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"Globalization and Volatility under Alternative Trade Structures"

Yunfang Hu, Kazuo Mino

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Globalization and Volatility under Alternative Trade Structures*

Yunfang Hu† and Kazuo Mino‡

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Abstract

This paper explores a dynamic two-country model with production externalities in which capital goods are not traded and international lending and borrowing are allowed. Unlike the integrated world economy model based on the Heckscher-Ohlin setting, our model yields indeterminacy of equilibrium under a wider set of parameter values than in the corresponding closed economy model. Our finding demonstrates that the assumption on trade structure would be a relevant determinant in considering the relation between globalization and economic volatility.

Keywords: two-country model, non-traded goods, equilibrium indeterminacy, social constant returns

JEL classification: F43, O41

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†Graduate School of International Cultural Studies, Tohoku University, 41 Kawauchi, Aoba-ku, Sendai, 980-8576 Japan

‡*Corresponding Author:* Institute of Economic Research, Kyoto University, Yoshida Honmachi, Sakyo-ku, Kyoto, 606-8051 Japan. E-mail: mino@kier.kyoto-u.ac.jp

1 Introduction

Does globalization enhance economic volatility? The equilibrium business cycle theory based on indeterminacy and sunspots has presented two different answers to this question. On the one hand, Meng (2003), Meng and Velasco (2003 and 2004) and Weder (2001) show that small-open economies with production externalities yield indeterminacy of equilibrium under a wider set of parameter values than in the corresponding closed economy model. Hence, according to these studies, globalization of an economy may increase economic volatility.[1] Nishimura and Shimomura (2002), on the other hand, examine a dynamic Hecksher-Ohlim model of the two-country world in which there are country-specific production externalities. They show that the world economy has the same conditions for equilibrium (in)determinacy as those for a closed economy counterpart. In addition, Sim and Ho (2007) find that if one of the two counties has no production externalities in Nishimura and Shimomura's model, then the equilibrium path of the world economy would be determinate even though the country with production externalities exhibits autarkic indeterminacy. These studies indicate that globalization does not necessarily enhance economic fluctuations.

At the first sight, the opposite results shown above seem to stem from the difference in the analytical frameworks used by the foregoing studies. The small-open economy models are based on the partial equilibrium analysis in which behavior of the rest of the world is exogenously given. In contrast, the models of world economy employ the general equilibrium approach that treats the world economic system as a closed economy consisting of multiple countries. Therefore, the behavior of an integrated world economy would be close to the behavior of closed economy counterpart. One may conjecture that such a difference would generate the contrasting views as to the destabilizing effect of globalization.

The purpose of this paper is to reveal that the difference in conclusions mentioned above mainly comes from the assumptions on trade structures rather than from the modelling strategies. To confirm this, we re-examine the world economy model under alternative trade structures. In particular, we focus on the case where capital goods are not internationally

[1] Lahiri (2001) also examines indeterminacy in a small-open economy model. Since he uses a framework different from the one used by Meng (2003) and others, his model needs a relatively high degree of external increasing returns to yield indeterminacy. Yong and Meng (2004) and Zhang (2008) also discuss equilibrium indeterminacy in small-open economies.

traded but there are international lending and borrowing. Our main finding is that the equilibrium indeterminacy conditions for the world economy with non-traded investment goods and financial transactions are similar to the stability conditions for the small-open economy models that have the same trade structure. More specifically, we show that our model may exhibit indeterminacy regardless of the restrictions on the preference structure. The closed-economy version of our model, which is essentially the same as the world economy model examined by Nishimura and Shimomura (2002) needs a high elasticity of intertemporal substitution in consumption to hold indeterminacy. We also show that if investment goods are tradable but consumption goods are not traded, then the dynamic behavior of the world economy is essential the same as that of the Heckscher-Ohlin framework. In this sense, the structure of international trade is a relevant determinant for the relation between globalization and volatility.

2 The Model

Consider a world economy consisting of two countries, home and foreign. Both countries have the same production technologies. In each country there is a continuum of identical, infinitely-lived households. All the agents in both countries have an identical time discount rate and the same form of instantaneous felicity function. The only difference between the two countries is the initial stock of wealth held by the households in each country.

2.1 Production

The home country has two production sectors. The first sector ($i = 1$) produces investment goods and the second sector ($i = 2$) produces pure consumption goods. The production function of i-th sector is specified as

$$Y_i = A_i K_i^{a_i} L_i^{b_i} \bar{X}_i, \qquad a_i > 0,\ b_i > 0,\ \ 0 < a_i + b_i < 1, \quad i = 1, 2$$

where Y_i, K_i and L_i are i-th sector's output, capital and labor input, respectively. Here, $\bar{X}_i$ denotes the sector and country-specific production externalities. We define:

$$\bar{X}_i = \bar{K}_i^{\alpha_i - a_i} \bar{L}_i^{1-\alpha_i-b_i}, \quad a_i < \alpha_i < 1, \quad \alpha_i + b_i < 1 \quad i = 1, 2.$$

If we normalizes the number of producers to one, then it holds that $\bar{K}_i = K_i$ and $\bar{L}_i = L_i$ $(i = 1, 2)$ in equilibrium. This means that the i-th sector's social production technology that internalizes the external effects is:

$$Y_i = A_i K_i^{\alpha_i} L_i^{1-\alpha_i}, \quad i = 1, 2. \tag{1}$$

Hence, the social technology satisfies constant returns to scale, while the private technology exhibits decreasing returns to scale.[2]

The factor and product markets are competitive, so that the private marginal product of each production factor equals its real factor price. These conditions are given by the following:

$$r = pa_1 \frac{Y_1}{K_1} = a_2 \frac{Y_2}{K_2}, \tag{2a}$$

$$w = pb_1 \frac{Y_1}{L_1} = b_2 \frac{Y_2}{L_2}, \tag{2b}$$

where w is the real wage rate, r is the rental rate of capital and p denotes the price of investment good in terms of the consumption good.

The production technologies of the foreign country are the same as those of the home country. The factor prices in the foreing country thus satisfy:

$$r^* = p^* a_1 \frac{Y_1^*}{K_1^*} = a_2 \frac{Y_2^*}{K_2^*}, \tag{3a}$$

$$w^* = p^* b_1 \frac{Y_1^*}{L_1^*} = b_2 \frac{Y_2^*}{L_2^*}, \tag{3b}$$

where variables with an asterisk denote foreign variables.

2.2 Households

We assume that the households in the home country access the international financial market where foreign bonds are freely traded. By trading bonds, the households in the home country can borrow from or lend to the foreign households. The representative household in the home country maximizes

$$U = \int_0^\infty \frac{C^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t} dt, \quad \sigma > 0, \quad \rho > 0$$

[2] This specification of production technology was first introduced by Behbhabib and Nishimura (1998). Benhabib et al. (2000), Meng (2003), Meng and Velasco (2003, 2004), Mino (2001) and Nishimura and Shimomura (2002) utilize the same functional forms.

subject to the flow budget constraint

$$\dot{\Omega} = R\Omega + w + \pi_1 + \pi_2 - C, \tag{4}$$

together with the no-Ponzi-game condition

$$\lim_{t\to\infty} \exp\left(-\int_0^t R_s ds\right)\Omega_t \geq 0$$

and the initial value of Ω_0. In the above, C is consumption, R denotes interest rate, π_i is the excess profits in the i-th sector[3] and Ω is the net wealth (in terms of the consumption goods). The net wealth of held by the household consists of domestic capital and foreign bonds:

$$\Omega = pK + B,$$

where B denotes the stock of foreign bonds (in terms of the consumption goods). When selecting its optimal consumption plan, the household take the sequences of $\{R_t, w_t,\ \pi_{1,t}, \pi_{2,t}, p_t\}_{t=0}^{\infty}$ as given.

