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Optimal Sharing Rules in Repeated Partnerships*

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Abstract

We study a simple model of repeated partnerships with noisy outcomes. Two partners first choose a sharing rule, under which they start their repeated interaction. We characterize the sharing rule which supports the most efficient equilibrium, and show that it suffices to consider two particular sharing rules. One is an asymmetric sharing rule, which induces only a more productive partner to work. It is optimal for impatient or less productive partners. The other treats them more evenly, and prevails for more productive and patient partners. Those results indicate how technological parameters and patience determine the role of a more productive partner. If the partners become more productive or more patient, the productive partner ceases to be a residual claimant and sacrifices his own share, in order to foster teamwork.

JEL Classification Numbers: C72, C73, L23, P13.

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1 Introduction

Free-riding is a major obstacle to efficient production in partnerships. Since it is inevitable that contributions of one partner's effort leak to other partners, it is generally impossible to design a budget-balancing sharing rule which attains efficiency (Holmstrom (1982)).

To the extent that partners engage in production repeatedly, one might expect that efficient production is sustainable due to the folk theorem. However, a classic paper by Radner, Myerson and Maskin (1986) shows that there could remain considerable efficiency loss even under repeated play, if the output of partnership is merely a noisy indicator of their effort levels. Indeed, the negative result by Radner, Myerson and Maskin (1986) has stimulated research on general repeated games with imperfect monitoring, and it has turned out to crucially depend on the assumption that the set of possible output levels is too small in comparison with the set of effort levels (Matsushima (1989), Fudenberg, Levine and Maskin (1994)). Nevertheless, their model is simple and tractable, and quite intuitive as to why partnerships are subject to efficiency loss even under long-term relationships.

The model of Radner, Myerson and Maskin (1986) is limited to identical partners and equal sharing rules. Our objective in this paper is to extend their model to the case where heterogenous partners can choose and commit to a sharing rule of their outputs at the beginning of their repeated interaction. We analyze a simple model with two partners, each of whom has only two alternatives of working or shirking in each period. The consequence of their choice of actions is binary, good or bad, which is stochastically decided. Since the outcome is only an aggregate, noisy signal of their efforts, incentives are difficult to provide, as is first pointed out by Alchian and Demsetz (1972). Using this model, we ask what is the most efficient outcome partners can sustain as an equilibrium, and what sharing rule supports it.

We prove that, unless the partnership is so productive that the partners can be induced to work even in a one-shot environment, either one of the following two particular sharing rules is always optimal. One is an asymmetric sharing rule, where only a partner whose effort has a greater value to the team is induced to work. The asymmetric sharing rule is optimal if the partners are impatient or if the partnership is not productive enough, and it is the best arrangement in one-shot partnerships. In other words, repeated play does not help at all in this case. The other sharing rule treats partners more evenly, and the period game under the sharing rule forms a prisoners' dilemma. In particular, it becomes equal sharing if the partners are identical. This symmetry-oriented sharing rule prevails if the partnership is more productive and if the partners are sufficiently patient, and achieves an outcome which cannot be sustained without repeated play.

Those results imply that the role of a more productive partner crucially depends on technological parameters and patience. If partners are neither patient nor productive as a whole, then the productive partner becomes a residual claimant of the team; he makes sole efforts and has the entire share. However, if the partners become more productive or more patient, he ceases to be a residual claimant and sacrifices his own share, in order to foster teamwork.

The symmetry-oriented sharing rule and the corresponding second-best outcome have two empirically interesting properties. First, suppose one partner's effort is superior to the other's, in terms of both the incremental probability of a good outcome and the cost of efforts. Then the more efficient partner has a *smaller* share than the other. Furthermore, the partner receives less in the second-best equilibrium. This is because the more efficient partner has a weaker short-run incentive to shirk. Thus it achieves an efficient allocation of incentives to make his share smaller. The result implies that, if the outside option to the more efficient partner is more favorable, he must receive some compensation, such as monetary transfer or status. This seems to be consistent with Frank's (1984) empirical finding that workers with apparently greater productivity in working organizations receive less than their marginal products, possibly being compensated with status.

Second, if the partners with equal effort costs become more homogeneous in the sense that their difference in the incremental probabilities of success shrinks, then the payoff sum under the second-best equilibrium increases. Namely, homogeneity between partners results in a better performance of the team. Hansmann (1996) argues that viability of the employee ownership crucially depends on its members' homogeneity, since it reduces the costs of collective decision making. Our result implies that the provision of incentives works best under homogeneity, and reinforces Hansmann's observation.

Our model assumes that partners commit to a sharing rule with budget balance, which is in force every period. However, our methodology covers the case where they can commit to a dynamic arrangement where the sharing rule of a given period may depend on the past and may exhibit budget-breaking. We show that such extensions never improve what sufficiently patient partners can sustain under the basic model. However, we also show that moderately patient partners benefit from the arrangement where they may break budget depending on the past outcomes. We later discuss implications of this observation on the corporate governance problem.

Since our model has only two actions and two signals in the period game, the folk theorem by Fudenberg, Levine and Maskin (1994) does not apply. Our equilibrium analysis rather shows that efficiency loss is unavoidable even if the partners are very patient. At the same time, our analysis clearly identifies what is the second-best outcome and what sharing rule sustains it, for any level of discount factor.

Our model is closely related to that of Rayo (2007), who also analyzes repeated team production problems. A main difference is that Rayo (2007) assumes that (i) individual signals for partners' efforts are additionally available, and (ii) since those signals are unverifiable, they can provide incentives only through relational contracts (Levin (2003)). Rayo (2007) makes a similar observation that shares can be quite different depending on productivity and observability, but the analysis is mainly focused on interactions between explicit sharing rules and implicit relational contracts. By comparison, our approach is much more explicit on the form of optimal sharing rules and its dependence on various parameters.[1]

The rest of this paper is organized as follows. In Section 2, we introduce a model. In Section 3, we characterize what the partners can do when their relationship is just

[1]Under the framework of repeated multi-agency with individual signals, Che and Yoo (2001) prove that, depending on parameters, the optimal contract exhibits either joint evaluation or relative-performance evaluation. All those results suggest a set of characteristics of optimal incentive schemes that exploit the repeated interaction of agents.

one-shot. We consider the case of repeated interaction and prove our main results in Section 4. Economic implications of the results are presented in Section 5, and discussions on possible extensions are provided in Section 6.

2 Model

Two risk-neutral partners, Partner 1 and Partner 2, are engaged in joint production. Each period, they simultaneously decide whether to work hard for the team or not. We denote their sets of actions by $\{W, S\}$, where W denotes "Work" and S denotes "Shirk." For each Partner i, it costs $c_i > 0$ to choose W, while it is costless to choose S. The outcome of the team production is binary, which is either good (success) or bad (failure), and it stochastically depends on actions of the two partners. We denote the set of outcomes by $\{G, B\}$, where G denotes "Good" and B denotes "Bad."

Let p^a be the probability of success when the two partners play $a = (a_1, a_2) \in \{W, S\} \times \{W, S\}$. Here we assume

$$\begin{aligned}
p^{W,W} &= \alpha + \gamma, \\
p^{W,S} &= \beta_1 + \gamma, \\
p^{S,W} &= \beta_2 + \gamma, \\
p^{S,S} &= \gamma.
\end{aligned}$$

Namely, γ is a basic probability of success, which applies even if no partner works. β_i is the incremental success probability by Partner i's sole effort, and α is the one by both partners' efforts.

