o)}{(2+\rho)(1+r_{t+1})} \right] v + \left[ \frac{w_t^l - (d - vd^o)}{2+\rho} - \frac{(1+\rho)d}{(2+\rho)(1+r_{t+1})} \right] (1-v).$$

Using the profit maximization conditions (16), (17) and (18), we have

$$\begin{aligned} k_{t+1} &= \left[ \frac{\beta Av^{\beta-1}(1-v)^\gamma k_t^\alpha - (d - vd^o)}{2+\rho} - \frac{(1+\rho)(d-d^o)}{(2+\rho)(1+\alpha Av^\beta(1-v)^\gamma k_{t+1}^{\alpha-1})} \right] v \\ &+ \left[ \frac{\gamma Av^\beta(1-v)^{\gamma-1} k_t^\alpha - (d - vd^o)}{2+\rho} - \frac{(1+\rho)d}{(2+\rho)(1+\alpha Av^\beta(1-v)^\gamma k_{t+1}^{\alpha-1})} \right] (1-v) \\ &= \frac{\beta Av^\beta(1-v)^\gamma k_t^\alpha - (d - vd^o)v}{2+\rho} - \frac{(1+\rho)(d-d^o)v}{(2+\rho)(1+\alpha Av^\beta(1-v)^\gamma k_{t+1}^{\alpha-1})} \\ &+ \frac{\gamma Av^\beta(1-v)^\gamma k_t^\alpha - (d - vd^o)(1-v)}{2+\rho} - \frac{(1-v)(1+\rho)d}{(2+\rho)(1+\alpha Av^\beta(1-v)^\gamma k_{t+1}^{\alpha-1})} \end{aligned}$$

Dividing both sides by  $2+\rho$ , we have

$$\begin{aligned} (2+\rho)k_{t+1} &= \beta Av^\beta(1-v)^\gamma k_t^\alpha - (d - vd^o)v - \frac{(1+\rho)(d-d^o)v}{1+\alpha Av^\beta(1-v)^\gamma k_{t+1}^{\alpha-1}} \\ &+ \gamma Av^\beta(1-v)^\gamma k_t^\alpha - (d - vd^o)(1-v) - \frac{(1-v)(1+\rho)d}{1+\alpha Av^\beta(1-v)^\gamma k_{t+1}^{\alpha-1}} \end{aligned}$$

Then, we obtain the dynamics of capital stock.

$$d - vd^o + \frac{(1+\rho)(d-vd^o)}{1+\alpha Av^\beta(1-v)^\gamma k_{t+1}^{\alpha-1}} + (2+\rho)k_{t+1} = (\beta + \gamma)Av^\beta(1-v)^\gamma k_t^\alpha. \quad (21)$$

The left-hand side of (21) is represented by  $\Phi(k_{t+1})$  as a function of  $k_{t+1}$ , and the right-hand side is represented by  $\Psi(k_t)$  as a function of  $k_t$ . The dynamic equation (21) can be rewritten as the following:

$$\Phi(k_{t+1}) = \Psi(k_t).$$

The function  $\Psi(k_t)$  is a concave that passes through the origin and increases monotonically (Figure 2).

The expression  $d - vd^o$ , in the function  $\Phi(k_{t+1})$ , is the intercept, and  $(2+\rho)k_{t+1}$  is represented by a straight line with gradient  $2+\rho$ . To verify the graph shape of the fractional part  $\frac{(1+\rho)(d-vd^o)}{1+\alpha Av^\beta(1-v)^\gamma k_{t+1}^{\alpha-1}}$ , we calculate the first-order and the second-order

derivatives:

$$\frac{d}{dk_{t+1}} \frac{(1+\rho)(d-vd^o)}{1+\alpha Av^\beta(1-v)^\gamma k_{t+1}^{\alpha-1}} = \frac{-(1+\rho)(d-vd^o)(\alpha-1)\alpha Av^\beta(1-v)^\gamma k_{t+1}^{\alpha-2}}{(1+\alpha Av^\beta(1-v)^\gamma k_{t+1}^{\alpha-1})^2} > 0$$