The definition of net wealth yields $\dot{\Omega} = p\dot{K} + \dot{p}K + \dot{B}$. Thus, the flow budget constraint (4) can be rewritten as

$$\dot{B} = RB + \left(R - \frac{\dot{p}}{p}\right)pK + w + \pi_1 + \pi_2 - C - p\dot{K}.$$

The non-arbitrage condition between holding capital and bond means that the net rate of return to capital equals the real interest on bonds:

$$\frac{r}{p} - \delta = R - \frac{\dot{p}}{p}, \tag{5}$$

where $\delta \in [0, 1)$ denotes the rate of capital depreciation. As a consequence, the optimization problem for the representative household in the home country is to maximize U by controlling C and I subject to the following constraints:

$$\dot{B} = RB + rK + w + \pi_1 + \pi_2 - C - pI, \tag{6}$$

$$\dot{K} = I - \delta K, \tag{7}$$

[3]Remember that the private technology exhinits decreasing returns to scale with respect to capital and labor.

together with the initial holdings of K_0 and B_0. In this reformulation, the no-Ponzi-game condition is given by

$$\lim_{t\to\infty} \exp\left(-\int_0^t R_s ds\right) B_t \geq 0. \tag{8}$$

Set up the Hamiltonian function for the optimization problem:

$$H = \frac{C^{1-\sigma}-1}{1-\sigma} + \lambda\left[RB + rK + w + \pi_1 + \pi_2 - C - pI\right] + q\left(I - \delta K\right),$$

where λ and q respectively denote the implicit price of the foreign bonds and domestic capital. Focusing on an interior solution, we see that the necessary conditions for an optimum are:

$$C^{-\sigma} = \lambda \tag{9a}$$

$$p\lambda = q, \tag{9b}$$

$$\dot{\lambda} = \lambda\left(\rho - R\right), \tag{9c}$$

$$\dot{q} = q\left(\rho + \delta\right) - \lambda r = q\left(\rho + \delta - \frac{r}{p}\right). \tag{9d}$$

The optimization conditions also involve the transverslity conditions on holding B and K: $\lim_{t\to\infty} \lambda e^{-\rho t} B = 0$ and $\lim_{t\to\infty} q e^{-\rho t} K = 0$.

Since the foreign households have the same preference structure, their optimization conditions corresponding to $(9a)$, $(9b)$, $(9c)$ and $(9d)$ are as follows:

$$C^{*-\sigma} = \lambda^*, \tag{10a}$$

$$p^*\lambda^* = q^*, \tag{10b}$$

$$\dot{\lambda}^* = \lambda^*\left(\rho - R\right), \tag{10c}$$

$$\dot{q}^* = q^*\left(\rho + \delta - \frac{r^*}{p^*}\right). \tag{10d}$$

It is to be noted that while the interest rate, R, is common for both countries, the real rate of return to capital in the foreign country, r^*/p^*, may differ from r/p, because in our framework the factor-price equalization fails to hold out of the steady state.

2.3 Market Equilibrium Conditions

We assume that consumption goods are internationally traded but investment goods are non-tradables.[4] Although such an assumption is restrictive one, it helps to elucidate the role of trade structure in a dynamic world economy. Moreover, a large portion of investment goods are construction and structures, so that the investment goods sector shares a larger part of non-tradables than the consumption good sector.[5] Since investment goods are traded in the domestic markets alone and consumption goods are internationally traded, the market equilibrium conditions for investment and consumption goods are respectively given by

$$Y_1 = I, \qquad Y_1^* = I^*, \tag{11}$$

$$Y_2 + Y_2^* = C + C^*, \tag{12}$$

where I and I^* are gross investment expenditures in the home and foreign countries, respectively. Physical capital in each country accumulates according to

$$\dot{K} = I - \delta K, \qquad \dot{K}^* = I^* - \delta K^*. \tag{13}$$

As for the factor markets, we follow the standard Heckscher-Ohlin modelling: it is assumed that capital and labor are perfectly shiftable between the production sectors within a country, but they cannot move across the borders. Therefore, the full-employment conditions for production factors in each country are the following:

$$K = K_1 + K_2, \qquad 1 = L_1 + L_2, \tag{14a}$$

$$K^* = K_1^* + K_2^*, \qquad 1 = L_1^* + L_2^*. \tag{14b}$$

We assume that labor supply in each country is fixed and normalized to one.

Finally, the equilibrium condition for the bond market is

$$B + B^* = 0,$$

[4] The structure of our model is one of the dependent economy models discussed in open-economy macroeconomics literature. Turnovsky and Sen (1995) treat a small-open economy model with non-tradable capital and Turnovsky (1997, Chapter 7) studies a neoclassical two-country, two-sector model in which capital goods are not traded. Mino (2008) also discusses the similar two-country model with external increasing returns. See also Chapter 5 in Turnousky (2009) for a brief literature review.

[5] Bems (2008) finds that the share of investment expenditure on non-traded goods is about 60% and that this figure has been considerably stable over the last 50 years both in developed and developing countries.

which means that $\Omega+\Omega^* = K+K^*$. Bonds are IOUs between the home and foreign households and, hence, the aggregate stock of bonds is zero in the world financial market at large.

3 Volatility of the World Economy

3.1 Dynamic System

In equilibrium it holds that $\bar{K}_i = K_i$, $\bar{L}_i = L_i$, $\bar{K}_i^* = K_i^*$ and $\bar{L}_i^* = L_i$ $(i = 1, 2)$. From $(2a)$, $(2b)$, $(3a)$ and $(3b)$ the factor prices in each country satisfy the following:

$$r = pa_1 A_1 k_1^{\alpha_1 - 1} = a_2 A_2 k_2^{\alpha_2 - 1}, \tag{15a}$$

$$w = pb_1 A_1 k_1^{\alpha_1} = b_2 A_2 k_2^{\alpha_2}, \tag{15b}$$

$$r^* = p^* a_1 A_1 k_1^{*\alpha_1 - 1} = a_2 A_2 k_2^{*\alpha_2 - 1}, \tag{15c}$$

$$w^* = p^* b_1 A_1 k_1^{*\alpha_1} = b_2 A_2 k_2^{*\alpha_2}, \tag{15d}$$

where $k_i = K_i/L_i$ and $k_i^* = K_i^*/L_i^*$ $(i = 1, 2)$. By use of $(15a)$, $(15b)$, $(15c)$ and $(15d)$, we can express the optimal factor intensity in each production sector as a function of relative price:

$$k_1 = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2-\alpha_1}} \left(\frac{a_1}{a_2}\right)^{\frac{\alpha_2}{\alpha_2-\alpha_1}} \left(\frac{b_1}{b_2}\right)^{\frac{\alpha_2-1}{\alpha_1-\alpha_2}} p^{\frac{1}{\alpha_2-\alpha_1}} \equiv k_1(p),$$

$$k_1^* = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2-\alpha_1}} \left(\frac{a_1}{a_2}\right)^{\frac{\alpha_2}{\alpha_2-\alpha_1}} \left(\frac{b_1}{b_2}\right)^{\frac{\alpha_2-1}{\alpha_1-\alpha_2}} p^{*\frac{1}{\alpha_2-\alpha_1}} \equiv k_1(p^*),$$

$$k_2 = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2-\alpha_1}} \left(\frac{a_1}{a_2}\right)^{\frac{\alpha_1}{\alpha_2-\alpha_1}} \left(\frac{b_1}{b_2}\right)^{\frac{\alpha_1-1}{\alpha_1-\alpha_2}} p^{\frac{1}{\alpha_2-\alpha_1}} \equiv k_2(p),$$

$$k_2^* = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2-\alpha_1}} \left(\frac{a_1}{a_2}\right)^{\frac{\alpha_1}{\alpha_2-\alpha_1}} \left(\frac{b_1}{b_2}\right)^{\frac{\alpha_1-1}{\alpha_1-\alpha_2}} p^{*\frac{1}{\alpha_2-\alpha_1}} \equiv k_2(p^*).$$

These expressions show that

$$\text{sign}\,[k_1(p) - k_2(p)] = \text{sign}\,[k_1(p^*) - k_2(p^*)] = \text{sign}\left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right), \tag{17}$$

$$\text{sign}\,k_i'(p) = \text{sign}\,k_i'(p^*) = \text{sign}\,(\alpha_2 - \alpha_1), \quad i = 1, 2. \tag{18}$$

In the above, the sign of $a_1/b_1 - a_2/b_2$ represents the factor intensity ranking from the private perspective, while sign $(\alpha_1 - \alpha_2)$ expresses the factor intensity ranking from the social perspective.

In this paper we restrict our attention to the interior equilibrium in which both countries imperfectly specialize. To ensure this restriction, we assume that relative price in each country satisfies the following condition:

$$L_1 = \frac{K - k_2(p)}{k_1(p) - k_2(p)} \in (0,1), \qquad L_1^* = \frac{K^* - k_2(p^*)}{k_1(p^*) - k_2(p^*)} \in (0,1). \tag{19}$$

Using functions $k_1(p)$ and $k_2(p)$, we see that capital accumulation equation in each country is written as

$$\dot{K} = y^1(K,p) - \delta K, \tag{20a}$$

$$\dot{K}^* = y^1(K^*,p^*) - \delta K^*, \tag{20b}$$

where $y^1(K,p)$ and $y^1(K^*,p^*)$ express the supply functions of investment goods given by

$$y^1(K,p) \equiv \frac{K - k_2(p)}{k_1(p) - k_2(p)} A_1 k_1(p)^{\alpha_1},$$

$$y^1(K^*,p^*) \equiv \frac{K^* - k_2(p^*)}{k_1(p^*) - k_2(p^*)} A_1 k_1(p^*)^{\alpha_1}.$$

It is easy to see that these supply functions satisfy:

$$\text{sign } y^1_K(K,p) = \text{sign } y^1_{K^*}(K^*,p^*) = \text{sign } \left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right), \tag{21a}$$

$$\text{sign } y^1_p(K,p) = \text{sign } y^1_{p^*}(K^*,p^*) = \text{sign } \left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right)(\alpha_1 - \alpha_2). \tag{21b}$$

The shadow values of capital in both countries change according to

$$\dot{q} = q[\rho + \delta - \hat{r}(p)], \tag{22a}$$

$$\dot{q}^* = q^*[\rho + \delta - \hat{r}(p^*)], \tag{22b}$$

where $\hat{r}(p) \equiv r/p = a_1 A_1 k_1(p)^{\alpha_1 - 1}$ and $\hat{r}(p^*) \equiv r^*/p^* = a_1 A_1 k_1(p^*)^{\alpha_1 - 1}$. Dynamic equations $(20a)$, $(20b)$, $(22a)$ and $(22b)$ depict behaviors of capital stocks and their implicit prices in the home and foreign countries.

The optimization conditions $(9c)$ and $(10c)$ mean that λ/λ^* stays constant over time and, therefore, from $(9a)$ and $(10a)$ the relative consumption, C/C^*, also stays constant even out of the steady state. Let us denote $C^*/C = (\lambda^*/\lambda)^{-1/\sigma} = \bar{m}\,(> 0)$. Then the world market equilibrium condition for consumption goods given by (12) becomes

$$(1 + \bar{m})\,\lambda^{-\frac{1}{\sigma}} = y^2(K,p) + y^2(K^*,p^*), \tag{23}$$

where

$$y^2(K,p) = \frac{k_1(p) - K}{k_1(p) - k_2(p)} A_2 k_2(p)^{\alpha_2},$$

$$y^2(K^*,p^*) = \frac{k_1(p^*) - K^*}{k_1(p^*) - k_2(p^*)} A_2 k_2(p^*)^{\alpha_2}.$$

The supply functions of consumption goods satisfy the following:

$$\text{sign } y^2_K(K,p) = \text{sign } y^2_{K^*}(K^*,p^*) = -\text{ sign } \left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right), \tag{24a}$$

$$\text{sign } y^2_p(K,p) = \text{sign } y^2_{p^*}(K^*,p^*) = -\text{ sign } \left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right)(\alpha_1 - \alpha_2). \tag{24b}$$

In view of (23), we see that λ is expressed as a function of capital stocks, prices and $\bar{m}$:

$$\begin{aligned} \lambda &= (1+\bar{m})^{\sigma} \left[y^2(K,p) + y^2(K^*,p^*)\right]^{-\sigma} \\ &\equiv \lambda(K,K^*,p,p^*;\bar{m}). \end{aligned} \tag{25}$$

Thus optimization conditions $(9b)$ and $(10b)$ yield

$$p = \frac{q}{\lambda(K,K^*,p,p^*;\bar{m})}, \quad p^* = \frac{q^*}{\lambda(K,K^*,p,p^*;\bar{m})}.$$

Solving these equations with respect to p and p^* presents the following expressions:

$$p = \pi(K,K^*,q,q^*;\bar{m}), \quad p^* = \pi^*(K,K^*,q,q^*;\bar{m}). \tag{26}$$

Substituting (26) into $(20a)$, $(20b)$, $(22a)$ and $(22b)$, we obtain a complete dynamic system of K, K^*, q and q^*.

3.2 Conditions for Equilibrium Indeterminacy

To discuss equilibrium determinacy of the world economy, we first confirm that the equilibrium paths of capital stocks and the relative prices in both coutries are independent of the level of $\bar{m}$.