If the outcome is good, it yields a monetary income of $X > 0$, which is to be divided between the two partners. A bad outcome means a zero monetary income. The partners commit to a *sharing rule*, which prescribes a division of the monetary income brought about by a successful outcome. Following the standard literature on partnerships (for example, Holmstrom (1982)), we assume budget balance on the sharing rule.[2] Thus we define a sharing rule as a pair (s_1, s_2) such that:

$$\begin{aligned}
&s_1 + s_2 = X, \\
&\min\{s_1, s_2\} \geq 0.
\end{aligned} \tag{1}$$

For simplicity, we exclude negative shares.

Given a sharing rule $s = (s_1, s_2)$, the partners' strategic interaction within a period is represented by the following normal-form game.

		Partner 2: W	Partner 2: S
Partner 1	W	$s_1(\alpha+\gamma) - c_1, s_2(\alpha+\gamma) - c_2$	$s_1(\beta_1+\gamma) - c_1, s_2(\beta_1+\gamma)$
	S	$s_1(\beta_2+\gamma), s_2(\beta_2+\gamma) - c_2$	$s_1\gamma, s_2\gamma$

Figure 1: The Period Game under a Sharing Rule s

We denote the payoff function of Partner i of this game with a sharing rule s by

[2] Subsection 6.2 considers the case where the partners may break budget.

$u_i^s(a_1, a_2)$, where a_k is Partner k's action. We define $u^s(a_1, a_2) = (u_1^s(a_1, a_2), u_2^s(a_1, a_2))$, and $U(a_1, a_2) = u_1^s(a_1, a_2) + u_2^s(a_1, a_2)$. Note that $U(a_1, a_2)$ does not depend on s.

In what follows, we maintain the following assumptions.

Assumption 1 *We assume:*

$$\alpha \geq \beta_1 + \beta_2, \tag{2}$$

$$X\beta_i > c_i, \quad i = 1, 2, \tag{3}$$

$$1 > \alpha + \gamma, \quad \gamma > 0, \tag{4}$$

$$X\beta_1 - c_1 \geq X\beta_2 - c_2. \tag{5}$$

(2) is our central assumption of complementarity, which states that both partners' combined efforts have greater effects on the increment of probability of success than the sum of their individual efforts. This assumption embodies the benefit of team production in the spirit of Alchian and Demsetz (1972).[3] (3) is, together with (2), a standard assumption that each partner's effort is socially efficient, which implies that it is efficient for both partners to work. (4) implies that the outcome is merely a noisy signal of their actions, which highlights our assumption of imperfect observations. Finally, (5) is assumed without loss of generality, which states that Partner 1 is labeled as more (or equally) productive, in the sense that the total surplus he single-handedly generates is greater than that by Partner 2's sole effort.

The two partners engage in this team production in each period $t = 0, 1, 2, \cdots$, under a sharing rule they commit to at the beginning of period 0.[4] That amounts to playing the normal-form game in Figure 1 every period. For each partner, a strategy of this infinitely repeated game is a mapping which determines a (randomized) action in each period, depending on what she observed in the past. We assume that each partner cannot observe the other partner's action directly. We also assume that a public randomization device is available at the beginning of each period. Hence past observations consist of the outcomes (success or failure) and that partner's own actions in all previous periods, together with realizations of the sunspots, including the one at the beginning of the current period. A pair of strategies defined as above generates a sequence of expected period payoffs, and we assume that each partner's overall utility from the strategy pair is the average discounted sum of the period payoffs. Formally, if a strategy pair $\sigma = (\sigma_1, \sigma_2)$ generates an expected period payoff sequence of $(u_i(t))_{t=0}^{\infty}$ for Partner i, then his payoff of the repeated game is:

$$g_i(\sigma) = (1 - \delta) \sum_{t=0}^{\infty} \delta^t u_i(t),$$

where $\delta \in (0, 1)$ is a common discount factor of the partners.

In this paper, we consider equilibrium by strategies which depend entirely on past outcomes of the production and sunspots, not on past actions. Those strategies are called *public strategies*, because they depend only on public information. The public strategy equilibrium payoffs admit a convenient characterization by the method of

[3] Subsection 6.3 discusses what happens if the complementarity condition is violated.

[4] The case where they can change sharing rules depending on the past is discussed in Subsection 6.1.

dynamic programming (Abreu, Pearce and Stacchetti (1990)). Under the full support assumption (4), a Nash equilibrium by public strategy is a sequential equilibrium, which is a solution concept satisfying a natural requirement of perfection.[5] Hence we adopt public strategy equilibrium as our solution concept, and hereafter refer it simply as "equilibrium."

Before proceeding to analysis, we point out that the partners commit to not only a sharing rule but also their repeated interaction; they cannot quit from the partnership. This is justified by assuming that their outside option values are smaller than the values $u_i^s(S,S)$'s. Note that this additional assumption is more reasonable under the assumption $\gamma > 0$.

3 Static Implementation

As our starting point, this section considers what the partners can sustain when the team forms a one-shot relationship.

Our first observation is that if the partnership is very productive, then under a reasonable sharing rule both partners have incentives to work even in the static environment.

Proposition 1 *If*

$$\frac{c_1}{\alpha - \beta_2} + \frac{c_2}{\alpha - \beta_1} \leq X \tag{6}$$

holds, then there exists a sharing rule under which (W, W) *is a Nash equilibrium of the period game.*

Proof. Under a sharing rule $s = (s_1, s_2)$, (W, W) is a period-game equilibrium if

$$s_i \geq \frac{c_i}{\alpha - \beta_j} \tag{7}$$

for each i and $j \neq i$. By (6), there exists a sharing rule satisfying (7) for each i and $j \neq i$. Q.E.D.

In general, (W, W) may not be a *unique* period-game equilibrium under a sharing rule which sustains it. It is possible that (S, S) is also an equilibrium. Here we are not concerned with the issue of unique implementation, because our primary subject of research is repeated games, which typically possess multiple equilibria. Thus we are always concerned with the most efficient equilibrium (namely, an equilibrium maximizing the sum of payoffs), whether the situation is static or dynamic.

Proposition 1 implies that if (6) holds, then efficient production is sustainable simply as a repetition of static equilibrium, whether the partners are patient or not. That is, there is no positive role in repeated play in this case; it just suffices to choose an appropriate sharing rule.

[5] Moreover, any pure strategy sequential equilibrium has an outcome-equivalent public strategy equilibrium. Therefore, if one limits attention to pure strategy equilibria, restriction to public strategies loses no generality in terms of equilibrium paths/payoffs. This point is relevant, because our main results hold even if one limits attention to pure strategies.

Next we consider the case where the condition for the first best is not satisfied;

$$\frac{c_1}{\alpha - \beta_2} + \frac{c_2}{\alpha - \beta_1} > X. \tag{8}$$

Proposition 2 *If* (8) *holds, then there exists a sharing rule under which* (W, S) *is a Nash equilibrium of the period game. Moreover, under no sharing rule, the corresponding period game has a Nash equilibrium whose payoff sum exceeds that of* (W, S).

Proof. First, consider the sharing rule $s = (X, 0)$. It is clear that S is a dominant action for Partner 2, and by (3), Partner 1's best reply against S is W. This proves the first part.