$$\frac{d^2}{dk_{t+1}^2} \frac{(1+\rho)(d-vd^o)}{1+\alpha Av^\beta(1-v)^\gamma k_{t+1}^{\alpha-1}} = \frac{\omega(k_{t+1})}{(1+\alpha Av^\beta(1-v)^\gamma k_{t+1}^{1+\alpha})^4}$$

where

$$\begin{aligned}\Omega(k_{t+1}) = & -(\alpha - 2)(1 + \rho)(d - vd^o)(\alpha - 1)\alpha Av^\beta(1 - v)^\gamma k_{t+1}^{\alpha-3} (1 + \alpha Av^\beta(1 - v)^\gamma k_{t+1}^{\alpha-1})^2 \\ & - \{-(1 + \rho)(d - vd^o)(\alpha - 1)\alpha Av^\beta(1 - v)^\gamma k_{t+1}^{\alpha-1}\} \\ & \times \{2(1 + \alpha Av^\beta(1 - v)^\gamma k_{t+1}^{\alpha-1})(\alpha - 1)\alpha Av^\beta(1 - v)^\gamma k_{t+1}^{\alpha-2}\}.\end{aligned}$$

The sign of the first-order derivative of the fractional part is positive. However, it is difficult to determine the sign of the second-order derivative, so the shape of the graph is not confirmed generally. Nevertheless, we can expect that the graph of  $\Phi(k_{t+1})$  will be as shown in Figure 2 by the simulation analysis described in the next section.

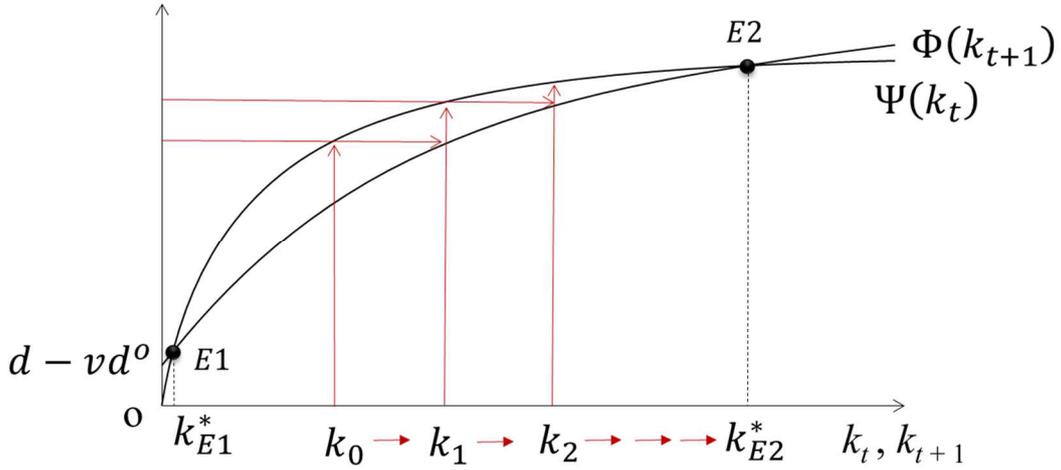


Figure 2: The steady-states and transitional process

Figure 2 shows the steady-states and transitional process of this model. The steady-states correspond to the intersections of  $\Phi(k_{t+1})$  and  $\Psi(k_t)$ . Figure 2 depicts a case where two steady-states (E1, E2) exist. The capital stocks corresponding to each steady state are  $k_{E1}^*$  and  $k_{E2}^*$ . We can verify that  $k_{E1}^*$  is an unstable steady-state and  $k_{E2}^*$  is a stable steady-state. The former is referred to as a lower steady-state equilibrium, and the latter is referred to as a higher steady-state equilibrium.

Next, let us analyze the effect of the government manipulating  $d^o$ , which is the premium paid by the high-income old, on the higher steady-state equilibrium. Differentiating the function  $\Phi$  by  $d^o$ , we can verify the sign is negative.

$$\frac{d\Phi}{dd^o} = -v + \frac{-(1 + \rho)v}{1 + \alpha Av^\beta(1 - v)^\gamma k_{t+1}^{\alpha-1}} < 0,$$

which means that the graph of  $\Phi$  shifts downward as  $d^o$  increases (Figure 3).