Lemma 1 *The steady-state levels of K, K^*, q and q^* are uniquely given and they are independent of the level of $\bar{m}$,*

Proof. See Appendix 1. ■

We first characterize the stationary equilibrium of the world economy. The steady state of the dynamic system derived above is established when $\dot{K} = \dot{K}^* = \dot{q} = \dot{q}^* = 0$. From (26) the relative price in the home and foreign countries, p and p^*, also stay constant in the steady-state equilibrium. As for the existence of a feasible steady state, we can confirm the following:

Proposition 2 *Suppose that both countries imperfectly specialize in the steady state. Then the steady-state values of K, K^*, p and p^* are uniquely determined. Additionally, if $\bar{m}$ is fixed, the steady-state levels of q and q^* are uniquely given as well.*

Proof. See Appendix 2. ■

It is worth noting that while the steady-state levels of K, K^*, p and p^* are independent of $\bar{m}$, the steady-state values of q and q^* depend on $\bar{m}$. Therefore, the presence of a unique set of steady state levels of q and q^* critically depends upon our assumption that the value of $\bar{m}$ is exogenously given. To complete our analysis on the steady-state equilibrium, we should consider how $\bar{m}$ is determined. Before discussing this problem, let us explore the local determinacy of the steady-state equilibrium under a given level of $\bar{m}$.

Proposition 3 *Under a given level of $\bar{m}$, the steady-state equilibrium of the world economy is locally indeterminate, if the investment good sector is more capital intensive than the consumption good sector from the social perspective but it is less capital intensive from the private perspective.*[6]

Proof. See Appendix 3 . ■

Proposition 2 claims that in our model equilibrium indeterminacy may emerge regardless of the magnitude of σ. This is in contrast to the conclusion of the Nishimura and Shimomura (2002) who show that, in addition to the conditions given in Proposition 2, the elasticity of intertemporal substitution in consumption, $1/\sigma$, should be high to hold intermediacy.[7]

[6] We can also show that, as well as in the NS model, our model holds equilibrium determinacy, if the factor-intensity rankings are the same both from private and social perspectives.

[7] More precisely, the indeterminacy conditions in the Nishimura-Shimumura model include the following:

$$\frac{1}{\sigma} > \max\left\{1,\ \frac{(1-\alpha_1)a_2 b_1(\rho+\delta) + \alpha_1 a_1\left[\rho b_2 + \delta b_1 a_2 + (1-a_1)b_2\delta\right]}{(a_2 b_1 - a_1 b_2)(\alpha_1 - \alpha_2)\left[\rho + \delta(1-a_1)\right]}\right\}.$$

Since the closed economy version of our model is the same as the integrated world economy model dicussed by Nishimura and Shimomura (2002), we need the same condition for holding indeterminacy if our model economy is closed. Hence, our result shows that the financially integrated world with non-tradable capital goods may produce indeterminacy under a wider range of parameter spaces than in the closed economy counterpart. In this sense, our model indicates that globalization may enhance the possibility of sunspot-driven economic fluctuations.[8]

We now consider how to determine $\bar{m}$. Using the market equilibrium condition for the investment goods in (11) and the factor income distribution relation such that $pY_1 + Y_2 = rpK + w + \pi_1 + \pi_2$ and $p^*Y_1^* + Y_2^* = r^*p^*K^* + w^* + \pi_1^* + \pi_2^*$, we find that the dynamic equation of foreign bonds are expressed as

$$\dot{B} = RB + Y_2 - C, \qquad \dot{B}^* = RB^* + Y_2^* - C^*.$$

These equations represent the current accounts of both countries. In view of the no-Ponzi game and the transversality conditions, the intertemporal constraint for the current account of each country is respectively given by the following:

$$\int_0^\infty \exp\left(-\int_0^t R_s ds\right) C_t dt = \int_0^\infty \exp\left(-\int_0^t R_s ds\right) y^2\left(K_t, p_t\right) dt + B_0,$$
$$\int_0^\infty \exp\left(-\int_0^t R_s ds\right) C_t^* dt = \int_0^\infty \exp\left(-\int_0^t R_s ds\right) y^2\left(K_t^*, p_t^*\right) dt + B_0^*.$$

Since it holds that $C_t^* = \bar{m}C_t$ for all $t \geq 0$, the above equations yield

$$\bar{m} = \frac{\int_0^\infty \exp\left(-\int_0^t R_s ds\right) y^2\left(K_t^*, p_t^*\right) dt + B_0^*}{\int_0^\infty \exp\left(-\int_0^t R_s ds\right) y^2\left(K_t, p_t\right) dt + B_0}. \tag{27}$$

[8] The indeterminacy conditions in Proposition 2 require that constant returns prevail in each production sector and that the external effects associated with capital are larger in the investment good sector than in the consumption good sector. Several investigations on scale economies have suggested that our indeterminacy conditions are not unrealistic. For example, the well-cited study by Basu and Fernald (1997) find that most industries in the US approximately exhibit constant returns to scale, which may support our assumption of social constant returns. Using the US data, Harrison (2003) claims that returns to scale of the consumption goods sector are close to be constant, while the investment goods sector exhibits weak increasing returns. In addition, she reveals that external effects may be larger in the investment good sector than in the consumption good sector. However, the existing studies do not present direct empirical evidence for our discussion. Since the indeterminacy conditions in Proposition 2 are frequently used in the literature, it is a relevant task to find more convincing empirical support.

Equation (27) demonstrates that $\bar{m}$ depends on the initial holdings of bonds, B_0 and B_0^*, as well as on the discounted present value of consumption goods produced in each country. It is to be noticed that the discounted present values of consumption goods are independent of $\bar{m}$. To see this, we differentiate both sides of (25) logarithmically with respect to time, which yields

$$\frac{\dot{\lambda}}{\lambda} = -\sigma \left[\frac{Y_K^2 K}{Y^2} \frac{\dot{K}}{K} + \frac{Y_{K^*}^2 K^*}{Y^2} \frac{\dot{K}^*}{K^*} + \frac{Y_p^2 p}{Y^2} \frac{\dot{p}}{p} + \frac{Y^2 p^*}{Y^2} \frac{\dot{p}^*}{p^*} \right], \tag{28}$$

where $Y^2 \equiv y^2(K,p) + y^2(K^*,p^*)$ denotes the aggregate supply of consumption goods in the world market. Note that from $(9b)$, $(9c)$, $(9d)$, $(10b)$, $(10c)$ and $(10d)$, we obtain:

$$\frac{\dot{p}}{p} = \frac{\dot{q}}{q} - \frac{\dot{\lambda}}{\lambda} = R + \delta - \hat{r}(p), \tag{29a}$$

$$\frac{\dot{p}^*}{p^*} = \frac{\dot{q}^*}{q^*} - \frac{\dot{\lambda}^*}{\lambda^*} = R + \delta - \hat{r}(p^*). \tag{29b}$$

Substituting $(20a)$, $(20b)$, $(29a)$, and $(29b)$ into (28) yields the following:

$$\begin{aligned} \rho - R \quad = \quad & -\sigma \left[\frac{Y_K^2 K}{Y^2} \left(\frac{y^1(K,p) - \delta K}{K} \right) + \frac{Y_{K^*}^2 K^*}{Y^2} \left(\frac{y^2(K^*,p) - \delta K^*}{K^*} \right) \right. \\ & \left. + \frac{Y_p^2 p}{Y^2} (R + \delta - \hat{r}(p)) + \frac{Y_{p^*}^2 p^*}{Y^2} (R + \delta - \hat{r}(p^*)) \right]. \end{aligned}$$

