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“A Tradable Permit System in an Intertemporal Economy: A General Equilibrium Approach”

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A Tradable Permit System in an Intertemporal Economy: A General Equilibrium Approach

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Abstract

The creation of an artificial market through a tradable permit system as a remedy against market failure is gaining popularity among analysts and policymakers. We show that in an intertemporal competitive economy, a tradable permit system may not achieve efficiency without setting appropriate permit interest rates (rewards for holding permits), and to find them, we must know in advance the path of efficient permit prices, which is difficult or impossible to obtain. We deal with this problem in two ways. First, we seek a special case in which the permit interest rates are given by a simple rule. Second, we propose a mechanism by which the permit interest rates are generated endogenously. The determinacy of an equilibrium under a tradable permit system is also examined.

Keywords: artificial market, tradable permit system, general equilibrium, permit interest rate, permit bank, indeterminacy

JEL Classification: H23; K32; Q58.

1 Introduction

To control pollution efficiently, Crocker (1966) and Dales (1968) independently developed the idea of transferable discharge permits. The first application was the Emission Trading Program by the US Environmental Protection Agency, which started from 1974 and dealt with a number of air pollutants. Since then, a tradable permit system as a means of remedying market failure has been gaining popularity among analysts and policymakers. Successful experiences are accumulated in the practice of environmental policies (Tietenberg, 2006). The US 1990 Clean Air Act Amendments initiating the sulfur dioxide (SO_2)/acid rain trading program is a notable case (Carlson et al., 2000). As another example, the European Union initiated its Emissions Trading Scheme in January 2005, which is the largest emissions trading market in the world (Kruger and Pizer, 2004). The invention of a tradable permit system is one of the most important contributions of environmental economics to the environmental policy.

Montgomery (1972) formally proved that the creation of an artificial market through a tradable permit system can restore efficiency at equilibrium in a competitive economy. While his analysis was conducted in a static environment, some researchers have recently extended their analyses to an intertemporal competitive economy. Except for the earliest two works of Cronshow and Kruse (1996) and Rubin (1996), studies by Kling and Rubin (1997), Leiby and Rubin (2001), and Yates and Cronshaw (2001) found that if the permits are bankable (i.e., saved for future use), a tradable permit system would not achieve efficiency without setting appropriate permit interest rates (i.e., rewards of holding permits).

The key point is as follows: Consider a tradable permit system to control pollution emissions. Let a firm hold permits of amount $B > 0$. Denote by $r(t)$ the equilibrium interest rate at time $t \geq 0$. For any $t, t' \geq 0$ in the valid term of the permits, the equilibrium price of the permits $p^e(t)$ satisfies the arbitrage condition:

$$p^e(t)B = p^e(t')B \exp\left(-\int_t^{t'} r(s)ds\right).$$

This equation with initial value $p(0)$ determines the entire path of equilibrium prices. On the other hand, the *efficient* prices of permits $p^*(t)$ are determined by the marginal damage cost of pollution emissions at the social optimum. Because this efficient pricing rule is independent of the arbitrage condition, there is no reason for the equilibrium prices and the efficient prices to have the same values.

To achieve efficiency in the tradable permit system, we need to introduce a permit interest rate $a(t)$. Then, the amount of permits at time t' becomes $B(t') = B \exp\left(\int_t^{t'} a(s)ds\right)$ and the arbitrage condition above is satisfied with $p^e(t) = p^*(t)$ as:

$$p^*(t)B = p^*(t')B \exp\left(\int_t^{t'} a(s)ds\right) \exp\left(-\int_t^{t'} r(s)ds\right).$$

By differentiating this with respect to t' and evaluating at $t' = t$, we have the formula of the permit interest rate:

$$a(t) = \frac{\dot{B}(t)}{B(t)} = r(t) - \frac{\dot{p}^*(t)}{p^*(t)}. \tag{1}$$

Notice that the equation is a very familiar intertemporal arbitrage condition: the sum of the return and capital gain of an asset coincides with the interest rate at each point in time, $\dot{B}/B + \dot{p}/p = r$.

A regulatory authority needs to exogenously set the appropriate permit interest rates, defined in (1), for a tradable permit system to achieve efficient pollution control. However, a problem of information collection arises: the exact calculation of permit interest rates requires information on the efficient permit prices $p^*(t)$. This could be a heavy burden, greatly reducing the merit of a tradable permit system, because it is expected, in a static framework, that the efficient prices are spontaneously and posteriorly found through an artificial market created by a tradable permit system.

While less focus on this problem has been provided in the literature,[1] it is a crucial problem because the virtue of a market-based solution or market per se is its potential to achieve efficiency without collecting information scattered over an economy. Notice, also, that a Pigovian tax, an alternative market-based policy measure, needs the same information on the efficient prices and thus shares the same information problem.

This paper analyzes this problem. We identify a set of conditions under which the permit interest rate is obtained by a simple rule. We also show that there is a mechanism by which the permit interest

[1] The reason is in part that the problem relates to the arbitrage condition that does not appear in a static analysis which is still a major framework in environmental economics. The seminal paper on the use of an artificial market by Montgomery (1972) is written in a static framework. The book by Baumol and Oates (1988), a classic work on economic analysis of environmental policies, spares only one chapter for a dynamic analysis and the chapter discusses natural resource management. The literature survey on environmental economics by Cropper and Oates (1992) is restricted to static analyses.

rate is generated through a market spontaneously, although the mechanism has a flaw that may lead to the indeterminacy of equilibria.

Because a tradable permit system is a policy measure that minimizes the cost of achieving an exogenously given policy target, most studies use a partial equilibrium approach and examine the cost effectiveness. On the contrary, this paper takes a general equilibrium approach and examines the efficiency. The main reason is that we seek rules of thumb on the appropriate permit interest rate and the richer structure of a general equilibrium model, compared with partial equilibrium and cost minimization models, is expected to give us clues. Furthermore, unlike most studies that use a finite time horizon model reflecting that tradable permit systems in practice have a finite planning period, we use an infinite time horizon, except for a special model for global warming. This choice can be theoretically supported because a policy measure is required whenever market failure exists. Correspondingly, we assume that a permit can be banked and withdrawn at any time.

However, one may suspect that this assumption exaggerates the problem of intertemporal arbitrage. That is, if the term of a permit is reasonably short, then without the permit interest rate, a tradable permit system may achieve an almost cost-effective or efficient outcome. Indeed, it is true for the limit case where the term of a permit is infinitesimal. Stokey (1998) proved that a tradable permit system with one period permits can achieve efficiency in a general equilibrium framework. With a perishable permit, the permit interest rate is irrelevant. In practice, however, permits are usually bankable.[2]

Global warming is a good example to show that even if the term of a permit is sufficiently short, the permit interest rate is not negligible. It is said that the consequences of greenhouse gas emissions by the present generation have appeared after more than a half century. Because the practical planning period of a tradable permit program (that would be some years) is sufficiently short relative to the problem, we tend to think that it is simply total emissions over the planning period that matters. Then, cost-effective pollution abatement is achieved by emission permits without setting the permit interest rate. However, note that what affects future generations is not the total emissions, but the concentration of the greenhouse gasses in the atmosphere. Then, the emission path under a zero permit interest rate could be suboptimal. We will formally show that the appropriate

[2] An exception is the Lead Trading Program in the United States (1982–1987), where no banking was allowed initially. However, it was allowed in late 1984.

rate is not zero for our global warming model. As such, even for global warming, the permit interest rate should be considered when a tradable permit system is designed.