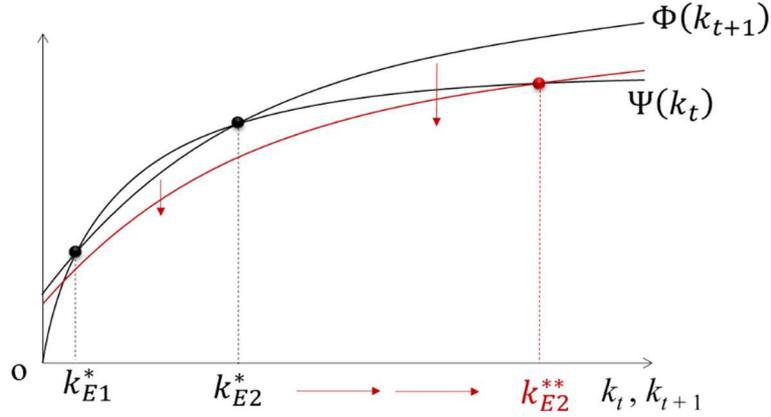


Figure 3: Changes in higher steady state equilibrium when  $d^o$  increases

From Figure 3, increasing  $d^o$  shifts the graph of  $\Phi(k_{t+1})$  downward, the stable steady-state  $k_{E2}^*$  shifts to the right, and the steady-state capital stock increases. Conversely, when the premiums for the high-income old decrease, the capital stock decreases.

In addition to analyzing the model which imposes insurance premiums on the high-income old, we also investigated the case where only the young generation pays all insurance premiums. This situation corresponds to the special case of  $d^o = 0$ . When there is a higher steady-state equilibrium in the case of  $d^o = 0$ , it is easy to verify that the level of capital stock is smaller than in the case of  $d^o > 0$ .

Why does an increase in premiums  $d^o$  for high-income old increase higher steady-state capital stock levels? The reason is explained as follows. As high-income old pay insurance premiums, the insurance contribution  $d - vd^o$  for the young decreases, which increases the disposable income of the young and increases the savings level from (13) and (14) (ie,  $\frac{ds_t^h}{dd^o} > 0, \frac{ds_t^l}{dd^o} > 0$ ). The increase in savings leads to an increase in capital stock through the fund market equilibrium formula (20).

From the above discussion, we obtain the following proposition.

**【Proposition】** By paying the premiums for pay-as-you-go pensions for the high-income old as well as the young generation, the stable steady-state capital stock level increases compared to the conventional case.

## 5. Numerical analysis

In this section, we examine numerical simulation analysis on points that could not be clarified by the theoretical research so far. That is, is there a parameter set in which a higher equilibrium and a lower equilibrium co-exist? In particular, this section confirms the existence, and changes, of the higher and lower equilibria in relation to the premium  $d^o$  for the high-income old. In addition, simulation analysis will be performed on the lifetime utility of high-income and low-income in the higher steady-state equilibrium.

First, we set the values of the exogenous parameters as follows.

$$\alpha=0.4, \quad \beta=0.4, \quad \gamma=0.2, \quad \rho=0.9, \quad A=10, \quad v=0.01, \quad d=0.2.$$

Concerning the value of  $\alpha$ , we specify  $\alpha = 0.4$ . This value means  $\beta + \gamma = 0.6$ , which is consistent with most empirical estimates of the labor income share.<sup>3</sup>

Under these exogenous values, let us examine how endogenous variables change when the value of  $d^o$  is increased. The following figures show the results.