Observe that each side of the above equation does not involve $\bar{m}$. Solving the above with respect to R, we find that the equilibrium level of the world interest rate can be expressed as a function of K, K^*, p and p^* :

$$R = R(K, K^*, p, p^*). \tag{30}$$

Consequently, by use of $(20a)$, $(20b)$, $(29a)$, $(29b)$ and (30), we obtain an alternative expression of the complete dynamic system of (K, K^*, p, p^*) in such a way that

$$\left. \begin{aligned} \dot{K} &= y^1(K,p) - \delta K, \\ \dot{K}^* &= y^1(K^*,p^*) - \delta K^*, \\ \dot{p} &= p\left[R(K,K^*,p,p^*) + \delta - \hat{r}(p)\right], \\ \dot{p}^* &= p^*\left[R(K,K^*,p,p^*) + \delta - \hat{r}(p^*)\right]. \end{aligned} \right\} \tag{31}$$

From $(9c)$ the steady-state level of interest rate satisfies $R = \rho$.[9] Since the dynamic system (31) does not involve $\bar{m}$, if the steady state is locally determinate (i.e. the linearized dynamic

[9] We can show that dynamic analysis of (31) presents the same conclusion as that stated in Proposition 2. However, since function (30) is rather complex, stability analysis is more cumbersome than that shown in Appendix 2.

system has two stable roots), then the equilibrium path of p_t and p_t^* are uniquely expressed as functions of K_t and K_t^* on the two-dimensional stable manifold. When we denote the relation between the relative prices and capital stocks on the stable saddle path as $p = \phi(K, K^*)$ and $p^* = \phi^*(K, K^*)$, the behaviors of capital stocks on the saddle path are expressed as

$$\begin{aligned} \dot{K} &= y^1 (K, \phi(K, K^*)) - \delta K, \\ \dot{K}^* &= y^1 (K^*, \phi^*(K, K^*)) - \delta K^*. \end{aligned}$$

These differential equations show that once the initial capital stocks, K_0 and K_0^*, are specified, the paths of $\{K_t, K_t^*\}_{t=0}^{\infty}$ are uniquely determined. As a result, the paths of $\{p_t, p_t^*, R_t\}_{t=0}^{\infty}$ are also uniquely given under the specified levels of K_0 and K_0^*. This means that when equilibrium determinacy holds, the level of $\bar{m}$ given by (27) is also uniquely selected under the given initial levels of K_0, K_0^*, B_0 and B_0^*.

In contrast, if the converging path of (31) is indeterminate (that is, the linearly approximated dynamic system of (31) has three or four stable roots), then the given initial levels of K_0 and K_0^* alone cannot pin down the equilibrium paths of p_t and p_t^*. Therefore, the level of $\bar{m}$ determined by (27) becomes indeterminate as well. In this situation, we should specify expectations formation of agents to select a particular path leading to the steady state. Once we specify a particular trajectory of the world economy with self-fulfilling expectations, we can determine the value of $\bar{m}$ that satisfy (27) . However, such an equilibrium path may fluctuate if a sunspot shock hits the world economy, so that $\bar{m}$ is also affected by expectations-driven fluctuations.

In the steady state it holds that $\dot{B} = \dot{B}^* = 0$ and $R = \rho$. Thus the steady-state level of bond holdings in both countries are given by

$$B = \frac{y^2 (K, p) - C}{\rho} = \frac{\bar{m} - 1}{\rho(1 + \bar{m})} y^2 (K, p), \tag{32a}$$

$$B^* = \frac{y^2 (K, p) - \bar{m} C}{\rho} = \frac{1 - \bar{m}}{\rho(1 + \bar{m})} y^2 (K, p). \tag{32b}$$

The above expressions show that when $\bar{m}$ is selected, the long-run asset position of each country is also determined. It is obvious that whether the home country becomes a creditor or a debtor in the long run depends solely on whether or not $\bar{m}$ exceeds one. As (27) demonstrates, if the equilibrium path is determinate and if the initial stocks of capital and bonds held by the home households are relatively large, then the home country tends to be a creditor in the long-run equilibrium. However, if there is a continuum of covering path around the steady state, the value of $\bar{m}$ determined by (27) is affected by the expectations formation of agents. This implies that in the presence of equilibrium indeterminacy, the initial holding of wealth in each country does not necessarily determine the asset position of that country in the long-run equilibrium.

To sum up, we have shown:

Proposition 4 *If the steady-state equilibrium of the world economy is locally determinate (indeterminate), then the steady-state level of asset position of each country is determinate (indeterminate).*

4 The Role of Trade Structure

In this section we compare indeterminacy conditions for the model under alternative trade structures. The purpose of this section is to give an intuitive implication for the reason why the shape of utility functions do not relate to the indeterminacy condition when investment goods are not internationally traded. We first consider the Heckscher-Ohlin modelling and then reconsider our model. Finally, we examine the opposite setting to our model where consumptions goods are not traded but investment goods are tradable.

4.1 The Heckscher-Ohlin Setting

In the Hecksher-Ohlin framework used by Nishimra and Shimomura (2002), both investment and consumption goods are internationally traded but there is no financial exchange. We focus on the case where neither home nor foreign countries specialize. Since both goods are traded, both countries face the same relative price. This means that under the assumption of symmetric technologies between the two countries, both home and foreign firms in each production sector select the same capital intensity as long as both countries imperfectly specialize. Hence, it holds that $k_i(p) = k_i^*(p) \quad (i = 1, 2)$ for all $t \geq 0$. Thus the world market equilibrium condition for investment goods, $\dot{K} + \delta K + \dot{K}^* + \delta K^* = Y_1 + Y_1^*$, yields the capital formation equation such that

$$\dot{K}_w = \frac{K_w - 2k_2(p)}{k_1(p) - k_2(p)} - \delta K_w, \tag{33}$$

where $K_w = K + K^*$ denotes the world level of capital stock. In addition, since firms in both countries choose the same capital intensities, the factor prices are also equalized between the two countries, that is, $\hat{r}^*(p) = \hat{r}(p)$ and $w^*(p) = w(p)$, implying that the dynamic equation of the shadow value of capital in the foreign country (22b) is replaced with $\dot{q}^* = q^*(\rho + \delta - \hat{r}(p))$. Consequently, (22$a$) shows that q^*/q stays constant over time and that the complete dynamic system of the world economy is given by (22a), (33) and the world market equilibrium condition for consumption goods:

$$Y_2 + Y_2^* = C + C^* \Rightarrow (1 + \bar{m})\, q = \frac{2k_1(p) - K_w}{k_1(p) - k_2(p)}, \tag{34}$$

where $\bar{m}$ is a positive constant defined as $\bar{m} = C/C^* = (q/q^*)^{-1/\sigma}$. Given $\bar{m}$, the equilibrium relative price can be expressed as $p = \pi(K_w, q; \bar{m})$. Substituting this function into (22a) and (33), we obtain a complete dynamic system with respect to K_w and q. This aggregate dynamic system is essentially the same as the closed economy model in Benhabib and Nishimura (1998).[10] As a consequence, the intuitive implication of the indeterminacy conditions for the Nishimura and Shimomura (2002) is essentially the same as those for the case of a closed economy.