Second, fix a sharing rule $s = (s_1, s_2)$ arbitrarily. (8) implies that for some i and $j \neq i$, it holds that

$$s_i(\alpha - \beta_j) < c_i.$$

Together with (2), this implies that S is a dominant action for Partner i in the corresponding period game. Therefore, no period-game equilibrium puts a positive probability on the pair (W, W). Since (W, S) is the most efficient among $\{(W, S), (S, W), (S, S)\}$ due to (3) and (5), the second part also follows. Q.E.D.

Propositions 1 and 2 show that the efficient production is possible if and only if (6) is satisfied. Indeed, if (8) rather holds, it is impossible to give sufficient shares to both partners simultaneously. However, since each partner's effort is socially efficient, they can make exactly one partner work, by letting the partner become a residual claimant. Since we assume that Partner 1 produces more, the best static arrangement is to let him work, as Proposition 2 claims.[6]

The sharing rule in the proof of Proposition 2 uniquely implements (W, S), though this is not a unique sharing rule which (uniquely) sustains it. The two partners' incentive conditions at (W, S) under $s = (X, 0)$ are strict. Hence we have other sharing rules which give smaller shares to Partner 1 but still make (W, S) an equilibrium. If the team production is one-shot, then the partners might agree on a rule with a smaller share for Partner 1, possibly from considerations for fairness. However, while those different sharing rules correspond to different equilibrium payoffs, the efficiency level (namely, their sum) is always the same.

4 Optimal Sharing Rules under Repeated Play

In this section, we analyze the infinitely repeated game given a sharing rule s. We assume (8) in what follows, so that the static implementation allows only one partner to work.

[6] Legros and Matsushima (1991) consider a more general model of one-shot partnerships, and provide a necessary and sufficient condition for efficient production. In our model, (6) is equivalent to their necessary and sufficient condition, so that Proposition 1 corresponds to their sufficiency result. Proposition 2 implies their necessity result, but it is more explicit on the second-best outcome.

4.1 Equilibrium Candidates

Following the idea of Radner, Myerson and Maskin (1986), we first fix a sharing rule s, and then look for a candidate for an equilibrium that maximizes the sum of each partner's payoffs. We first limit attention to pure strategy equilibria. Suppose that the maximum payoff sum exceeds that of the profile (W, S). Then (W, W) must be the initial-period actions of that equilibrium. Let $f(y) = (f_1(y), f_2(y))$ denote the continuation payoff pair from the next period on, when the outcome in the current production is $y \in \{G, B\}$. Then the equilibrium payoff pair, denoted $v = (v_1, v_2)$, can be decomposed into:

$$v = (1-\delta)u^s(W, W) + \delta\big[(\alpha+\gamma)f(G) + (1-\alpha-\gamma)f(B)\big]. \tag{9}$$

For this to be an equilibrium, it is necessary that each partner does not want to choose S instead of W. That is, if we define

$$\Delta_1^s = u_1^s(S, W) - u_1^s(W, W),$$
$$\Delta_2^s = u_2^s(W, S) - u_2^s(W, W),$$

we have

$$v_i \geq (1-\delta)\big(\Delta_i^s + u_i^s(W, W)\big) + \delta\big[(\beta_j+\gamma)f_i(G) + (1-\beta_j-\gamma)f_i(B)\big] \tag{10}$$

for $i = 1, 2$ and $j \neq i$.

By (9) and (10), we obtain

$$f_i(B) \leq f_i(G) - \frac{1-\delta}{\delta}\frac{\Delta_i^s}{\alpha-\beta_j}$$

for each i and $j \neq i$. Substituting it to (9) yields:

$$v_i \leq (1-\delta)\Big[u_i^s(W, W) - \frac{1-\alpha-\gamma}{\alpha-\beta_j}\Delta_i^s\Big] + \delta f_i(G) \tag{11}$$

for each i and $j \neq i$. Since $f(G)$ is also a (pure) equilibrium payoff pair, we have $v_1 + v_2 \geq f_1(G) + f_2(G)$. Substituting it to the sum of (11) for $i = 1, 2$, and rearranging it, we have

$$v_1 + v_2 \leq V^* \equiv \Big[u_1^s(W, W) - \frac{1-\alpha-\gamma}{\alpha-\beta_2}\Delta_1^s\Big] + \Big[u_2^s(W, W) - \frac{1-\alpha-\gamma}{\alpha-\beta_1}\Delta_2^s\Big]. \tag{12}$$

Substituting the period-game payoffs into (12), we obtain

$$V^* = X - \frac{1-\beta_2-\gamma}{\alpha-\beta_2}c_1 - \frac{1-\beta_1-\gamma}{\alpha-\beta_1}c_2. \tag{13}$$

Since our supposition is that $v_1 + v_2 > U(W, S)$, our argument is valid only when $V^* > U(W, S)$ or, equivalently,

$$X > \frac{1-\alpha-\gamma}{1-\beta_1-\gamma} \cdot \frac{c_1}{\alpha-\beta_2} + \frac{c_2}{\alpha-\beta_1}. \tag{14}$$

Therefore, the partnership must be productive to some extent. Namely, X must be relatively so large as to satisfy (14), but not too large to violate (8).

Since V^* represented by (13) does not depend on s, it sets an upper bound on the pure strategy equilibrium payoff sums, independently of the sharing rule, if (14) is satisfied. It is easily seen that $V^* < U(W,W)$, so that the equilibrium is bounded away from full efficiency. This is similar in structure to the uniform inefficiency result by Radner, Myerson and Maskin (1986), on which we comment later.

We point out that the value V^* has a similar expression to a formula presented in Abreu, Milgrom and Pearce (1991), which characterizes the most efficient symmetric equilibrium payoff of symmetric prisoners' dilemma type games. Rearranging (12), we obtain:

$$V^* = \left(u_1^s(W,W) - \frac{\Delta_1^s}{\frac{1-\beta_2-\gamma}{1-\alpha-\gamma} - 1} \right) + \left(u_2^s(W,W) - \frac{\Delta_2^s}{\frac{1-\beta_1-\gamma}{1-\alpha-\gamma} - 1} \right). \tag{15}$$

Both terms in a bracket have the form of a partner's payoff from full cooperation minus some loss term. The loss term has the form of the gain from unilateral own deviation, divided by the likelihood ratio of the bad outcome between deviation and conformance minus unity. This structure also appears in the Abreu-Milgrom-Pearce formula, though their original formula consists of the first bracketed term only, since they consider symmetric equilibria. (15) is a generalization of their formula to asymmetric environments.[7]

While our argument provides a candidate for the payoff sum of the most efficient equilibrium, it also suggests what its payoff pair is. For V^* to be the greatest equilibrium payoff sum, (10) must hold with equality, and $v_1 + v_2 = f_1(G) + f_2(G)$ must hold. It is natural to suppose $v = f(G)$. Using these equations, we can solve v for:

$$v = v^s \equiv \left(s_1 - \frac{1-\beta_2-\gamma}{\alpha-\beta_2} c_1, s_2 - \frac{1-\beta_1-\gamma}{\alpha-\beta_1} c_2 \right). \tag{16}$$

The pair v^s is important, because the following result shows that it is the only candidate for an equilibrium payoff pair with the payoff sum V^*. The result also extends the argument to all (possibly mixed) equilibria.