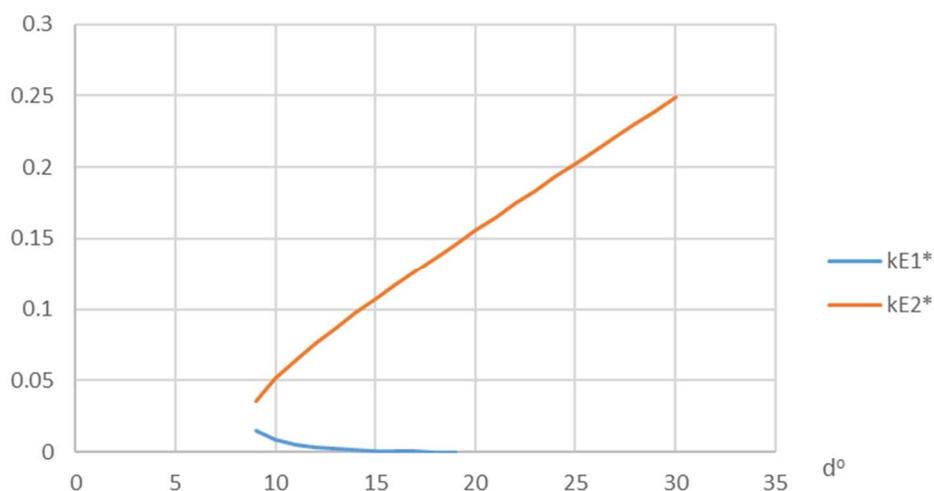


Figure 4: Effect of an increase in  $d^o$  on  $k_{E1}^*$  and  $k_{E2}^*$

In Figure 4, the relationship between  $d^o$  and the steady-state capital stock is depicted in the range of  $9 \leq d^o \leq 30$ . Here, we focus on  $k_{E2}^*$ , which corresponds to a higher steady-state equilibrium. As discussed in the theoretical analysis, this simulation revealed that increasing  $d^o$  also increases  $k_{E2}^*$ . Conversely, the lower steady-state equilibrium  $k_{E1}^*$  decreases and disappears in the range of  $20 < d^o$ .

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<sup>3</sup> Karabarbounis and Neiman (2014) show that labor shares in major developed countries such as the United States, Japan and Germany range from 0.57 to 0.65.

From (17) and (18), the wages of high-income and low-income also increase as the capital stock of the higher equilibrium increases. However, as we have already confirmed,  $\frac{w_t^h}{w_t^l}$  is constant in this model, so the wage gap does not change.

Next, Figures 5 and 6 show the relationship between changes in  $d^o$  and lifetime utilities of high-income and low-income for the higher equilibrium.

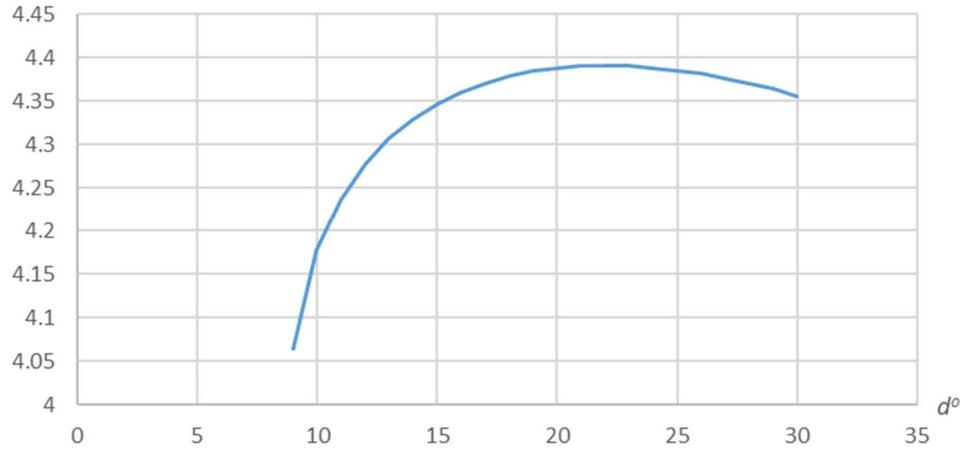


Figure 5: Effect of an increase in  $d^o$  on the lifetime utility of high-income