Now suppose that a sunspot shock hits the world economy and all the households in the world expect that the rates of return to their capital will increase. This raises the marginal values of capital, q and q^*. If σ is sufficiently small the intertermporal substitution effect dominates the income effect, then households reduce consumption and invest more, which leads to higher level of world capital, K_w. As we have assumed that the consumption good sector is more capital intensive from the private perspective, (34) shows that an increase in K_w raises consumption goods production relative to the investment goods. This increases the price of investment good p, and the firms select a lower capital intensity because the

[10] Nishimura and Shimomura (2002) show that the steady-state levels of K_w and p are uniquely given. Distribution of capital stock between the two countries depends on the initial holdings of capital if the equilibrium path of the world economy is determinate. They also confirm that the long-run distribution of capital (so the long-run trade patterns) becomes indeterminate, if the steady state of the dynamic system of the world economy is a sink.

social technology of the capital goods is more capital intensive than that of the consumption good sector (see (18)). Consequently, the rate of return to capital in the world economy will increase and the sunspot-driven expectations are self-fulfilled.

In contrast, if σ is large enough, the income effect dominates the intertemporal substitution effect and, hence, a sunspot deriven expected rise in the marginal value of capital may increase consumption. As a result, investment in the world will decrease and the relative price of investment goods, p, declines. In view of (18) , a lower p reduces the rate of return to capital so that the initial change in expectations is not self-fulfilled.

4.2 Non-Tradable Investment Goods

Unlike the Heckscher-Ohlin modelling, we have assumed that investment goods are not tradable and international lending and borrowing are allowed. Again, suppose that households in both countries expect that the rates of return to their capital will increase. Then households intend to rise their investment. In the Heckscher-Ohlin environment, this requires that households reduce their current consumption, and thus the magnitude of σ plays a pivotal role. In our model, however, households may increase their real investment by borrowing from foreign households rather than decrease their current consumption levels. If households in both countries try to borrow in the international financial market, the world interest rate, R, will increase. From (31) we see that a higher R raises the price of investment goods, p and p^*, in both countries. [11]Given the factor intensity ranking conditions in Proposition 2, higher levels of p and p^* decrease capital intensities and the rates of retune to capital, r and r^*, actually increase. In other words, the presence of bond market cuts off the direct link between the current consumption and real investment held in the Heckscher-Ohlin model (as well as in the closed economy model).

The above intuition is confirmed more clearly in the small-open economy model explored by Meng and Velasco (2004) and others. They also assume that investment goods are not traded but international lending and borrowing are possible. Since the world interest rate is

[11]Precisely speaking, a rise in the interest rate does not necessarily increases the relative prices, if the economy out of the steady state. We have restricted we restrict our attention to the local dynamic around the steady state, where it holds that $R = \hat{r}(p^*) - \delta = R - \hat{r}(p^* - \delta)$ at the outset.

given for a small country, the dynamic behavior of the small-open economy are described by

$$\dot{K} = y^1 (K, p) - \delta K,$$
$$\dot{p} = p \left[\bar{R} + \delta - \hat{r}(p) \right],$$

where $\bar{R}$ denotes a given world interest rate. Since the shadow value of capital follows $\dot{q} = q \left(\delta + \rho - \bar{R} \right)$, it is assumed that $\delta + \rho = \bar{R}$ to keep q at a finite level. As a result, the current level of consumption, which satisfies $C^{-\sigma} = q$, stays constant as well. In this extreme case, the interest rate will not respond to a rise in investment, there is no link between the current levels of savings and consumption, so that the dynamic behavior of the economy is independent of the preference structure. Meng and Velasco (2003 and 2004.) show that the factor-ranking conditions shown Proposition 2 are the necessary and sufficient conditions for indeterminacy. In our general equilibrium modelling, the world interest rate changes as investment in the world economy increases. This is why the factor-ranking conditions are sufficient but not necessary for indeterminacy in our model. However, since we have focused on the local behavior of the world economy around the symmetric steady state where both countries hold the same level of capital, our indeterminacy conditions are closed to those for the small-open economy with non-tradable investment goods.

4.3 Non-Tradable Consumption Goods

We now consider the opposite situation where the consumption goods are not internationally traded, but the investment goods are tradable and financial capital mobility is possible. In this case the commodity market equilibrium conditions are given by

$$I + I^* = Y_1 + Y_1^*, \quad C = Y_2, \quad C^* = Y_2^*.$$

We take the tradable investment good as a numareise. Then the net wealth held by the domestic household (in terms of investment good) is $\Omega = B + K$ and the flow budget constraint is written as

$$\dot{B} = R(B + K) + w + \pi_1 + \pi_2 - \hat{p}C - I.$$

where $\hat{p}$ $(= 1/p)$ denotes the domestic price of consumption good in terms of investment good. The Hamiltonina function for the households in the home country is given by

$$H = \frac{C^{1-\sigma} - 1}{1 - \sigma} + \lambda \left[RB + rK + w + \pi_1 + \pi_2 - \hat{p}C - I \right] + q (I - \delta K)$$

and the key first-order conditions for an optimum are:

$$C^{-\sigma} = \lambda \hat{p}, \tag{35a}$$

$$\lambda = q \tag{35b}$$

$$\dot{\lambda} = \lambda\left(\rho - R\right), \tag{35c}$$

$$\dot{q} = q\left(\rho + \delta - r\right). \tag{35d}$$

The key condition here is $(35b)$: since investment goods are internationally tradable, capital and international bonds are perfectly substitute each other so that their implicit prices have are the same. This means tha, in view of $(35c)$ and $(35d)$, $R = r - \delta$ for all $t \geq 0$.

Since households in both country face the same interest rate, R, the rate of returns to capital in the foreign country satisfies

$$r^* - \delta = R = r - \delta.$$

Therefore, $r\left(p\right) = r\left(p^*\right)$ holds in each moment, implying that p always equals p^*. Thus the world market equilibrium condition of investment good yields the dynamic equation of the aggregate capital given by (**??**) . In addition, from the equilibrium condition for consumption goods in each country we obtain

$$C^{-1/\sigma} = y^2\left(K, \hat{p}\right), \qquad C^{*-1/\sigma} = y^2\left(K^*, \hat{p}\right),$$

where it holds that $C^* = \bar{m}C$. The above equilibrium conditions present (34) . Therefore, the dynamic system of the world economy is the same as that of the Nishimura-Shimomura model.[12] This conclusion demonstrates that our assumption of the absence of investment goods trade plays a pivotal role for making our indeterminacy conditions diverge from those for the model with Hechscher-Ohlin properties.[13]

[12] Noteice that in this subsection $\hat{p}$ denotes the consumption good price in terms of investment goods. It is easy to see that the stability condition for the model in Section 4.1 are still the same if we replace p with $1/\hat{p}$.