This paper is organized as follows: We begin by deriving an optimal permit interest rate, using both models of flows and stocks of pollution. For each case, by specifying preferences and technology as well as the environment, we attempt to find some rules of thumb for an appropriate permit interest rate. We show that if the utility function, the production function, and/or the assimilation function of pollution have specific properties such as linearity, additive separability, and isoelasticity, then the appropriate permit interest rates are obtained from a simple rule. As a specific but important case, we also examine a model for global warming.

In Section 3, we modify a tradable permit system to overcome the problem of the permit interest rate. We create a bank of permits, which we refer to as a permit bank. Each permit holder has an account of permits in the bank. Deposits and withdrawals are made in terms of permits, whereas the balance is expressed in monetary terms by multiplying the quantity of permits held by their market price. The prevailing market interest rate is applied. The idea of a permit bank comes from the similarity and differences between tradable permits and money. A permit is a means of exchange and can be saved, which is similar to money. The differences compared with money are that a permit is exchangeable only for a specific nonmarket good such as a pollution emission right and a permit does not bear interest. By creating a bank of permits, a permit becomes more like money and a tradable permit system with a permit bank comes to be embedded into a market mechanism in a more seamless way. We show that a tradable permit system with a permit bank can achieve efficiency at equilibrium, where the permit interest rates could be determined by a market without governmental intervention.

Section 4 deals with the dynamics of equilibrium paths under a tradable permit system. While a tradable permit system with a permit bank has desirable features, the permit market allows the equilibrium paths to have more degrees of freedom than the case with the government exogenously setting the path of permit interest rates. Then, the concern with equilibrium indeterminacy arises. We show that an indeterminacy emerges under a tradable permit system with a permit bank. For a tradable permit system with exogenous permit interest rates, we show that a steady state equilibrium is saddle point stable and is thus locally determinate.

Section 5 provides concluding remarks that summarize and discuss the results. Although it is also important to determine the appropriate amount of issue of permits, this study focuses on the permit interest rate.

2 Permit interest rate

2.1 Flow pollution model

Consider a simple competitive economy. The production technology of the economy is described by a "pollution as an input" production function $F(K, X)$, where $K \geq 0$ is the capital stock and X denotes the pollution flow. We assume that the production function $F : \mathbb{R}_+^2 \to \mathbb{R}_+$ is concave and smooth with $F(0, X) = F(K, 0) = 0$, $F_K(K, X) > 0$, $F_X(K, X) > 0$, and $\lim_{K \to 0} F_K(K, X) = \lim_{X \to 0} F_X(K, X) = \infty$ for all $K > 0$ and $X > 0$.[3] There is an upper limit on pollution emissions X because pollution is in fact a byproduct of the production, and not an input. Assume that the upper limit proportionally depends on the amount of the capital stock K. Therefore, the effective domain of F is given by $\{(K, X) \in \mathbb{R}_+^2 : 0 \leq X \leq \phi K\}$ with a positive constant, ϕ. Given K, the potential product is $F(K, \phi K)$ and the corresponding pollution emission is ϕK. If the actual pollution emission is $X < \phi K$, then some resources are used for abatement activities and the actual output is less than the potential: $F(K, X) < F(K, \phi K)$. With this formulation, we can treat the pollution flow as an input. See Copeland and Taylor (2003, chapter 2) for an elaborate explanation of a pollution as an input in the production function. Throughout the paper, the initial endowment of capital stock is denoted by K_0. We assume no capital depreciation for simplicity.

There is a continuum of identical infinitely lived households of measure one. Let $\rho > 0$ be the time discount rate and let $u(c, X)$ be the instantaneous utility function, where c denotes the consumption rate. Assume that $u : \mathbb{R}_+^2 \to \mathbb{R} \cup \{-\infty\}$ is a concave and smooth function with $u_c(c, X) > 0$, $u_X(c, X) < 0$, $\lim_{c \to 0} u_c(c, X) = \infty$, and $\lim_{X \to 0} u_X(c, X) = 0$ for all $c > 0$ and $X > 0$. Here, we assume that pollution *flow* X affects utility. In the next subsection, we consider the case in which the *stock* of pollution affects utility.

[3] The notation f_x for a function f denotes the partial derivative with respect to its argument x: $f_x := \partial f / \partial x$.

The social planner's problem is formulated as:

$$\max_{c(t),X(t)} \int_0^\infty u(c(t),X(t))e^{-\rho t}dt \tag{2}$$

$$\text{subject to } \dot{K}(t) = F(K(t),X(t)) - c,\ 0 \leq X(t) \leq \phi K(t),\ K(0) = K_0 > 0\,. \tag{3}$$

The associated Hamiltonian is given by:

$$H(c,X,K,\lambda,\mu) = u(c,X) + \lambda[F(K,X) - c] + \mu\,(\phi K - X)\,.$$

If $(c^*(t), X^*(t), K^*(t))$ is an optimal path, then there exist the costate variable $\lambda^*(t)$ and the Lagrange multiplier $\mu^*(t)$, with which (3) and the following hold for each $t \geq 0$:

$$u_c(c^*(t), X^*(t)) - \lambda^*(t) = 0 \tag{4}$$

$$u_X(c^*(t), X^*(t)) + \lambda^*(t)F_X(K^*(t), X^*(t)) - \mu^*(t) = 0, \tag{5}$$

$$\dot{\lambda}^*(t)/\lambda^*(t) = \rho - F_K(K^*(t), X^*(t)) + \phi\mu^*(t)/\lambda^*(t), \tag{6}$$

$$\mu^*(t) \geq 0,\ \text{and}\ \mu^*(t)[\phi K^*(t) - X^*(t)] = 0, \tag{7}$$

and the transversality condition:

$$\lim_{t\to\infty} e^{-\rho t}\lambda^*(t)K^*(t) = 0. \tag{8}$$

We are interested in how the optimal path $(c^*(t), X^*(t), K^*(t))$ can be mimicked in a competitive economy when the regulatory authority implements a tradable permit system. Throughout this paper, it is assumed that the permit is emission-based, so that one unit of emissions are allowed in return for one permit. Let $B > 0$ be the total amount of permits that are distributed to firms once in the initial period. Denote the permit interest rate by $a(t)$, a measurable function on $[0,\infty)$. Then, a tradable permit system is defined by a pair $(B, a(t))$. Let the products be the numeraire and $p(t)$ be the price of the tradable permits. Denote by $r(t)$ the interest rate at time t. The representative firm's problem is given by:

$$\max_{K(t),X(t),y(t)} \int_0^\infty \pi(t)\exp\left(-\int_0^t r(s)ds\right)dt \tag{9}$$

$$\text{subject to } \pi(t) = F(K(t),X(t)) - r(t)K(t) - p(t)y(t)$$

$$\dot{B}(t) = a(t)B(t) - X(t) + y(t),\ B(0) = B,$$

$$K(t) \geq 0, X(t) \in [0, \phi K(t)],\ \text{and } \liminf_{t\to\infty} B(t)\exp\left(-\int_0^t a(s)ds\right) \geq 0,$$

where $y(t)$ is the amount of permits purchased (if $y(t) > 0$) or sold (if $y(t) < 0$) at time t, and the last constraint is a no-Ponzi game condition, without which the firm can buy permits unlimitedly and emit its pollution arbitrarily.

The representative household's problem is:

$$\max_{c(\cdot)\geq 0} \int_0^\infty u(c(t), \tilde{X}(t))e^{-\rho t}dt \tag{10}$$

$$\text{subject to } \dot{m}(t) = r(t)m(t) + \tilde{\pi}(t) - c,\ m(0) = K_0 > 0 \text{ given},$$

$$\liminf_{t\to\infty} m(t)\exp\left[-\int_0^t r(s)ds\right] \geq 0,$$

where $\tilde{X}(t)$ is the pollution flow, $m(t)$ is the balance of the assets, and $\tilde{\pi}(t)$ is the profit distribution from firms. $\tilde{X}(t)$ and $\tilde{\pi}(t)$ are exogenous for a household. The last inequality in the constraint is a no-Ponzi game condition.