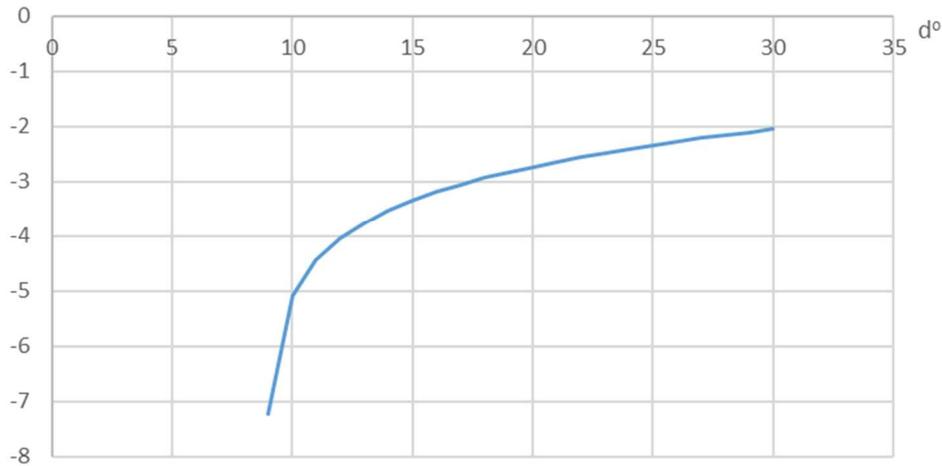


Figure 6: Effect of an increase in  $d^o$  on the lifetime utility of low-income

From Figure 5, there is an optimal insurance premium for the high-income old, in which the lifetime utility of the high-income maximizes, and this value is  $d^o = 45$ . On the other hand, we can confirm from Figure 6 that the lifetime utility of low-income continues to increase as  $d^o$  increases. It is easy to see that the lifetime utility increases

because the low-income population receives income transfers from the high-income old. But what is the reason for the increase in lifetime utility even for the high-income old? We can explain as follows.

	Type $h$	Type $l$
Young	$c^{yh}$	$c^{yl}$
Old	$c^{oh}$	$c^{ol}$

Figure 7: Consumption of 4 types of people in each period

In this model, there are young and old generations in each period, in addition, they can be divided into type  $h$  (high-income) and type  $l$  (low-income). Therefore, there are always four types of people in each period. Figure 7 shows the consumption of these four types of people. First, type  $l$  people receive more income transfers due to the increase in  $d^o$ , so consumption increases for both young and old. Next, for type  $h$  people, when  $d^o$  is increased, income is transferred from the old to the young, and the consumption of the young increases. However, the consumption of type  $h$  old will decrease. Until  $d^o = 45$ , the contribution of the increase in lifetime utility accompanying the increase in consumption of the young outweighs the contribution of the decrease in income of the old, and after that, the decrease in consumption of the old generation will outweigh the increase in consumption of the young. Therefore, the lifetime utility of high-income is convex upward with  $d^o = 45$  at the vertex.

The situation where  $d^o = d^*$ , such that the lifetime utility of the high-income population is maximized, is Pareto efficient from a social point of view. Let us consider the case where the level of  $d^o$  is gradually increased from a level smaller than  $d^*$ . Since the lifetime utilities of the low-income and high-income populations increase, all types agree with increasing  $d^o$ . But where  $d^o > d^*$ , the lifetime utility of the high-income population decreases, so type  $h$  opposes increases in  $d^o$ . Therefore, the Pareto efficient case for all types is  $d^o = d^*$ .

## 5. Concluding remarks

We introduced a pay-as-you-go pension system in which the high-income old, in addition to the young generation, also pay pension premiums in a two-generation overlapping model in which high-income and low-income households exist. As a result, we found that the stable steady-state capital stock level increases with the rich old generation's contribution to the pension system. Furthermore, using numerical simulations, we showed that there is a Pareto efficient premium level of rich old which maximizes the lifetime utility of high-income households.

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## Appendix

In this appendix, we derive (20). In the period  $t$ , the generation  $t$  of the young and the generation  $t-1$  of the old coexist. Each individual of each generation consumes goods. In order to distinguish between the old period ( $o$ ) and the young period ( $y$ ), populations  $H$  and  $L$  are subscripted accordingly. Then the goods market equilibrium is described by

$$Y_t = c_t^{yh} H^y + c_t^{oh} H^o + c_t^{yl} L^y + c_t^{ol} L^o + I_t,$$

where  $I_t$  stands for total investment.