Proposition 3 *Suppose* (14) *holds. Fix a sharing rule $s = (s_1, s_2)$ arbitrarily. Then any payoff pair $v = (v_1, v_2)$ such that*

$$v_1 + v_2 \geq V^*, \ \ v \neq v^s \tag{17}$$

is not an equilibrium payoff pair under any δ.

Proof. Let us fix δ and a sharing rule s. Suppose that a payoff pair $v = (v_1, v_2)$ such that (17) holds is an equilibrium payoff pair. Let Z be the convex hull of $\{v^s, u^s(W,S), u^s(S,W), u^s(S,S)\}$. Because (14) is equivalent to $V^* > U(W,S)$, the only $z \in Z$ such that $z_1 + z_2 \geq V^*$ is v^s. Therefore, we have $v \notin Z$. Hence the sepa-

[7]Other authors generalize the Abreu-Milgrom-Pearce formula to various directions. Kandori and Obara (2006) generalize it to equilibria by nonpublic strategies, which we discuss in Subsection 6.3. Kobayashi and Ohta (2007) develop a version for a model of repeated multimarket operations.

rating hyperplane theorem applies. Moreover, since the vector $(1,1)$ weakly separates v from Z, there exists a strictly positive vector $\lambda = (\lambda_1, \lambda_2) \gg 0$ such that

$$\lambda_1 v_1 + \lambda_2 v_2 > \lambda_1 z_1 + \lambda_2 z_2 \tag{18}$$

for any $z \in Z$.

Without loss of generality, we can assume that v maximizes the inner product with λ over all equilibrium payoff pairs.[8] Let

$$m = \big(\eta_1 W + (1-\eta_1)S, \eta_2 W + (1-\eta_2)S\big)$$

be the mixed action profile played in the initial period under this equilibrium.[9] Since the continuation payoff pairs do not have a greater inner product than v, we have:

$$\lambda_1 u_1^s(m) + \lambda_2 u_2^s(m) \geq \lambda_1 v_1 + \lambda_2 v_2 > \lambda_1 u_1^s(a) + \lambda_2 u_2^s(a)$$

for any $a \neq (W,W)$, where the second inequality follows from (18). Hence we have $\eta_i > 0$ for each i.

Next, let $f(y) = (f_1(y), f_2(y))$ be the continuation payoff pair from the next period on, if the outcome of the initial period is $y \in \{G, B\}$. Since $\eta_i > 0$ for each i, we have the following value equations and incentive conditions for $i = 1, 2$, and $j \neq i$:

$$v_i = (1-\delta)\Big(s_i q_i^W - c_i\Big) + \delta\Big[q_i^W f_i(G) + \Big(1 - q_i^W\Big) f_i(B)\Big] \tag{19}$$

$$\geq (1-\delta) s_i q_i^S + \delta\Big[q_i^S f_i(G) + \Big(1 - q_i^S\Big) f_i(B)\Big], \tag{20}$$

where $q_i^{a_i}$ is the probability of a good outcome if Partner i chooses a_i and Partner j plays m_j. (19) and (20) reduce to:

$$(1-\delta)\Big[c_i - s_i\Big(q_i^W - q_i^S\Big)\Big] \leq \delta\Big(q_i^W - q_i^S\Big)\big[f_i(G) - f_i(B)\big]. \tag{21}$$

Since $f(G)$ is an equilibrium payoff pair, we also have

$$\lambda_1 v_1 + \lambda_2 v_2 \geq \lambda_1 f_1(G) + \lambda_2 f_2(G).$$

Take the inner product of (19) for $i = 1, 2$ with λ, and then substitute all those inequalities. Then we obtain:

$$\begin{aligned}\lambda_1 v_1 + \lambda_2 v_2 &\leq \lambda_1\left(s_1 - \frac{1-q_1^S}{q_1^W - q_1^S} c_1\right) + \lambda_2\left(s_2 - \frac{1-q_2^S}{q_2^W - q_2^S} c_2\right)\\ &= \lambda_1\left[s_1 - \frac{1-\eta_2\beta_2-\gamma}{\eta_2(\alpha-\beta_1-\beta_2)+\beta_1} c_1\right] + \lambda_2\left[s_2 - \frac{1-\eta_1\beta_1-\gamma}{\eta_1(\alpha-\beta_1-\beta_2)+\beta_2} c_2\right],\end{aligned}$$

[8] Abreu's (1988) topological argument on the set of equilibrium payoffs also applies to our model, and the equilibrium payoff pair set can be shown to be compact. Hence existence of a maximizer is guaranteed.

[9] We use convex combinations in order to denote randomization over two elements.

which is increasing in η_1 and η_2 by (2). Therefore, we at last have:

$$\lambda_1 v_1 + \lambda_2 v_2 \leq \lambda_1 v_1^s + \lambda_2 v_2^s,$$

which is a contradiction against (18), because $v^s \in Z$. Q.E.D.

Proposition 3 shows that V^* is an upper bound on the sum of equilibrium payoffs (namely, the efficiency level of an equilibrium) which applies to any degree of patience of the partners, if the team is moderately productive so that the condition (14) holds. We relegate examination of whether and when the bound is tight to the next subsection, and just point out that this is an uniform inefficiency result which stems from the same logic as Radner, Myerson and Maskin (1986). Our simple two-action, two-signal model does not satisfy the pairwise full-rank condition, which is a sufficient condition for the folk theorem by Fudenberg, Levine and Maskin (1994). Due to the failure of pairwise full rank, we cannot attribute one bad signal to a particular partner's deviation. Thus the partners must punish each other upon realization of a bad outcome, which causes unavoidable efficiency loss.[10] Proposition 3 also states the loss term explicitly.

If (14) does not hold, then V^* is smaller than the bound on the payoff sum in one-shot partnerships (Proposition 2). The same argument as above proves that the latter bound is relevant in this case. In the following result, we just give a sketch of the proof.

Proposition 4 *Suppose* (14) *does not hold. Then under any sharing rule, any equilibrium has a payoff sum no greater than* $U(W, S)$.

Sketch of the Proof. Fix δ and s. Suppose that $v = (v_1, v_2)$ is an equilibrium payoff pair under δ and s, and that

$$v_1 + v_2 > U(W, S).$$

By a similar argument to Proposition 3, we can assume that v is an equilibrium with a greatest payoff sum, and (W, W) is played with a positive probability in the initial period under the equilibrium. Then the same argument establishes that we rather have

$$v_1 + v_2 \leq V^* \leq U(W, S),$$

a contradiction. Q.E.D.

Proposition 4 shows that if the partnership is not productive so much, repeated play does not help at all. Namely, it cannot improve what the partners can do under static arrangements.