If with a pair of prices $(r^e(t), p^e(t))$, the solutions to the problems (9) and (10) satisfy the market clearing condition and the household fulfils its expectation, they constitute a competitive equilibrium. Formally:

Definition 1: Competitive equilibrium. The tuple

$$(\pi^e(t), X^e(t), y^e(t), K^e(t), B^e(t), c^e(t), m^e(t), \tilde{\pi}^e(t), \tilde{X}^e(t); r^e(t), p^e(t); K_0, B, a(t))$$

is a competitive equilibrium if it satisfies the following conditions:

1. $(\pi^e(t), X^e(t), y^e(t), K^e(t), B^e(t))$ is a solution to the firm's problem (9) given $(r^e(t), p^e(t); B, a(t))$.

2. $(c^e(t), m^e(t))$ is a solution to the household's problem (10) given $(K_0, \tilde{\pi}^e(t), \tilde{X}^e(t), r^e(t))$.

3. $y^e(t) = 0$, $m^e(t) = K^e(t)$, $c^e(t) = F(K^e(t), X^e(t)) - \dot{K}^e(t)$, $X^e(t) = \tilde{X}^e(t)$, $\pi^e(t) = \tilde{\pi}^e(t)$.

The (current value) Hamiltonian for the firm's problem (9) is written as:

$$H^F(K, X, y, B, \zeta, \nu, t) = F(K, X) - r(t)K - p(t)y + \zeta\left[a(t)B - X + y\right] + \nu\left(\phi K - X\right).$$

Therefore, at equilibrium, the following hold:

$$\begin{aligned}&\text{(a) } F_K(K^e(t), X^e(t)) - r^e(t) + \nu(t)\phi = 0;\ \text{(b) } F_X(K^e(t), X^e(t)) - p^e(t) - \nu(t) = 0; \\ &\text{(c) } \dot{p}^e(t) = \left[r^e(t) - a(t)\right]p^e(t);\ \text{(d) } \nu(t)\left(\phi K^e(t) - X^e(t)\right) = 0, \nu(t) \geq 0.\end{aligned} \tag{11}$$

For the household's problem (10), the associated Hamiltonian is written as:

$$H^H(m, c, \lambda, t) = u(c, \tilde{X}(t)) + \lambda(r^e(t)m - \tilde{\pi}(t) - c).$$

At equilibrium, the following hold:

$$\text{(a) } u_c(c^e(t), \tilde{X}^e(t)) - \lambda(t) = 0;\ \text{(b) } \dot{\lambda}(t)/\lambda(t) = \rho - r^e(t). \tag{12}$$

The conditions 1 and 2 in Definition 1 are satisfied if (11) and (12) hold and the no-Ponzi game conditions for the firm and the household are satisfied with equality.

Now, we state the necessary conditions under which a tradable permit system $(B, a(t))$ can achieve the social optimum in a competitive economy.

Proposition 1 *A tradable permit system $(B, a(t))$ can achieve efficiency at a competitive equilibrium only if it satisfies:*

$$a(t) = \rho - \frac{d}{dt}\ln\left[-u_X(c^*(t), X^*(t))\right], \ \textit{and} \tag{13}$$

$$B = \limsup_{T\to\infty}\int_0^T X^*(t)\exp\left[-\int_0^t a(s)ds\right]dt. \tag{14}$$

If $X^*(t) < \phi K^*(t)$, then (13) can be replaced with:

$$a(t) = F_K(K^*(t), X^*(t)) - \frac{d}{dt}\ln\left[F_X(K^*(t), X^*(t))\right]. \tag{15}$$

Proof. Suppose that an equilibrium path is optimal, that is, $(c^e(t), X^e(t), K^e(t)) = (c^*(t), X^*(t), K^*(t))$. For the household, $\tilde{X}^e(t) = X^*(t)$ at equilibrium and (12 a) is satisfied when $\lambda(t) = \lambda^*(t)$ from (4). (12 b) is satisfied with the interest rate $r^e(t) = \rho - \dot{\lambda}^*(t)/\lambda^*(t)$. Because $m^e(t) = K^*(t)$ at equilibrium, the no-Ponzi game condition is satisfied with equality from (8):

$$\liminf_{t\to\infty} m(t) \exp\left[-\int_0^t r(s)ds\right] = \lambda^*(0)^{-1} \liminf_{t\to\infty} \lambda^*(t))K^*(t)e^{-\rho t} = 0. \tag{16}$$

For the firm, (11 a) is satisfied when $\nu(t) = \mu^*(t)/\lambda^*(t)$, which follows from (6) and (12 b):

$$r^e(t) = F_K(K^*(t), X^*(t)) - \frac{\mu^*(t)}{\lambda^*(t)}\phi = F_K(K^e(t), X^e(t)) - \nu(t)\phi.$$

(11 b) is satisfied when:

$$p^e(t) = -u_X(c^*(t), X^*(t))/\lambda^*(t), \tag{17}$$

by (5). With (13), (11 c) is satisfied. This is verified as follows:

$$a(t) = r^e(t) - \frac{\dot{p}^e(t)}{p^e(t)} = \left(\rho - \frac{\dot{\lambda}^*(t)}{\lambda^*(t)}\right) - \left(\frac{d \ln\left[-u_X(c^*(t), X^*(t))\right]}{dt} - \frac{\dot{\lambda}^*(t)}{\lambda^*(t)}\right).$$

With (14), the no-Ponzi game condition for the firm is satisfied with equality, because $\dot{B}^e(t) = a(t)B^e(t) - X^*(t)$ with $B^e(0) = B$ is solved as

$$B - B^e(t)\exp\left(-\int_0^t a(s)ds\right) = \int_0^T X^*(t)\exp\left[-\int_0^t a(s)ds\right]dt. \tag{18}$$

Finally, (15) is immediate from (11). ■

The following three comments are provided in relation to the proposition:

1. As indicated in the introduction, a tradable permit system is a useful policy tool only if the permit interest rate is set appropriately, as in (13) of Proposition 1.

2. Even if no abatement is optimal ($X^*(t) = \phi K^*(t)$), the price of permits is positive, as shown in (17). This indicates that even when pollution *control* is not required, pollution *pricing* is necessary—otherwise, some markets would be distorted. The principle is that pollution should be priced at the aggregate marginal damage at an efficient equilibrium, which is demonstrated for the Pigovian tax in a general equilibrium model by Baumol and Oates (1988, chapter 4). Hence, a permit system

should be implemented over all periods, despite the optimal level of pollution control. Stokey (1998) discovered this fact and showed the following: without pricing of pollution, the capital market is distorted so that it prevents an efficient outcome. Notice that this point does not appear in a static analysis or in a partial equilibrium analysis such as Leiby and Rubin (2001).

3. In general, it would be difficult to find an appropriate path of permit interest rates $a(t)$. However, Proposition 1 provides us with some cases where there is a simple rule for appropriate $a(t)$: (a) the marginal disutility of the pollution $-u_X(c,X)$ is constant. In this case, $a(t)=\rho$; (b) the elasticity of the marginal disutility of the pollution $-Xu_{XX}(c,X)/u_X(c,X)=\gamma>0$ is constant. In this case, $a(t)=\rho-\gamma\dot{X}^*(t)/X^*(t)$; (c) the pollution abatement is necessary for all periods ($X^*(t)<\phi K^*(t)$ for all $t\geq 0$), and the marginal abatement cost of the pollution $F_X(K,X)$ is constant.[4] In this case, $a(t)=r(t)$ and the permit price $p(t)$ will be constant;[5] (d) the pollution abatement is necessary for all periods and the elasticity of the marginal abatement cost $-XF_{XX}(K,X)/F_X(K,X)=\beta>0$ is constant. In this case, $a(t)=r-\beta\dot{X}^*(t)/X^*(t)$. Note that for the cases of (b) and (d), the permit interest rate is greater than the time discount rate and the market interest rate, respectively, if the optimal pollution is decreasing in time ($\dot{X}^*(t)<0$).