[13] In this paper we focus on the decentralized economies. Our entire discussion can be reformulated in terms of the pseudo-planning problems where the planner maximizes the world welfare, $W = \int_0^\infty e^{-\rho t}\left[\frac{C^{1-\sigma}}{1-\sigma} + \mu^* \frac{C^{*1-\sigma}}{1-\sigma}\right] dt$, by taking the sequences of external effects as given. The resource constraints for the planner are: (i) $Y_1 + Y_1^* = I + I^*$ and $Y_2 + Y_2^* = C + C^*$ in the Heckscher-Ohlin model, (ii) $Y_1 = I,\ Y_1^* = I^*$ and $Y_2 + Y_2^* = C + C^*$ in our model, and; (iii) $Y_1 + Y_1^* = I + I^*$, $Y_2 = C$ and $Y_2^* = C^*$ in the model with

5 A Final Remark

The world economy as a whole is a closed economy in which there are heterogeneous countries. Therefore, its model structure is similar to that of a closed, single economy model with heterogeneous agents. In particular, if consumption and saving decisions are made by the representative household in each country, the world economy model is closely connected to the closed economy model with heterogeneous households. There is, however, an important difference between the world economy models and the single country setting: when dealing with the world economy model, we should specify the trade structure between the countries. This paper has revealed that the assumption on trade structure may be critical for the presence of equilibrium indeterminacy even if there is no international heterogeneity in technologies and preferences. Several authors have explored recently how the presence of heterogeneous preferences and technologies alter the determinacy/indeterminacy conditions in the equilibrium business cycle models with market distortions. These studies have shown that the heterogeneity in preferences and technologies often affects stability condition in a critical manner.[14] In a similar vein, Sim and Ho (2007) find that introducing technological heterogeneity into the Nishimura-Shimomura model may produce a substantial change in equilibrium indeterminacy results. Those existing findings suggest that it is worth extending our model by considering further heterogeneity between the two countries.

Appendices

Appendix 1: Proof of Proposition 1.

When $\dot{q} = \dot{q}^* = 0$ in $(22a)$ and $(22b)$, it holds that

$$a_1 A_1 k_1 (p)^{\alpha_1 - 1} = a_1 A_1 k_1 (p^*)^{\alpha_1 - 1} = \rho + \delta.$$

Thus by use of $(15a)$ and $(15c)$, we find that

$$p = p^* = \left(\frac{A_2}{A_1}\right)\left(\frac{a_2}{a_1}\right)^{\alpha_2}\left(\frac{b_2}{b_1}\right)^{1-\alpha_2}\left(\frac{\rho+\delta}{a_1 A_1}\right)^{\frac{\alpha_2-\alpha_1}{\alpha_1-1}}.$$

non-tradable consumption goods. It is easy to confirm that the optimization problem, subject to (iii) yields the same equilibrium conditions as those for the optimization problem under (i). See Hu and Mino (2010) for the detail.

[14] See, for example, Ghiglino and Olszak-Duquenne (2005).

These conditions show that the steady-state levels of p and p^* are uniquely given and it holds that $p = p^*$ in the steady state. The steady-state levels of capital stocks satisfying $\dot{K} = \dot{K}^* = 0$ in (20a) and (20b) are determined by the following conditions:

$$\frac{K - k_2(p)}{k_1(p) - k_2(p)} A_1 k_1(p)^{\alpha_1} = \delta K,$$

$$\frac{K^* - k_2(p^*)}{k_1(p^*) - k_2(p^*)} A_1 k_1(p^*)^{\alpha_1} = \delta K^*.$$

Using the conditions for $\dot{p} = \dot{p}^* = 0$ and the fact that $p = p^*$ holds in the steady state, we confirm that the steady-state level of capital stock in each county has the same value, which is given by

$$K = K^* = \frac{(aA_1)^{\frac{1}{1-\alpha_1}} (\rho + \delta)^{\frac{\alpha_1}{\alpha_1 - 1}}}{\rho + \delta \left(1 - \delta + \frac{a_2 b_1}{b_2}\right)} \left(\frac{a_2 b_1}{a_1 b_2}\right),$$

which has a positive value. We also find that the steady-state values of labor allocation to the investment good sector are:

$$L_1 = L_1^* = \frac{a_1 \delta \left(\frac{a_2 b_1}{a_1 b_2}\right)}{\rho + (1 - a_1)\delta + a_1 \delta \left(\frac{a_2 b_1}{a_1 b_2}\right)} \in (0, 1).$$

Hence, (19) is fulfilled so that both countries imperfectly specialize. In addition, if $\bar{m}$ is fixed, from (23) the steady-state value of λ is uniquely determined as well, implying that $q = p\lambda$ and $q^* = p^*\lambda$ are also uniquely given in the steady state.

Appendix 2: Proof of Proposition 2

To prove Proposition 2, the following facts are useful:

Lemma 1 *In the symmetric steady state where $K = K^*$ and $q = q^*$, the following relations are satisfied:*

$$y_K^i(K, p) = y_{K^*}^i(K^*, p^*), \quad i = 1, 2,$$

$$y_p^i(K, p) = y_{p^*}^i(K^*, p^*), \quad i = 1, 2,$$

$$\pi_K(K, K^*, q, q^*) = \pi_K^*(K, K^*, q, q^*) = \pi_{K^*}(K, K^*, q, q^*) = \pi_{K^*}^*(K, K^*, q, q^*),$$

$$\pi_q(K, K^*, q, q^*) = \pi_{q^*}^*(K, K^*, q, q^*),$$

$$\pi_{q^*}(K, K^*, q, q^*) = \pi_q^*(K, K^*, q, q^*).$$

Proof. By the functional forms of $y^i_j(\cdot)$ $(i = 1, 2,\ j = K, K^*, p, p^*)$, it is easy to see that $y^i_K(K, p) = y^i_{K^*}(K^*, p^*)$ and $y^i_p(K, p) = y^i_{p^*}(K^*, p^*)$ are established when $p = p^*$ and $K = K^*$.As for the rest of the results, we may use $p\lambda(\cdot) = q$ and $p^*\lambda(\cdot) = q^*$ to derive the following:

$$\frac{\partial p}{\partial K} = \pi_K = -\frac{\lambda_K}{\lambda + p\lambda_P}, \quad \frac{\partial p}{\partial K^*} = \pi_{K^*} = -\frac{\lambda_{K^*}}{\lambda + p\lambda_P}, \tag{36a}$$

$$\frac{\partial p^*}{\partial K} = \pi^*_K = -\frac{\lambda_K}{\lambda + p^*\lambda_{P^*}}, \quad \frac{\partial p^*}{\partial K^*} = \pi^*_{K^*} = -\frac{\lambda_{K^*}}{\lambda + p^*\lambda_{P^*}}, \tag{36b}$$

$$\frac{\partial p}{\partial q} = \pi_q = \frac{\lambda + p\lambda_p}{\lambda(\lambda + 2p\lambda_p)}, \quad \frac{\partial p}{\partial q^*} = \pi_{q^*} = -\frac{p\lambda_p}{\lambda(\lambda + 2p\lambda_p)}, \tag{36c}$$

$$\frac{\partial p^*}{\partial q} = \pi^*_q = -\frac{p^*\lambda_{p^*}}{\lambda(\lambda + 2p^*\lambda_{p^*})}, \quad \frac{\partial p^*}{\partial q^*} = \pi^*_{q^*} = \frac{\lambda + p^*\lambda_{p^*}}{\lambda(\lambda + 2p^*\lambda_{p^*})}. \tag{36d}$$