4.2 Optimal Sharing Rules

Here we examine whether there exists an equilibrium whose payoff sum equals V^*, under some δ and s. Note first that the period game becomes a prisoners' dilemma

[10] Our model is free from other folk theorems in the literature. Kandori (2003) studies the same class of repeated games as Fudenberg, Levine and Maskin (1994), and shows that the folk theorem holds more generally if players can communicate each other. However, his folk theorem does not apply to any two-player game.

under appropriate sharing rules. In a repeated prisoners' dilemma, a natural equilibrium candidate is the one by trigger strategy. In our setting, the trigger strategy would start with working and then revert to perpetual shirking upon a single observation of B. Since the bad outcome always has a positive probability, the Nash reversion occurs on the equilibrium path. By Proposition 3, the only payoff pair with a sum V^* which can form an equilibrium under s is v^s. Therefore, a necessary condition for the trigger strategy to work is that v^s is represented as a convex combination of $u^s(W,W)$ and $u^s(S,S)$. Namely, we must have:

$$\frac{u_2^s(W,W)-v_2^s}{u_1^s(W,W)-v_1^s}=\frac{u_2^s(W,W)-u_2^s(S,S)}{u_1^s(W,W)-u_1^s(S,S)}. \tag{22}$$

The only sharing rule which satisfies (22) is the following one, which we denote by $s^*=(s_1^*,s_2^*)$.

$$s_1^*=\frac{1}{D}\Big[Xc_1(\alpha-\beta_1)\beta_2+c_1c_2(\beta_1-\beta_2)\Big],$$
$$s_2^*=\frac{1}{D}\Big[Xc_2(\alpha-\beta_2)\beta_1+c_1c_2(\beta_2-\beta_1)\Big],$$

where

$$D\equiv c_1(\alpha-\beta_1)\beta_2+c_2(\alpha-\beta_2)\beta_1>0.$$

Some computations yield that for each i and $j\neq i$,

$$s_i^*>0,$$
$$s_i^*\alpha>c_i>s_i^*(\alpha-\beta_j),$$

where the inequalities follow from (2), (3) and (8). Namely, s^* is indeed a sharing rule, each partner prefers (W,W) to (S,S), and it is optimal for each partner to select S against W. Together with (2), S is a dominant action for both partners, and the period game under s^* is a prisoners' dilemma. We will examine other properties of s^* in details in Subsection 4.3. Here we only point out that if the partners are identical (i.e., $\beta_1=\beta_2$ and $c_1=c_2$), we have $s_1^*=s_2^*$; s^* is an equal sharing rule.

Suppose that both partners adopt the trigger strategy under s^*, and let $v=(v_1,v_2)$ be the payoff pair of the trigger strategy profile. Then we have the following value equation for $i=1,2$.

$$\begin{aligned}v_i&=(1-\delta)[s_i^*(\alpha+\gamma)-c_i]+\delta\big[(\alpha+\gamma)v_i+(1-\alpha-\gamma)s_i^*\gamma\big]\\&=\frac{(1-\delta)[s_i^*(\alpha+\gamma)-c_i]+\delta(1-\alpha-\gamma)s_i^*\gamma}{1-\delta(\alpha+\gamma)}.\end{aligned} \tag{23}$$

The necessary and sufficient condition for equilibrium is for $i=1,2$ and $j\neq i$;

$$(1-\delta)\big[c_i-s_i^*(\alpha-\beta_j)\big]\leq\delta(\alpha-\beta_j)(v_i-s_i^*\gamma). \tag{24}$$

If we substitute (23) into (24), we can solve (24) for δ. While we have (23) and (24) for each i, the solution is the same:

$$\delta \geq \underline{\delta} \equiv \frac{c_1(\alpha-\beta_1)+c_2(\alpha-\beta_2)-X(\alpha-\beta_1)(\alpha-\beta_2)}{c_1(\beta_2+\gamma)(\alpha-\beta_1)+c_2(\beta_1+\gamma)(\alpha-\beta_2)-X(\alpha-\beta_1)(\alpha-\beta_2)\gamma}. \tag{25}$$

It is easy to verify that $0 < \underline{\delta} < 1$. Note also that (23) and (24) provide a simpler expression of $\underline{\delta}$ for $i = 1, 2$ and $j \neq i$:

$$\underline{\delta} = \frac{c_i - s_i^*(\alpha-\beta_j)}{(\beta_j+\gamma)c_i - s_i^*(\alpha-\beta_j)\gamma}. \tag{26}$$

If we evaluate v_i's at $\delta = \underline{\delta}$ using (26), we obtain:

$$v = v^* \equiv \left(s_1^* - \frac{1-\beta_2-\gamma}{\alpha-\beta_2}c_1, s_2^* - \frac{1-\beta_1-\gamma}{\alpha-\beta_1}c_2 \right).$$

Note that $v^* = v^{s^*}$. Namely, the trigger strategy profile is an equilibrium for any $\delta \geq \underline{\delta}$, and it achieves the upper bound payoff sum V^* at $\delta = \underline{\delta}$. Hence the upper bound in Proposition 3 is tight.

More generally, we have the following result.

Proposition 5 *Under s^*, for any $\delta \geq \underline{\delta}$, there exists an equilibrium with a payoff pair v^*.*

Proof. We have seen that the claim is true for $\delta = \underline{\delta}$. Thus fix $\delta > \underline{\delta}$, and let us modify the trigger strategy profile in the following way. Use sunspots so that at every period, partners forget all pasts and simply restart the trigger strategy profile with probability $1 - (\underline{\delta}/\delta)$. This makes the partners' effective discount factor $\underline{\delta}$. For the same reason that the trigger strategy profile is an equilibrium at $\underline{\delta}$, the modified profile is also an equilibrium at δ, with the same payoff pair v^*. Q.E.D.

Finally, we consider what happens if the partners commit to a sharing rule other than s^*. If they are sufficiently patient, it is possible that v^s is sustainable as an equilibrium under $s \neq s^*$. However, the following result demonstrates that any $s \neq s^*$ *never* supports the largest equilibrium payoff sum V^* for as wide a range of discount factors as s^* does. In fact, s^* is the *only* sharing rule which supports the payoff sum V^* at the critical discount factor $\underline{\delta}$. The result also considers the case $\delta < \underline{\delta}$.

Proposition 6 *Assume that* (14) *holds. Fix $\delta \leq \underline{\delta}$ and a sharing rule s. If there exists an equilibrium whose payoff sum exceeds $U(W, S)$, then we must have $\delta = \underline{\delta}$ and $s = s^*$.*

Proof. Fix $\delta \leq \underline{\delta}$ and s, and fix an equilibrium whose payoff sum exceeds $U(W, S)$. Without loss of generality, we can assume that under the equilibrium (W, W) is played with a positive probability in the initial period. Let $v = (v_1, v_2)$ be the payoff pair,

$$m = (\eta_1 W + (1-\eta_1)S, \eta_2 W + (1-\eta_2)S)$$

be the mixed action profile played in the initial period, and $f(y) = (f_1(y), f_2(y))$ be the continuation payoffs after an outcome $y \in \{G, B\}$. Then we have (21) for each i.

Since $q_i^W - q_i^S \leq \alpha - \beta_j$ by (2), we have:

$$(1-\delta)\left(\frac{c_1}{\alpha-\beta_2}+\frac{c_2}{\alpha-\beta_1}-X\right) \leq \delta\big[f_1(G)+f_2(G)-f_1(B)-f_2(B)\big].$$

Note that $\delta \leq \underline{\delta}$, $f_1(G)+f_2(G) \leq V^*$ (by Proposition 3), and $f_1(B)+f_2(B) \geq U(S,S)$. Hence we have:

$$(1-\underline{\delta})\left(\frac{c_1}{\alpha-\beta_2}+\frac{c_2}{\alpha-\beta_1}-X\right) \leq \underline{\delta}\big(V^* - U(S,S)\big).$$

However, by (25), it must hold with equality. Hence all inequalities which appear before must hold with equalities. In particular, we have $\delta = \underline{\delta}$. Since we also have $f_1(G) + f_2(G) = V^*$, $f(G) = v^s$ by Proposition 3. Since $f_1(B) + f_2(B) = U(S,S)$, $f(B) = u^s(S,S)$ by feasibility. Substitute all those equalities, including $q_i^W - q_i^S = \alpha - \beta_j$, into (21) for each i. Since (21) holds with equality, we obtain

$$(1-\underline{\delta})\big[c_i - s_i(\alpha-\beta_j)\big] = \underline{\delta}(\alpha-\beta_j)(v_i^s - s_i\gamma)$$

for each i and $j \neq i$. Comparing this with (24), which holds with equality at $\delta = \underline{\delta}$ and $v = v^*$, we see that $s = s^*$. Q.E.D.