2.2 Stock pollution model

In this subsection, we consider a case where the pollution stock affects the utility. Let $S(t)$ denote the pollution stock at time t. We assume that the evolution of the pollution stock is governed by

$$\dot{S}(t)=A(S(t),X(t)),$$

where $X(t)$ is the pollution flow as before. The assimilation function $A:\mathbb{R}^2_+\to\mathbb{R}$ is assumed to be concave with $A_X(S,X)>0$ for all $(S,X)>0$. The instantaneous utility function is modified as $u(c,S)$. The utility function is concave and smooth with $u_c\,(c,S)>0$, $u_S\,(c,S)<0$, $\lim_{c\to 0}u_c\,(c,S)=\infty$, and $\lim_{S\to 0}u_E\,(c,S)=0$ for all $c>0$ and $S>0$. The discount rate ρ and the production technology $F(K,X)$ are the same as before.

[4] Note that the marginal productivity F_X can be interpreted as the marginal abatement cost measured by units of the product.

[5] If the conditions in (a) and (c) are met, then the optimal path is composed of a bang-bang control and a singular control, because the associated Hamiltonian is linear in X. The interiority in (c) implies that the economy is at an interior steady state and $a(t)=r(t)=\rho$.

The social planner's problem is formulated as:

$$V(K_0, S_0) = \max_{c(t), X(t)} \int_0^\infty u(c(t), S(t))e^{-\rho t}dt \tag{19}$$

$$\text{subject to } \dot{K}(t) = F(K(t), X(t)) - c, \ 0 \le X(t) \le \phi K(t), \ K(0) = K_0 > 0, \tag{20}$$

$$\dot{S}(t) = A(S(t), X(t)), \ S(0) = S_0 > 0. \tag{21}$$

The associated Hamiltonian is given by:

$$H(c, X, K, S, \lambda, \xi, \mu) = u(c, S) + \lambda[F(K, X) - c] + \xi A(S, X) + \mu\left(\phi K - X\right).$$

If $(c^*(t), X^*(t), K^*(t), S^*(t))$ is an optimal path, then there exist the costate variables $\lambda^*(t)$ and $\xi^*(t)$ and the Lagrange multiplier $\mu^*(t)$, with which (20), (21) and the following equations hold for each $t \ge 0$:

$$u_c(c^*(t), S^*(t)) - \lambda^*(t) = 0 \tag{22}$$

$$\lambda^*(t)F_X(K^*(t), X^*(t)) - \xi^*(t)A_X(S^*(t), X^*(t)) - \mu^*(t) = 0, \tag{23}$$

$$\dot{\lambda}^*(t)/\lambda^*(t) = \rho - F_K(K^*(t), X^*(t)) + \phi\mu^*(t)/\lambda^*(t), \tag{24}$$

$$\dot{\xi}^*(t)/\xi^*(t) = \rho - A_S(S^*(t), X^*(t)) - u_S(c^*(t), S^*(t))/\xi^*(t) \tag{25}$$

$$\mu^*(t) \ge 0, \text{ and } \mu^*(t)[\phi K^*(t) - X^*(t)] = 0, \tag{26}$$

and the transversality condition:

$$\lim_{t\to\infty} e^{-\rho t}\left[\lambda^*(t)K^*(t) - \xi^*(t)S^*(t)\right] = 0. \tag{27}$$

In a decentralized economy with a tradable permit system $(B, a(t))$, the representative firm solves the problem (9) as in the previous subsection. The representative household's problem is:

$$\max_{c(\cdot)\geq 0}\int_0^{\infty} u(c(t),\tilde{S}(t))e^{-\rho t}dt \tag{28}$$

$$\text{subject to } \dot{m}(t) = r(t)m(t) + \tilde{\pi}(t) - c,\ m(0) = K_0 > 0 \text{ given,}$$

$$\liminf_{t\to\infty} m(t)\exp\left[-\int_0^t r(s)ds\right] \geq 0.$$

A competitive equilibrium in this subsection is defined as follows:

Definition 2: Competitive equilibrium. The tuple:

$$(\pi^e(t), X^e(t), y^e(t), K^e(t), B^e(t), c^e(t), m^e(t), \tilde{\pi}^e(t), \tilde{S}^e(t), S^e(t); r^e(t), p^e(t); K_0, S_0, B, a(t)),$$

is a competitive equilibrium if it satisfies the following conditions:

1. $(\pi^e(t), X^e(t), y^e(t), K^e(t), B^e(t))$ is a solution to the firm's problem (9) given $(r^e(t), p^e(t); B, a(t))$.
2. $(c^e(t), m^e(t))$ is a solution to the household's problem (28) given $(K_0, \tilde{\pi}^e(t), \tilde{S}^e(t), r^e(t))$.
3. $S^e(t)$ is the solution to the initial problem, $\dot{S}(t) = A(S(t), X^e(t))$ with $S(0) = S_0$
4. $y^e(t) = 0$, $m^e(t) = K^e(t)$, $c^e(t) = F(K^e(t), X^e(t)) - \dot{K}^e(t)$, $S^e(t) = \tilde{S}^e(t)$, $\pi^e(t) = \tilde{\pi}^e(t)$.

The counterpart of Proposition 1 is written as:

Proposition 2 *A tradable permit system $(a(t), B)$ can achieve efficiency only if it satisfies:*

$$a(t) = \frac{u_S(c^*(t), S^*(t))}{V_S(K^*(t), S^*(t))} + A_S(S^*(t), X^*(t)) - \frac{d}{dt}\ln\left[A_X(S^*(t), X^*(t))\right], \text{ and} \tag{29}$$

$$B = \limsup_{T\to\infty}\int_0^T X^*(t)\exp\left[-\int_0^t a(s)ds\right]dt, \tag{30}$$

where $V_S = \partial V(K,S)/\partial S$, the partial derivative of the value function defined by (19). If $X^*(t) < \phi K^*(t)$, then (29) can be replaced with (15) in Proposition 1.

Proof. The proof is quite similar to the proof of Proposition 1. Therefore, we address the different points. Suppose that an equilibrium path is optimal, that is, $(c^e(t), X^e(t), K^e(t), S^e(t)) =$

$(c^*(t), X^*(t), K^*(t), S^*(t))$. By (11 b) and (23), we have:

$$p^e(t) = F_X(K^*(t), X^*(t)) - \frac{\mu^*(t)}{\lambda^*(t)} = \frac{\xi^*(t)}{\lambda^*(t)} A_X(S^*(t), X^*(t)).$$

Then, (11 c) implies:

$$a(t) = \left(\rho - \frac{\dot{\lambda}^*(t)}{\lambda^*(t)}\right) - \left(\frac{\dot{\xi}^*(t)}{\xi^*(t)} + \frac{d \ln [A_X(S^*(t), X^*(t))]}{dt} - \frac{\dot{\lambda}^*(t)}{\lambda^*(t)}\right).$$

From (25), we have (29). Notice that $\xi^*(t) = V_S(K^*(t), S^*(t))$ by Léonard (1987). ■

Different from the case of flow pollution, the appropriate permit interest rates are determined not only by the preferences, but also by the shadow price of the pollution stock V_S. This complexity remains even if the assimilation function has a simple form such as $A(S, X) = X$. Therefore, for the case of stock pollution, we have simple rules only if the optimal path is interior ($X^* < \phi K^*$). Then, the rules 3(c) and 3(d) below Proposition 1 are also applicable to the pollution stock case.