Since $\lambda_K(\cdot) = \lambda_{K^*}(\cdot)$ and $\lambda_p(\cdot) = \lambda_{p^*}(\cdot)$ in the steady state where $K = K^*$ and $p = p^*$, we obtain $\pi_K = \pi^*_K = \pi_{K^*} = \pi^*$, $\pi_q = \pi^*_{q^*}$ and $\pi_{q^*} = \pi^*_q$. ■

Let us linealize the dynamic system of $(20a)$, $(20b)$, $(22a)$ and $(22b)$ at the steady state. The coefficient matrix of the linealized system is given by

$$J = \begin{bmatrix} y^1_K - \delta + y^1_p\pi_K & y^1_p\pi_{K^*} & y^1_p\pi_q & y^1_p\pi_{q^*} \\ y^1_{p^*}\pi^*_K & y^1_{K^*} - \delta + y^1_{p^*}\pi^*_{K^*} & y^1_{p^*}\pi^*_q & y^1_{p^*}\pi^*_{q^*} \\ -q\hat{r}'\pi_K & -q\hat{r}'\pi_{K^*} & -q\hat{r}'\pi_q & -q\hat{r}'\pi_{q^*} \\ -q\hat{r}'\pi^*_K & -q\hat{r}'\pi^*_{K^*} & -q\hat{r}'\pi^*_q & -q\hat{r}'\pi^*_{q^*} \end{bmatrix}.$$

By use of Lemma 1, we see that the characteristic equation of J is written as

$$\begin{aligned}
\Gamma(\eta) &= \det[\eta I - J] \\
&= \det \begin{bmatrix} \eta - (y_K^1 - \delta + y_p^1 \pi_K) & -y_p^1 \pi_K & -y_p^1 \pi_q & -y_p^1 \pi_{q^*} \\ -y_p^1 \pi_K & \eta - (y_K^1 - \delta + y_p^1 \pi_K) & -y_p^1 \pi_{q^*} & -y_p^1 \pi_q \\ q\hat{r}' \pi_K & q\hat{r}' \pi_K & \eta + q\hat{r}' \pi_q & q\hat{r}' \pi_{q^*} \\ q\hat{r}' \pi_K & q\hat{r}' \pi_K & q\hat{r}' \pi_q & \eta + q\hat{r}' \pi_q \end{bmatrix} \\
&= \det \begin{bmatrix} \eta - \left(y_K^1 - \delta\right) & 0 & \eta & 0 \\ 0 & \eta - (y_K^1 - \delta) & 0 & \eta \\ q\hat{r}' \pi_K & q\hat{r}' \pi_K & \eta + q\hat{r}' \pi_q & q\hat{r}' \pi_{q^*} \\ q\hat{r}' \pi_K & q\hat{r}' \pi_K & q\hat{r}' \pi_q & \eta + q\hat{r}' \pi_q \end{bmatrix} \\
&= \left[\eta - \left(y_K^1 - \delta\right)\right] \left[\eta + q\hat{r}'(\pi_q - \pi_{q^*})\right] \xi(\eta).
\end{aligned}$$

where η denotes the characteristic root of J and

$$\xi(\eta) \equiv \eta^2 + \left[q\hat{r}'(\pi_q + \pi_{q^*}) - \left(y_K^1 - \delta\right) - 2y_p^1 \pi_K\right] \eta - q\hat{r}' \left(y_K^1 - \delta\right) (\pi_q + \pi_{q^*}).$$

Our assumptions mean that $\frac{a_1}{b_1} - \frac{a_2}{b_2} < 0$ and $\alpha_1 - \alpha_2 > 0$. Thus from (24a) we see that $y_K^1 - \delta < 0$. In addition, equation (36c) shows that $\pi_q - \pi_{q^*} = 1/\lambda\,(>0)$. Hence, using $\hat{r}(p) \equiv a_1 A_1 k_1(p)^{\alpha_1 - 1}$, we obtain:

$$\hat{r}'(\pi_q - \pi_{q^*}) = a_1(a_1 - 1) A_1 (k_1(p))^{a_1 - 2} \frac{k_1'(p)}{\lambda} > 0.$$

As a consequence, at least two roots of $\Gamma(\eta) = 0$ have negative real parts. Equations in (36c) also show

$$\pi_q + \pi_{q^*} = \frac{1}{\lambda + 2p\lambda_p},$$

where

$$\begin{aligned}
\lambda_p &= \frac{\partial}{\partial p}(1 + \bar{m})^{\frac{1}{\sigma}} \left[y^2(K, p) + y^2(K^*, p^*)\right]^{-\frac{1}{\sigma}} \\
&= -\frac{y_p^2}{\sigma}(1 + \bar{m})^{\frac{1}{\sigma}} \left[y^2(K, p) + y^2(K^*, p^*)\right]^{-\frac{1}{\sigma} - 1} < 0.
\end{aligned}$$

Therefore, in the steady state equilibrium. the following holds:

$$\lambda + 2p\lambda_p = \frac{1}{\sigma}\left[\sigma - \frac{p y_p^2(K, p)}{y^2(K, p)}\right].$$

Notice that under our assumptions, it holds that $y_p^2(K,p) > 0$. Suppose that σ is small enough to satisfy $\sigma < py_p^2/y^2$. Then $\lambda_p + 2p\lambda_p > 0$ so that $\pi_q + \pi_{q^*} < 0$, which leads to

$$-q\hat{r}'\left(y_K^1 - \delta\right)\left(\pi_q + \pi_{q^*}\right) < 0.$$

This means that $\xi(\eta) = 0$ has one positive and one negative roots. As a result, $\Gamma(\eta) = 0$ has three stable roots. Hence, if σ is smaller than the price elasticity of supply function of consumption goods, then there locally exists a continuum of equilibrium paths converging to the steady state.

Now suppose that σ is larger than py_p^2/y^2. Then we obtain $\pi_q + \pi_{q^*} > 0$. Furthermore, it holds that

$$\begin{aligned} -2y_p^1\pi_K &= -2y_p^1\left(-\frac{p\lambda_K}{\lambda + 2p\lambda_p}\right) \\ &= -\frac{2py_p^1}{\lambda + 2p\lambda_p}y_K^2\left[\frac{(1+\bar{m})^{\sigma^{-1}}}{\sigma}\right]\left(2y^2\right)^{-\sigma^{-1}-1} > 0, \end{aligned}$$

because $y_p^1 < 0$ and $y_K^2 > 0$ under our assumptions. Consequently, the following inequalities are established:

$$-q\hat{r}'\left(y_K^1 - \delta\right)\left(\pi_q + \pi_{q^*}\right) > 0,$$

$$q\hat{r}'\left(\pi_q + \pi_{q^*}\right) - \left(y_K^1 - \delta\right) - 2y_K^1\pi_K > 0.$$

These conditions demonstrate that $\xi(\eta) = 0$ has two roots with negative real parts and, hence, all the roots of $\Gamma(\eta) = 0$ are stable ones. In sum, if $\frac{a_1}{b_1} - \frac{a_2}{b_2} < 0$ and $\alpha_1 - \alpha_2 > 0$, then the characteristic equation of the linearlized system involves at least three stable roots, implying that the converging path towards the steady state is locally indeterminate.

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