All those results, including Proposition 1, are summarized by the following theorems. Each of them characterizes the maximum payoff sum sustained by an equilibrium, and the corresponding sharing rule.

Theorem 1 *(i) If* (6) *holds, then the maximum payoff sum in the repeated game is* $U(W,W)$*, irrespective of* δ*.*

(ii) If (8) *and* (14) *hold, then the maximum payoff sum is* V^* *if* $\delta \geq \underline{\delta}$*, and* $U(W,S)$ *if* $\delta < \underline{\delta}$*.*

(iii) If (14) *does not hold, then* $U(W,S)$ *is the maximum payoff sum, irrespective of* δ*.*

Theorem 2 *Suppose* (8) *holds. Then the maximum payoff sum is always achieved by either* s^* *or* $s = (X,0)$*. Moreover,* s^* *is the only sharing rule supporting* V^* *whenever* V^* *is the maximum.*

Theorem 1 completely characterizes the best outcome this team can accomplish under suitable choice of sharing rules. Theorem 2 shows that, except the case where the partnership allows static implementation of the first best, we only need to consider two particular sharing rules as a candidate for the one which sustains the second best. One is an asymmetric sharing rule, which induces only one partner to work. The other is s^*, which treats partners more evenly. Moreover, for any sharing rule other than s^*, there exists a discount factor under which it does not sustain V^*, while s^* does. In this sense, s^* is the unique optimal sharing rule for patient partners. Since s^* corresponds to equal sharing if the partners are identical, the theorem demonstrates optimality of equal sharing in symmetric environments.

The theorems indicate how the optimal sharing rule can change dramatically, depending on parameters on technology and patience. Let us examine it from a perspective of Partner 1's share. If partners are not so patient or not so productive as a whole, then Partner 1 becomes a residual claimant of the team; he is the only person who makes efforts, in return for the entire share. However, if the partners become more productive or more patient, he no longer assumes such a role and rather sacrifices his own share, in order to foster teamwork.

4.3 Properties of the Second-Best Outcome

In this subsection, we examine properties of s^* and v^*, which are the optimal sharing rule and the second-best equilibrium payoff pair for patient partners. First, we study how the shares depend on technological parameters.

Proposition 7 *(i) If $\beta_1 > \beta_2$, then $s_1^* < Xc_1/(c_1+c_2)$ and $s_2^* > Xc_2/(c_1+c_2)$.*

(ii) If $\beta_1 = \beta_2$, then $s_1^* = Xc_1/(c_1+c_2)$ and $s_2^* = Xc_2/(c_1+c_2)$.

(iii) If $\beta_1 < \beta_2$, then $s_1^* > Xc_1/(c_1+c_2)$ and $s_2^* < Xc_2/(c_1+c_2)$.

Proof. By simple calculation, we obtain

$$\begin{aligned}\frac{Xc_1}{c_1+c_2} - s_1^* &= \frac{c_1 s_2^* - c_2 s_1^*}{c_1+c_2} \\ &= \frac{c_1 c_2(\beta_1-\beta_2)(X\alpha - c_1 - c_2)}{D(c_1+c_2)}.\end{aligned}$$

Since $X\alpha - c_1 - c_2 > 0$ by (2) and (3), the claim for s_1^* follows. The claim for s_2^* is obvious, because $s_2^* = X - s_1^*$. Q.E.D.

Proposition 7 implies that if $\beta_1 = \beta_2$, we have $s_2^*/s_1^* = c_2/c_1$. Namely, the partners' shares are proportional to their costs. Since $\beta_1 = \beta_2$ means that the partners' efforts are technologically the same, it makes sense from a normative viewpoint to make the shares proportional to the effort costs. Therefore, the optimal sharing rule s^* admits a normative interpretation in this case.

If $\beta_1 \neq \beta_2$, however, incentive consideration comes in, and s^* no longer has such equity appeal. In particular, Proposition 7 says that Partner i with greater β_i has a *smaller* share than the one suggested by equity consideration. Greater β_i means greater ability when he works alone. This implies that the other partner has a stronger incentive to free-ride. Hence the efficient provision of incentives must weaken that temptation, and this is possible by making the other partner's share greater than the normative argument prescribes.

Proposition 7 can be intuitively understood by using (22). For an arbitrarily given s, if one increases s_1 and decreases s_2 by the same amount, then the LHS of (22) increases while its RHS decreases. Therefore, if a given s does not satisfy (22) because, for example, the LHS is greater than the RHS, then one can conclude that s gives Partner 1 a greater share than s^*. Applying this argument to $s = (Xc_1/(c_1+c_2), Xc_2/(c_1+c_2))$ is an alternative proof of Proposition 7.

If $\beta_1 > \beta_2$, Proposition 7-(i) implies that $s_2^*/s_1^* > c_2/c_1$. This in turn implies that

$$\frac{u_2^{s^*}(S,S)}{u_1^{s^*}(S,S)} = \frac{s_2^*}{s_1^*} = \frac{u_2^{s^*}(W,W)+c_2}{u_1^{s^*}(W,W)+c_1} < \frac{u_2^{s^*}(W,W)}{u_1^{s^*}(W,W)},$$

which is depicted in Figure 2. Since v^* is a convex combination of $u^{s^*}(W,W)$ and

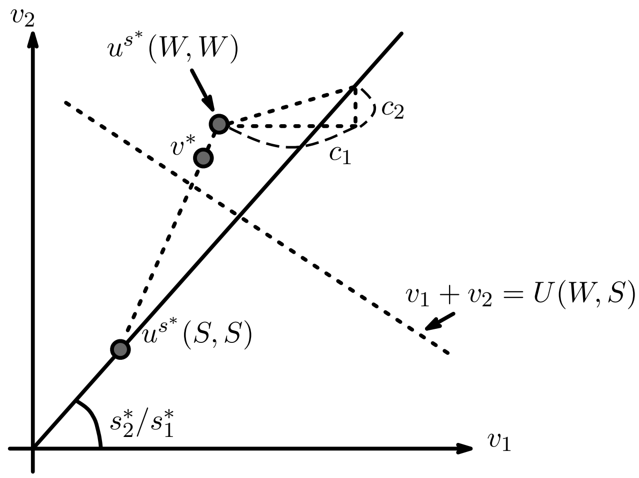

Figure 2: A Graphic Explanation of Proposition 8

$u^{s^*}(S,S)$, this observation leads to the following result.