2.3 Global warming model

As an important variation of the stock pollution model, this subsection considers a global warming model. The carbon market is an artificial market established under the Kyoto Protocol of the UNFCCC adopted in 1997. It is the largest market in the world for the control of emissions: The size of the global carbon market was over \$60 billion in 2007 and is expected to grow up to \$3.12 trillion by 2020, according to the Point Carbon News (3/11 and 5/22, 2008). A remarkable feature of global warming is the time lag over a few decades between the cause (greenhouse gas emissions) and the consequence (the global climate change), mainly because of the long life of greenhouse gases and the slow response of the oceans to atmospheric warming. There will be "unavoidable climate change" over the next half century, regardless of actions to curb emissions from now (see IPCC 2007, chapter 10). The time lag is far beyond a practical planning period. Reflecting this, we assume that the planning horizon of a tradable permit system is finite $T < \infty$ and the damages suffered within the period are predetermined, whereas the pollution emissions in the period only affect the future generations after the planning period. Formally, we consider the following social planner's problem:

$$\max \int_0^T u(c(t),t)e^{-\rho t}dt + U(K(T),S(T),T)e^{-\rho T}$$

$$\text{subject to } \dot{K}(t) = F(K(t),X(t)) - c,\ 0 \leq X(t) \leq \phi K(t),\ K(0) = K_0 > 0,$$

$$\dot{S}(t) = A(S(t),X(t)),\ S(0) = S_0 > 0.$$

where the value function U values the discounted sum of utilities after period T. We assume that $U(\cdot,\cdot,T)$ is concave and differentiable. The optimal path is characterized by:

$$u_c(c^*(t),t) - \lambda^*(t) = 0 \tag{31}$$

$$\lambda^*(t)F_X(K^*(t),X^*(t)) - \xi^*(t)A_X(S^*(t),X^*(t)) - \mu^*(t) = 0, \tag{32}$$

$$\dot{\lambda}^*(t)/\lambda^*(t) = \rho - F_K(K^*(t),X^*(t)) + \phi\mu^*(t)/\lambda^*(t), \tag{33}$$

$$\dot{\xi}^*(t)/\xi^*(t) = \rho - A_S(S^*(t),X^*(t)) \tag{34}$$

$$\mu^*(t) \geq 0,\ \mu^*(t)[\phi K^*(t) - X^*(t)] = 0,\ \text{and} \tag{35}$$

$$\lambda^*(T) = U_K(K^*(T),S^*(T),T),\ \xi^*(T) = U_S(K^*(T),S^*(T),T). \tag{36}$$

In the decentralized economy with a tradable permit system $(B,a(t))$, the representative firm solves:

$$\max_{K(t),X(t),y(t)} \int_0^T \pi(t)\exp\left(-\int_0^t r(s)ds\right)dt \tag{37}$$

$$\text{subject to } \pi(t) = F(K(t),X(t)) - r(t)K(t) - p(t)y(t)$$

$$\dot{B}(t) = a(t)B(t) - X(t) + y(t),\ B(0) = B,$$

$$K(t) \geq 0, X(t) \in [0,\phi K(t)],\ \text{and } B(T) \geq 0.$$

The representative household solves:

$$\max_{c(\cdot)\geq 0} \int_0^T u(c(t),t)e^{-\rho t}dt + U(m(T),\tilde{S}(T),T)e^{-\rho T} \tag{38}$$

$$\text{subject to } \dot{m}(t) = r(t)m(t) + \tilde{\pi}(t) - c,\ m(0) = K_0 > 0 \text{ given.}$$

At equilibrium, (11) and the following hold:

$$B^e(T) \geq 0, \ p^e(T)B^e(T) = 0 \tag{39}$$

$$\text{(a) } u_c(c^e(t),t) - \lambda(t) = 0, \text{ (b) } \dot{\lambda}(t)/\lambda(t) = \rho - r^e(t), \text{ (c) } \lambda(T) = U_K(K^e(T), \tilde{S}^e(T), T). \tag{40}$$

Therefore, we have:

Proposition 3 *A tradable permit system $(B, a(t))$ can achieve efficiency only if it satisfies:*

$$a(t) = A_S(S^*(t), X^*(t)) - \frac{d}{dt}\ln\left[A_X(S^*(t), X^*(t))\right], \ \textit{and} \tag{41}$$

$$B = \int_0^T X^*(t)\exp\left[-\int_0^t a(s)ds\right]dt. \tag{42}$$

The proof is almost the same as the proof of Proposition 2 and is thus omitted.

Now the permit interest rate $a(t)$ depends only on the assimilation function $A(S, X)$. A simple rule on $a(t)$ is obtained if we can approximate $A(S, X)$ by a linear function:

$$A(S, X) = \theta(S - \bar{S}) + X,$$

where the positive constant θ is the assimilation factor and $\bar{S}$ is a threshold at which the assimilation mechanism changes from a negative feedback to a positive feedback when S exceeds $\bar{S}$. The rule is to set $a(t) = \theta$. In particular, if $\theta = 0$, then we have $a(t) = 0$, which is used in practice. Furthermore, notice that if $\theta = 0$, then $S(T) = \int_0^T X(t)dt$. That is the case where the total emissions aggregated over the planning period matter. However, for the case of global warming, $\theta \neq 0$ and it is not the total emissions over time, but the concentration of greenhouse gases that affects the global climate system.[6] Therefore, the appropriate permit interest rates to a carbon emission allowance cannot be zero.

[6] Nordhaus (1999) presented informative simulation results comparing the costs incurred by various policy targets such as the emissions, the concentrations, and the global temperatures.

3 Tradable permit system with a permit bank

The virtue of a tradable permit system is to fully make use of a market as a device for efficient resource allocation and to save government intervention for achieving efficiency. From this point of view, it would be desirable if we could design a tradable permit system without setting permit interest rates $a(t)$. We can show that this is possible if we establish a permit bank as follows. In the bank, each permit holder has an account of permits. Deposits and withdrawals are made in terms of permits, whereas the balance is expressed in monetary terms by multiplying the quantity of permits and the market price. The prevailing market interest rates are applied.

A tradable permit system with a permit bank is represented by the initial distribution of the permits B, because now the permit interest rate $a(t)$ does not need to be set exogenously. Denote by $Q(t)$ the monetary value of the permits held by the representative firm at time t, i.e., $Q(t) = p^e(t)B(t)$. The state equation and the no-Ponzi game condition in the firm's problem (9) are replaced with their monetary units counterpart:

$$\dot{Q}(t) = r(t)Q(t) - p(t)X(t) + p(t)y(t), \; Q_0 = Q(0) = p(0)B \tag{43}$$

and

$$\liminf_{t\to\infty} Q(t) \exp\left(-\int_0^t r(s)ds\right) \geq 0$$

The associated Hamiltonian is written as:

$$H^F(K, X, y, B, \zeta, \nu, t) = F(K, X) - r(t)K - p(t)y + \eta\left[r(t)Q(t) - p(t)X(t) + p(t)y(t)\right] + \nu\left(\phi K - X\right).$$

Therefore, the following hold at equilibrium:

$$\begin{aligned}
&\text{(a) } F_K(K^e(t), X^e(t)) - r^e(t) + \nu(t)\phi = 0; \text{ (b) } F_X(K^e(t), X^e(t)) - p^e(t) - \nu(t) = 0; \\
&\text{(c) } \nu(t)\left(\phi K^e(t) - X^e(t)\right) = 0, \nu(t) \geq 0, \text{ (d) } \liminf_{t\to\infty} Q^e(t)\exp\left(-\int_0^t r^e(s)ds\right) = 0; \\
&\text{(e) } y^e(t) = 0.
\end{aligned} \tag{44}$$