Assuming that the capital depreciation rate is 0,  $I_t = K_{t+1} - K_t$ , so the equation above can be rewritten as follows:

$$Y_t = c_t^{yh} H^y + c_t^{oh} H^o + c_t^{yl} L^y + c_t^{ol} L^o + (K_{t+1} - K_t) \quad (\text{A.1})$$

Combining (19) and (A.1), we have

$$\begin{aligned} r_t K_t + w_t^h H^y + w_t^l L^y \\ = c_t^{yh} H^y + c_t^{oh} H^o + c_t^{yl} L^y + c_t^{ol} L^o + (K_{t+1} - K_t) \end{aligned} \quad (\text{A.2})$$

Solving for  $K_{t+1}$ , we have

$$K_{t+1} = (w_t^h - c_t^{yh}) H^y + (w_t^l - c_t^{yl}) L^y + (1 + r_t) K_t - c_t^{oh} H^o - c_t^{ol} L^o \quad (\text{A.3})$$

Substituting individual budget constraint equations (5), (6), (7) and (8) into (A.3), we obtain the following equation.

$$\begin{aligned} K_{t+1} = s_t^h H^y + s_t^l L^y + (1 + r_t) K_t - (1 + r_t) s_{t-1}^h H^o - (1 + r_t) s_{t-1}^l L^o \\ + (d - vd^o) H^y + (d - vd^o) L^y - (d - d^o) H^o - d L^o \end{aligned} \quad (\text{A.4})$$

Note that  $(d - vd^o) H^y + (d - vd^o) L^y - (d - d^o) H^o - d L^o$  on the right-hand side of (A.4) equals zero. Because of the balance of the pay-as-you-go pensions, payment of insurance premiums equals receipt of pensions, which can be expressed by  $(d - vd^o) H^y + (d - vd^o) L^y + H^o d^o = H^o d + L^o d$ .

Using this condition, we can rewrite (A.4) to

$$K_{t+1} = s_t^h H^y + s_t^l L^y + (1 + r_t) K_t - (1 + r_t) s_{t-1}^h H^o - (1 + r_t) s_{t-1}^l L^o \quad (\text{A.5})$$

Next, we substitute budget constraints (5) and (7) into (A.3), and set  $t = 0$ . Then we have

$$\begin{aligned} K_1 &= [s_0^h + (d - vd^o)] H^y + [s_0^l + (d - vd^o)] L^y + (1 + r_0) K_0 - c_0^{oh} H^o - c_0^{ol} L^o \\ &= s_0^h H^y + s_0^l L^y + (1 + r_0) K_0 + (d - vd^o) H^y + (d - vd^o) L^y - c_0^{oh} H^o - c_0^{ol} L^o \\ &= s_0^h H^y + s_0^l L^y + (1 + r_0) K_0 - c_0^{oh} H^o - c_0^{ol} L^o + H^y d - v H^y d^o + L^y d - v L^y d^o \end{aligned}$$

Using  $v = \frac{H^o}{H^y + L^y}$  and  $H^y + L^y = 1$ , we have

$$K_1 = s_0^h H^y + s_0^l L^y + (1 + r_0) K_0 - c_0^{oh} H^o - c_0^{ol} L^o + d - H^o d^o \quad (\text{A.6})$$

The first old whose period 0 have assets of  $K_0$  as a whole and gain their asset income  $r_0 K_0$ , and those who have high asset income pay premium  $d^o H^o$ . The total amount of

the balance plus the pension  $d(H^0 + L^0)$  received by all the old in the period 0 is exhausted for consumption. Then we obtain the following budget constraint:

$$(1 + r_0)K_0 + d(H^0 + L^0) - d^o H^o = c_0^{oh} H^o + c_0^{ol} L^o$$

Note that  $H^0 + L^0 = 1$  and substitute this equation into the right-hand side of (A.6) to get

$$K_1 = s_0^h H^y + s_0^l L^y.$$

Similarly, we obtain the budget constraint for  $t = 1$  as follows:

$$K_2 = s_1^h H^y + s_1^l L^y.$$

Repeating this discussion becomes

$$K_t = s_{t-1}^h H^y + s_{t-1}^l L^y.$$

Then we have the fund market equilibrium condition:

$$K_{t+1} = s_t^h H + s_t^l L. \tag{20}$$