Proposition 8 *Suppose* (14) *holds. Then*

(i) if $\beta_1 > \beta_2$*, then* $v_2^*/v_1^* > s_2^*/s_1^*$*,*

(ii) if $\beta_1 = \beta_2$*, then* $v_2^*/v_1^* = s_2^*/s_1^*$*, and*

(iii) if $\beta_1 < \beta_2$*, then* $v_2^*/v_1^* < s_2^*/s_1^*$*.*

Proof. Suppose $\beta_1 \geq \beta_2$. Then by the definition of v^*, we have:

$$\begin{aligned} s_1^* v_2^* - s_2^* v_1^* &= s_2^* c_1 \frac{1-\beta_2-\gamma}{\alpha-\beta_2} - s_1^* c_2 \frac{1-\beta_1-\gamma}{\alpha-\beta_1} \\ &= \frac{1}{D} c_1 c_2 (\beta_1 - \beta_2)\Big(V^* - U(S,S)\Big) \geq 0, \end{aligned}$$

where the inequality follows from $\beta_1 \geq \beta_2$ and (14). Moreover, it holds with equality if and only if $\beta_1 = \beta_2$. Since (14) implies $v_1^* > 0$, this proves the parts (i) and (ii). The case $\beta_1 < \beta_2$ is similar, so we omit the proof. Q.E.D.

Figure 2 describes the proposition for the case $\beta_1 > \beta_2$. (14) ensures that v^* is above the line $v_2/v_1 = s_2^*/s_1^*$, so that the claim follows.

Combined with Proposition 7, Proposition 8 implies that Partner i with greater β_i has a smaller share than his effort cost ratio, and receives a much smaller share to the equilibrium payoff sum V^*. We consider economic implications of the results in Subsection 5.1.

4.4 Comparative Statics

This subsection reports comparative statics results for the most efficient equilibrium payoff sum V^* and the lowest discount factor supporting the outcome, $\underline{\delta}$. Since all those results are a consequence of tedious but elementary calculations, we just state the results without a proof.

Proposition 9 *(i) V^* is strictly increasing in α, γ and X, and is strictly decreasing in β_1, β_2, c_1 and c_2.*

(ii) $\underline{\delta}$ is strictly decreasing in α, γ and X, and is strictly increasing in β_1, β_2, c_1 and c_2.

The result for α, γ and X is natural; its increase simply increases the productivity of partnership, and therefore improves efficiency and the critical discount factor. Note also that the change makes (14) more likely to hold. A similar logic works for the result on c_1 and c_2. In contrast, an increase either in β_1 or β_2 has diametrically opposite effects on patient partners. It worsens the second-best outcome, increases the critical discount factor and makes (14) less likely to hold. This is because the change increases what a partner receives when he shirks, which makes cooperation harder to sustain.[11]

5 Economic Implications

In this section, we assume (8), (14) and $\delta \geq \underline{\delta}$, so that s^* is the optimal sharing rule with its corresponding equilibrium payoff pair v^*.

5.1 Partners' Productivity and Shares

We have already seen in Propositions 7 and 8 how partners are treated under s^*. The following proposition is a simple application of those results.

Proposition 10 *Suppose $\beta_1 \geq \beta_2$ and $c_1 \leq c_2$. Then we have $s_1^* \leq s_2^*$ and $v_1^* \leq v_2^*$. Moreover, both inequalities hold with equality if and only if $\beta_1 = \beta_2$ and $c_1 = c_2$.*

Proof. The statements straightforwardly follow from Propositions 7 and 8. Q.E.D.

If the condition for Proposition 10 is met, Partner 1's effort is more efficient. Nevertheless, the proposition reveals that the more efficient partner has a smaller share for additional monetary income of success, and in fact he is worse off in the team. The reason for this inequality is that a less efficient Partner 2 has a stronger incentive to deviate, so the efficient provision of incentives requires his stake from cooperation to be greater.[12]

The condition for Proposition 10 is a special case of (5), and (5) itself does not imply the result. If, for instance, $\alpha = 0.9$, $\beta_1 = 0.4$, $\beta_2 = 0.5$, $\gamma = 0.01$, $c_1 = 1$,

[11] However, impatient partners benefit from an increase in β_1, because it increases the value under static implementation (Proposition 2).

[12] Harrington (1989) offers a similar idea in a model of cartels by firms with heterogenous discount factors. In his cartel scheme, colluding firms share the market unequally, so that less patient firms enjoy larger market shares. This mitigates their temptations to deviate.

$c_2 = 1.41$, and $X = 4$, then all assumptions are satisfied and $s_1^* \simeq 1.806 < s_2^* \simeq 2.193$, but we have $v_1^* \simeq 0.581 > v_2^* \simeq 0.530$.

Proposition 10 is a somewhat excessive confirmation of the empirical finding by Frank (1984), who argues that workers with apparently greater productivity receive less than their marginal products. Consequently, if we also believe that the outside option of more productive workers should be more favorable, then they must receive some compensation. This implication suggests that the more productive workers should be compensated by greater fixed wage or, like the hypothesis by Frank (1984), rewarded by status.

5.2 Homogeneity of Partners

As an ownership structure of enterprises, partnerships (or employee ownership) are not so common, only found in some particular industries such as law firms, plywood manufacturers, mutual funds, and so on. Hansmann (1996) argues that one difficulty in employee ownership is costs of collective decision making within the organization, which can be prohibitively high when partners have diverse interests. Hansmann (1996) thus maintains that partnerships are viable only in enterprises whose members are homogeneous, as is the case in the industries mentioned above.

Our model does not deal with the issue of preferences of partners, and their homogeneity or heterogeneity is evaluated only through their technological abilities. Still it offers an idea that partnerships with homogeneous (or similar) members perform better than those with more heterogenous partners, reinforcing Hansmann's argument which favors homogeneity. To make our point, suppose initially that the partners are identical; $\beta_1 = \beta_2 = \beta$ and $c_1 = c_2 = c$. Let us change β_1 and β_2 so that now they are replaced with $\beta + \eta$ and $\beta - \eta$ respectively, where $\eta > 0$. Namely, Partner 1 now becomes more productive at the expense of Partner 2, which we interpret as introduction of heterogeneity. To make comparison, let us denote the corresponding value of V^* by $V^*(\eta)$.

Proposition 11 *$V^*(\eta)$ is strictly decreasing in η.*

Proof. Since we initially have $\beta_1 = \beta_2 = \beta$ and $c_1 = c_2 = c$, we can write $V^*(\eta)$ as

$$V^*(\eta) = X - 2\frac{(1-\beta-\gamma)(\alpha-\beta) - \eta^2}{(\alpha-\beta)^2 - \eta^2}c.$$

It is easy to see that it is decreasing in η, so the claim follows. Q.E.D.

Proposition 11 shows that homogeneity in abilities is another ingredient for successful partnerships, since it facilitates provision of incentives based on repeated interaction.

6 Discussions

This section discusses some extensions of our model.