For the household, there exists the costate variable $\lambda(t)$ and the following hold at equilibrium as before:

$$\begin{aligned} &\text{(a) } u_c - \lambda(t) = 0; \text{ (b) } \dot{\lambda}(t)/\lambda(t) = \rho - r^e(t); \\ &\text{(c) } \liminf_{t\to\infty} K^e(t) \exp\left(-\int_0^t r(s)ds\right) = 0, \end{aligned} \tag{45}$$

where $u_c = u_c(c^e(t), X^e(t))$ for the flow pollution model and $u_c = u_c(c^e(t), \tilde{S}^e(t))$ for the stock pollution model. Then, we have:

Proposition 4 *With a permit bank, a tradable permit system B can achieve an optimal path for both flow and stock pollution models if the initial distribution of permits B satisfies:*

$$B = \frac{1}{p^*(0)} \limsup_{T\to\infty} \int_0^T p^*(t) X^*(t) \exp\left[-\int_0^t r^*(s)ds\right] dt.$$

The interest rates $r^*(t)$ and the permit prices $p^*(t)$ are given by:

$$r^*(t) = \rho - \frac{d}{dt} \ln u_c$$

and

$$p^*(t) = \begin{cases} -u_X/u_c & \textit{for the flow pollution model} \\ \xi^*(t) A_X & \textit{for the stock pollution model} \end{cases},$$

where all functions are evaluated at the optimum and $\xi^*(t)$ is defined by the system (22)–(27).

Proof. Let r^* and p^* be the equilibrium prices. Suppose that the household expects the path of pollution as $\tilde{X}(t) = X^*(t)$ for the flow pollution model and $\tilde{S}(t) = S^*(t)$ for the stock pollution model. For the flow pollution model, there are the costates $\lambda^*(t)$ and $\mu^*(t)$ and the optimal controls $c^*(t)$ and $X^*(t)$ satisfies (5)–(8). Then, it is easy to see that the optimal path satisfies the sufficient conditions for the individual optimality of the firm (44) and the household (45). Notice that the firm's transversality condition is satisfied because (43) implies:

$$Q(T) \exp\left(-\int_0^T r^*(t)dt\right) = Q_0 - \int_0^T p^*(t) X^*(t) \exp\left(-\int_0^t r^*(s)ds\right) dt.$$

The same argument is applied to the stock pollution model. ■

Proposition 4 is a desirable result. Once a permit bank is established, then the government need not be bothered with finding appropriate permit interest rates. Instead of the government, a market finds them. The idea of a permit bank comes from the analogy between permits and money mentioned in the introduction. With a permit bank, tradable permits are similar to money. The above equilibrium model somewhat resembles macro models with money such as cash-in-advance and money-in-utility models. (see, for example, Woodford, 1994 and Benhabib et al., 2001).

4 Equilibrium dynamics

4.1 Model

The previous two sections examined how a tradable permit system can achieve efficiency. This section examines whether it can be done *with certainty*. We analyze the equilibrium dynamics when a tradable permit system is introduced in a competitive economy. Because the analysis in the general setup is difficult,[7] we adopt a specific flow pollution model. The instantaneous utility is additively separable and isoelastic:

$$u(c, X) = \frac{c^{1-\sigma} - 1}{1-\sigma} - \beta \frac{X^{\gamma+1}}{\gamma+1}, \text{ where } \sigma > 0, \beta > 0, \text{ and } \gamma > 0. \tag{46}$$

The production technology is Cobb-Douglas:

$$F(K, X) = K^{\alpha} X^{1-\alpha} \text{ and } X \leq \phi K, \text{ where } 0 < \alpha < 1 \text{ and } \phi \geq (\rho/\alpha)^{1/(1-\alpha)}. \tag{47}$$

Thus, the social planner's problem becomes:

$$\max \int_0^{\infty} \left[\frac{c^{1-\sigma} - 1}{1-\sigma} - \beta \frac{X^{\gamma+1}}{\gamma+1} \right] e^{-\rho t} dt \tag{48}$$

$$\text{subject to } \dot{K}(t) = K^{\alpha} X^{1-\alpha} - c, \ X \leq \phi K, \text{ and } K(0) = K_0 \text{ given.}$$

The detailed analysis of this model can be found in Stokey (1998). Throughout this section, we consider only an interior equilibrium path and ignore the pollution constraint $X \leq \phi K$.

[7] As seen below, for the case of a tradable permit system with a permit bank, the dynamics are not completely described by a system of differential equations.

4.2 Case of a tradable permit system with a permit bank

We first examine a tradable permit system with a permit bank. By (43), (44) and (45), the equilibrium dynamics are given by:

$$\begin{aligned}
&\text{(a) } \dot{K}(t) = F(K(t), X(t)) - c(t),\ K(0) = K_0; \\
&\text{(b) } \liminf_{t\to\infty} K(t) \exp\left(-\int_0^t F_K(K(s), X(s))ds\right) = 0; \\
&\text{(c) } \dot{c}(t)/c(t) = \sigma^{-1}\left[F_K(K(t), X(t)) - \rho\right]; \\
&\text{(d) } \dot{Q}(t) = F_K(K(t), X(t))Q(t) - F_X(K(t), X(t))X(t),\ Q(0) = F_X(K_0, X(0))B_0; \\
&\text{(e) } \liminf_{t\to\infty} Q(t) \exp\left(-\int_0^t F_K(K(s), X(s))ds\right) = 0.
\end{aligned} \tag{49}$$

These equilibrium dynamics are unusual because they lack a differential equation for the evolution of the pollution flow $X(t)$. Instead, the evolution of $X(t)$ is governed by an integral equation:

$$B_0 = F_X(K_0, X(0))^{-1} \int_0^\infty F_X(K(t), X(t))X(t) \exp\left(-\int_0^t F_K(K(s), X(s))ds\right) dt, \tag{50}$$

which is obtained from (49 d) and (49 e).

To investigate the equilibrium dynamics, let $r = F_K(K, X)$. Then, $F(K, X) = \alpha^{-1}rK$ and $F_X(K, X) = [(1-\alpha)/\alpha]r(K/X)$. Using these, the dynamics are:

$$\begin{aligned}
&\text{(a) } \dot{K}(t) = \alpha^{-1}r(t)K(t) - c(t),\ K(0) = K_0; \\
&\text{(b) } \liminf_{t\to\infty} K(t) \exp\left(-\int_0^t r(t)ds\right) = 0; \\
&\text{(c) } \dot{c}(t)/c(t) = \sigma^{-1}\left[r(t) - \rho\right],
\end{aligned} \tag{51}$$

and

$$B_0 = \frac{r(0)X(0)}{K_0} \int_0^\infty r(t)K(t) \exp\left(-\int_0^t r(s)ds\right) dt. \tag{52}$$

Given a path of interest rates $r(t)$, the system (51) determines $c(t)$ and $K(t)$. The equilibrium path of $X(t)$ is determined by the equation $r = F_K(K, X) = \alpha(X/K)^{1-\alpha}$, i.e.,

$$X(t) = \left(\frac{r(t)}{\alpha}\right)^{\frac{1}{1-\alpha}} K(t). \tag{53}$$

To keep (53) compatible with (52), we need to adjust $r(0)$, which does not affect the equilibrium paths. This suggests that with a tradable permit system with a bank, a competitive equilibrium may be indeterminate. The formal statement and its proof are as follows.

Let $g(t) = \dot{c}(t)/c(t)$. We impose the following growth condition on g which is necessary for a household to have a finite discount sum of utilities:

$$\text{Condition G: } \int_0^\infty \rho - (1-\sigma)g(s)ds = \infty.$$

Then, we have:

Proposition 5 *For any measurable function $g(t)$ satisfying the condition G, there is an equilibrium path satisfying (49 a)–(49 e) that is characterized by:*

$$c(0) = c_0 := \frac{K_0}{\int_0^\infty \exp\left(-\int_0^t \frac{r(s)-\alpha g(s)}{\alpha} ds\right) dt}, \quad c(t) = c_0 \exp \int_0^t g(s)ds, \tag{54}$$

$$K(t) = \left[K_0 - \int_0^t c(s) \exp\left(-\int_0^s \frac{r(v)}{\alpha} dv\right)\right] \exp\left(\int_0^t \frac{r(s)}{\alpha} ds\right) \text{ for all } t > 0, \tag{55}$$

$$X(t) = \left(\frac{r(s)}{\alpha}\right)^{\frac{1}{1-\alpha}} K(t) \text{ for all } t > 0, \tag{56}$$

$$X(0) = X_0 := \left[\frac{(1-\alpha)B_0}{\int_0^\infty c_0 \exp\left(-\int_0^t r(s) + \sigma g(s)ds\right) dt - K_0}\right]^{\frac{1}{\alpha}} K_0, \tag{57}$$

where

$$r(t) = \rho + \sigma g(t). \tag{58}$$

Proof. We show that (54)–(57) are a solution of the system of (49 a)–(49 e). Take arbitrarily $g(t)$ and let $c(t) = c_0 \exp \int_0^t g(s)ds$ as in the proposition. Because $F_K = r$ by (49 c) and (58), the initial value problem (49 a) is solved as (55). $K(t)$ must satisfy the transversality condition (49 b). This is equivalent to:

$$\lim_{t\to\infty} \left(K_0 - \int_0^t c_0 \exp\left(-\int_0^s \frac{r(v)}{\alpha} - g(v)dv\right) ds\right) \exp\left(\int_0^t \frac{(1-\alpha)r(s)}{\alpha} ds\right) = 0.$$

As a necessary condition for this equation, we have (54). Indeed, (54) is sufficient, too. To see this, apply l'Hopital's rule:

$$\lim_{t\to\infty}\frac{K_0-\int_0^t c_0\exp\left(-\int_0^s\frac{r(v)}{\alpha}-g(v)dv\right)ds}{\exp\left(-\int_0^t\frac{1-\alpha}{\alpha}r(s)ds\right)}=\lim_{t\to\infty}\frac{\alpha c_0}{(1-\alpha)\,r(t)}\exp\left(-\int_0^t r(s)-g(s)ds\right)=0.$$

Note that $\lim_{t\to\infty} r(t) = 0$ is ruled out by the condition G. (56) follows from $r = F_K(K, X) = \alpha(X/K)^{1-\alpha}$. The rest of the proof shows that (57) is equivalent to (50). Denote by $\tilde{C}$ the lifetime discount sum of consumptions, i.e.,

$$\tilde{C}=\int_0^\infty c(t)\exp\left(-\int_0^t r(s)ds\right)dt.$$

Let:

$$Q=\int_0^\infty F_X(K(t),X(t))X(t)\exp\left(-\int_0^t F_K(K(s),X(s))ds\right)dt.$$

Because $F_X X = (1-\alpha)F = (1-\alpha)(\dot{K}+c)$, $F_K = r$, and $rK = \alpha F$ by (47), (49 a), and (49 c), this equation is modified as:

$$\begin{aligned}
Q&=(1-\alpha)\int_0^\infty F(K(t),X(t))\exp\left(-\int_0^t r(s))ds\right)dt\\
&=(1-\alpha)\left\{\int_0^\infty \dot{K}(t)\exp\left(-\int_0^t r(s))ds\right)dt+\tilde{C}\right\}\\
&=(1-\alpha)\left\{\lim_{t\to\infty}K(t)\exp\left(-\int_0^t r(s))ds\right)-K_0+\int_0^\infty r(t)K(t)\exp\left(-\int_0^t r(s))ds\right)dt+\tilde{C}\right\}\\
&=(1-\alpha)\left\{-K_0+\alpha\int_0^\infty F(K(t),X(t))\exp\left(-\int_0^t r(s))ds\right)dt+\tilde{C}\right\}\\
&=(1-\alpha)\left\{-K_0+\frac{\alpha}{1-\alpha}Q+\tilde{C}\right\},
\end{aligned}$$

where the third line uses integration by parts and the fourth line uses (49 b). Therefore, we have $Q = \tilde{C} - K_0$. Because $F_X(K, X) = (1-\alpha)(K/X)^\alpha$, (50) is equivalent to (58). Note that (50) is equivalent to (49 d and e). ■

In Proposition 5, only the growth condition G is imposed on the consumption growth rate g. Therefore, there are uncountable equilibrium paths. This indeterminacy is sometimes called global indeterminacy, distinguished from local indeterminacy; the indeterminacy of the convergent paths to a unique steady state. Under a global indeterminacy, we cannot ensure efficiency of an equilibrium path even in the long run.

4.3 Tradable permit system with permit interest rates

For the case of a tradable permit system with permit discount rates, regulatory authority sets a path of permit interest rates $a(t)$. By (43), (44) and (45), the equilibrium dynamics are given by:

$$\begin{aligned}
&\text{(a) } \dot{K}(t) = F(K(t), X(t)) - c(t),\ K(0) = K_0; \\
&\text{(b) } \liminf_{t\to\infty} K(t)\exp\left(-\int_0^t F_K(K(s), X(s))ds\right) = 0; \\
&\text{(c) } \dot{c}(t)/c(t) = \sigma^{-1}\left[F_K(K(t), X(t)) - \rho\right]; \\
&\text{(d) } (d/dt)(\ln F_X(K(t), X(t)) = F_K(K(t), X(t)) - a(t); \\
&\text{(e) } B_0 - \liminf_{t\to\infty} \int_0^t X(s)\exp\left(-\int_0^s a(v)dv\right) ds = 0,
\end{aligned} \tag{59}$$

where $a(t)$ and B_0 are given by (13) and (14) in Proposition 1, respectively. In contrast to the equilibrium dynamics (49) for a tradable permit system with a permit bank, this system contains a differential equation for $X(t)$ in (59 d).

We examine the uniqueness for a special case where the initial capital stock equals an optimal steady state of the social planner's problem (48). The optimal steady state is uniquely given by:

$$K_{ss} = \left[\frac{1-\alpha}{\beta}\left(\frac{\rho}{\alpha}\right)^{-\frac{\alpha(1-\sigma)+(\sigma+\gamma)}{1-\alpha}}\right]^{\frac{1}{\sigma+\gamma}},\ X_{ss} = \left[\frac{1-\alpha}{\beta}\left(\frac{\rho}{\alpha}\right)^{-\frac{\alpha(1-\sigma)}{1-\alpha}}\right]^{\frac{1}{\sigma+\gamma}}. \tag{60}$$

Notice that $F_K(K_{ss}, X_{ss}) = \rho$ holds. Therefore, the regulatory authority designs the tradable permit system $(B_0, a(t))$ to satisfy:

$$a(t) = \rho \text{ and } B_0 = X_{ss}/\rho. \tag{61}$$

Let $z(t) = K(t)/X(t)$ and $z_{ss} = K_{ss}/X_{ss} = (\rho/\alpha)^{-1/(1-\alpha)}$. Then, (59 c) and (59 d) with (61) imply that the consumption path is written as $c(t) = \omega z(t)^{\alpha/\sigma}$ with $\omega > 0$. Now the equilibrium dynamics are:

$$\begin{aligned}
&\text{(a) } \dot{K}(t) = z(t)^{-(1-\alpha)}K(t) - \omega z(t)^{\alpha/\sigma},\ K(0) = K_{ss}; \\
&\text{(b) } \lim_{t\to\infty} K(t)\exp\left(-\alpha \int_0^t z(s)^{-(1-\alpha)}ds\right) = 0; \\
&\text{(c) } \dot{z}(t)/z(t) = z(s)^{-(1-\alpha)} - \rho/\alpha; \\
&\text{(d) } B_{ss} - \int_0^\infty [K(t)/z(t)]\, e^{-\rho t}dt = 0.
\end{aligned} \tag{62}$$

The system of differential equations (62 a) and (62 c) has a unique steady state (K_∞, z_∞) which depends on the value of ω:

$$(K_\infty, z_\infty) = \left(\omega z_{ss}^{(1-\alpha+\alpha/\sigma)}, z_{ss}\right). \tag{63}$$

It is easily verified that the steady state is a saddle point. Figure 1 depicts the phase diagram. If the initial value is not on the stable manifold, either the capital stock is exhausted in a finite time period or grows unboundedly. For the former case, there is no production after the capital stock is used up, which cannot be chosen as an equilibrium path. For the latter case, from (62 a) the growth rate of capital $\dot{K}/K$ is approximately equal to $z_{ss}^{-(1-\alpha)}$ in the long run. Then, for a sufficient large T:

$$\begin{aligned}
[K(t)/z(t)]\, e^{-\rho t} &\approx [K(T)/z_{ss}]\exp\left[\left(-\rho + z_{ss}^{-(1-\alpha)}\right)(t-T)\right] \\
&= [K(T)/z_{ss}]\exp\left[\frac{(1-\alpha)\,\rho}{\alpha}(t-T)\right] \to \infty \text{ as } t\to\infty.
\end{aligned}$$

Therefore, the transversality condition (62 d) is violated. As a result, an equilibrium path must be on the stable manifold and converges to the steady state (K_∞, z_∞).

Using these results, we can show that the equilibrium path satisfying (62 a)–(62 d) is unique.

Proposition 6 *Suppose that the initial stock of capital is K_{ss} and the regulatory authority adopts a tradable permit system $(B_0, a(t))$ given by (61). Then a competitive equilibrium is unique and coincides with the socially optimal path.*

Proof. Recall that the steady state (K_∞, z_{ss}) of the system of differential equations (62 a) and (62 c) depends on ω. If $\omega = K_{ss}/z_{ss}^{(1-\alpha+\alpha/\sigma)}$, then $K_\infty = K_{ss} = K_0$ and the equilibrium path keeps the initial stock level, which is socially optimal. If $\omega > K_{ss}/z_{ss}^{(1-\alpha+\alpha/\sigma)}$, then $K_\infty > K_{ss}$. This case is

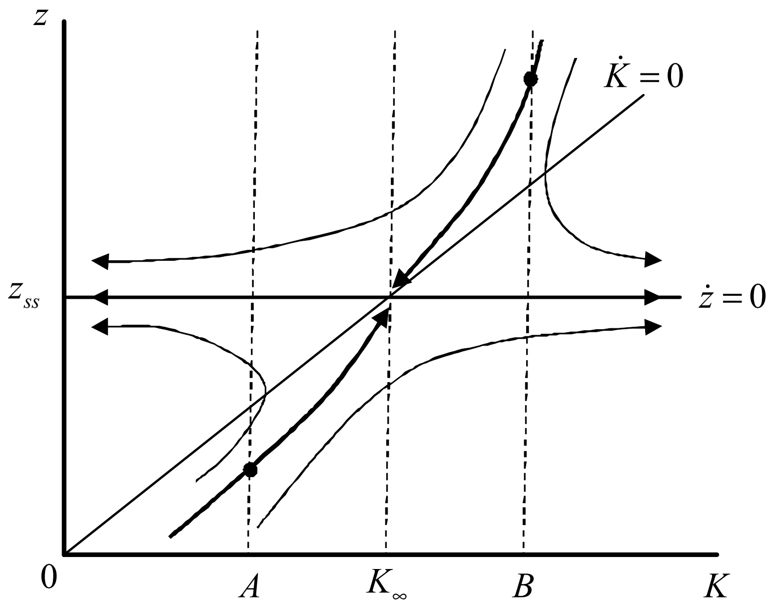

Figure 1: Figure 1 Phase diagram of the equilibrium paths

illustrated in Figure 1 where point A is K_{ss}. Because the equilibrium path goes up along the stable manifold, $z(t) < z_{ss}$ and $K(t) > K_{ss}$ for all $t \geq 0$. This implies that (62 d) is not satisfied:

$$\int_0^\infty [K(t)/z(t)]\, e^{-\rho t} dt > \int_0^\infty (K_{ss}/z_{ss})\, e^{-\rho t} dt = B_{ss}. \tag{64}$$

Therefore, this case is ruled out. Consider the case of $\omega < K_{ss}/z_{ss}^{(1-\alpha+\alpha/\sigma)}$. This time, point B on Figure 1 is K_{ss}. The symmetric argument concludes that this case is ruled out, too. Therefore, we have a unique $\omega = K_{ss}/z_{ss}^{(1-\alpha+\alpha/\sigma)}$ and a unique equilibrium path satisfying (62) that coincides with the optimal path of the social planner's problem. ■

5 Concluding remarks

The creation of an artificial market by means of a tradable permit system is a prospective prescription against market failure. However, in an intertemporal economy, we need an additional device to ensure that the efficient prices of permits satisfy the intertemporal arbitrage condition. We dealt with this problem in two ways: to extrapolate the appropriate permit interest rates and to create a permit bank.

For the former, we sought a set of conditions under which the appropriate permit interest rates are given by a simple rule. We obtained the following results:

1. If pollution flows affect utility and the marginal disutility is constant, then the permit interest rates equal the time discount rate ($a(t) = \rho$).

2. If pollution flows affect utility and the elasticity of the marginal disutility is constant $\gamma > 0$, then $a(t) = \rho - \gamma \dot{X}^*(t)/X^*(t)$, where $X^*(t)$ is the optimal path of the pollution flows. In particular, if $\dot{X}^*(t) < 0$, then $a(t) > \rho$.

3. If the optimal path of pollution flows is interior over time and the marginal abatement cost of pollution is constant, then the permit interest rates equal the interest rates ($a(t) = r(t)$). This rule is applicable to both cases where pollution flows and pollution stocks affect utility.

4. If the optimal path of pollution flows is interior over time and the elasticity of the marginal abatement cost is constant $\beta > 0$, then $a(t) = r(t) - \beta \dot{X}^*(t)/X^*(t)$. In particular, if $\dot{X}^*(t) < 0$, then $a(t) > r(t)$. This rule is applicable to both cases where pollution flows and pollution stocks affect utility.

5. For the global warming case, if the natural assimilation function can be approximated by a linear function, then $a(t) = \theta$. Here, a positive constant θ is the assimilation factor to the greenhouse gasses in the atmosphere.

We showed that a tradable permit system with a permit bank can achieve efficiency without governmental intervention such as setting the permit interest rates. However, indeterminacy of equilibria may arise. While the indeterminacy can be a serious problem because one cannot ensure an efficient equilibrium, it is not indigenous to this policy measure. The indeterminacy can emerge in a laissez faire economy as well as an economy regulated with an environmental tax. (see Itaya, 2008.) How to tame the indeterminacy is a future research agenda.

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