6.1 History-Dependent Sharing Rules

We have assumed that the partners commit to a sharing rule at the beginning of their repeated interactions, which is in force every period. In this subsection, we introduce a possibility that they may change future sharing rules depending on what happened in the past.[13]

Now we assume that the partners commit to a dynamic agreement at the beginning of their relationships, which determines the sharing rule of each period as a function of past outcomes. Formally, we define a *dynamic sharing rule* as a pair of sharing rule s^0 and a sequence of functions $(\psi^t)_{t=1}^{\infty}$, where each ψ^t maps a t-length sequence of G or B to a sharing rule. If the partners commit to a dynamic sharing rule $(s^0, (\psi^t)_{t=1}^{\infty})$, the sharing rule in the initial period is s^0, and the sharing rule in period t when the past outcome is $y^t = (y(\tau))_{\tau=0}^{t-1}$, where $y(\tau) \in \{G, B\}$, is $\psi^t(y^t)$. Note that if they agree upon some dynamic sharing rule, it induces a dynamic game with period payoffs which depend on corresponding period sharing rules. The game is no longer a standard repeated game, because period payoffs are different each period.[14]

One may be tempted to think that introduction of dynamic sharing rules will expand partners' strategic possibilities, and therefore it may increase the value partners can achieve as an equilibrium. This is not true, however. In fact, we can prove that Theorem 1 continues to hold. Namely, history dependence of sharing rules never increases the payoff sum partners can sustain in a self-enforcing manner, nor it improves the range of discount factors under which the outcome is sustained. To see that, fix a dynamic sharing rule which maximizes the sum of partners' payoffs, and let s^0 be its sharing rule in period 0. Then the statement of Proposition 3 holds if we set $s = s^0$. To see that, let us redefine $f(y)$ as the continuation payoff pair if future sharing rules are determined according to the dynamic sharing rule. Then the argument in the proof of Proposition 3 goes through. Given the result, we can also generalize Proposition 6 and show that any dynamic sharing rule which induces an equilibrium under some $\delta \leq \underline{\delta}$ with a payoff sum greater than $U(W, S)$ must have $s^0 = s^*$. Since we also have $\delta = \underline{\delta}$ as a generalization of the proposition, we conclude that Theorem 1 generalizes.

While any dynamic sharing rule that supports V^* at $\underline{\delta}$ must adopt s^* in any cooperative stage, this does not mean that the optimal dynamic sharing rule is unique. Indeed, we have flexibility over choice of sharing rules in the punishment phase. Suppose partners play a trigger strategy profile or its modification presented in Proposition 5. Any dynamic sharing rule sustains the second best, as far as it has the continuation value of $s_i^*\gamma$ at the beginning of the punishment path for each Partner i. Hence two dynamic sharing rules with different paths of period sharing rules in the punishment phase are both optimal, if they achieve the same continuation payoffs.

That said, we also point out that dynamic sharing rules may not be enforceable, to the extent that the production outcome is observable only between the partners, and not to the court.

[13]For expositional simplicity, we here assume that future sharing rules do not depend on past sunspots. However, it is easy to accommodate sunspots into analysis.

[14]We do not allow a possibility that the dynamic sharing rule depends on other information than production outcomes, that is, partners' own actions. Since past actions are a partner's private information, it would be difficult to solicit them in an incentive-compatible manner.

6.2 Budget-Breaking

One source of inefficient production in static partnerships is the requirement of budget balance on sharing rules. This subsection examines an extension where partners can credibly break the budget, under our environment of repeated play and noisy production outcomes. We limit attention to the case where partners can dispose some fraction of the gain brought about by successful production. That is, we redefine a sharing rule as a pair such that $s_1 \geq 0$, $s_2 \geq 0$ and $s_1 + s_2 \leq X$.

When the partners can break budget, the analysis crucially depends on whether they can commit to a dynamic sharing rule or only commit to a fixed sharing rule every period. If the latter is the case, then selecting a sharing rule with $s_1 + s_2 = \tilde{X} < X$ is exactly the same as considering our model in the previous sections with X replaced by $\tilde{X}$. By Proposition 9, it is clear that commitment to budget-breaking simply hurts.

Therefore, suppose that the partners can commit both to a dynamic sharing rule and period sharing rules with budget-breaking. In this case, we can consider the following type of dynamic sharing rules, combined with the trigger strategy or its modification described in Proposition 5: (i) under the cooperative phase, the partners employ a sharing rule s with budget balance, and (ii) under the punishment phase, the partners employ a sharing rule $(0, 0)$. Under this arrangement, partners commit to an extreme of budget-breaking and throw away all additional gain from successful production in the punishment phase.

The dynamic game induced by this dynamic sharing rule is closely related to our previous model where the productivity parameter set $(\alpha, \beta_1, \beta_2, \gamma)$ is replaced with $(\alpha', \beta_1', \beta_2', \gamma') = (\alpha + \gamma, \beta_1 + \gamma, \beta_2 + \gamma, 0)$.[15] Under the new parameter set, we have the same p^a as the original parameter set for each $a \in \{(W, W), (W, S), (S, W)\}$, and the following period game for a given sharing rule s. It is easy to see that the trigger

		Partner 2	
		W	S
Partner 1	W	$s_1(\alpha + \gamma) - c_1, s_2(\alpha + \gamma) - c_2$	$s_1(\beta_1 + \gamma) - c_1, s_2(\beta_1 + \gamma)$
	S	$s_1(\beta_2 + \gamma), s_2(\beta_2 + \gamma) - c_2$	$0, 0$

Figure 3: The Period Game under the New Parameter Set

strategy profile in the dynamic game induced by the above dynamic sharing rule is an equilibrium if and only if the same profile is an equilibrium of the repeated prisoners' dilemma with the period payoffs in Figure 3. Hence we can analyze the original dynamic game, by using the payoff matrix in Figure 3.

If we substitute $(\alpha', \beta_1', \beta_2', \gamma')$ into (13) and (25), we see that this parameter change keeps V^* unchanged but reduces $\underline{\delta}$. In other words, budget-breaking never improves what sufficiently patient partners can sustain. However, budget-breaking under the punishment stage makes punishment severer and therefore sustains the second best for a wider range of discount factors.[16] This result shows that on some intermediate range of discount factors, budget-breaking can (considerably) increase efficiency of partnership production.

[15] The following argument is not affected by the fact that the new model does not satisfy (4).

[16] As for the sharing rule s^*, s_i^* decreases by the change if $\beta_i > \beta_j$. If $\beta_1 = \beta_2$, s^* is unaffected by the change.

This argument has a direct implication on what happens if we introduce a principal as a budget breaker, as in Holmstrom (1982). The argument implies that (i) a principal is beneficial only to the partnership whose members are moderately patient, and (ii) the principal should break budget only in the punishment phase of the equilibrium the partners play. Here the principal's role is to inflict additional punishment on the partners after a bad outcome, thereby providing stronger incentives in the cooperative phase.

In the literature of corporate governance, this idea of a principal who breaks budget only after a bad performance also appears in the *contingent governance* structure, studied in Aoki (1994). Our argument can be regarded as offering a dynamic foundation to Aoki's static model. Aoki (1994) also claims that the Japanese firms used to have that type of principal in their main banks, whose control over lending firms was most apparent when the firms' performance was not good. In relation to our model, this claim is intriguing, because the main-bank system had its heyday in the rapid economic growth periods, typically characterized by large interest rates and therefore relatively heavy discounting.

6.3 Non-Complementarity

In Section 2, we emphasized that the assumption of complementary efforts (2) plays a central role in our analysis. One important implication of the assumption is that a bad outcome more strongly indicates a partner's shirking if the other partner works than if he shirks, in terms of the likelihood ratio:

$$\frac{1-\beta_2-\gamma}{1-\alpha-\gamma} > \frac{1-\gamma}{1-\beta_1-\gamma}. \tag{27}$$

(27) guarantees that no equilibrium with randomization is second-best, because randomization simply reduces both period payoffs and statistical detectability of deviations.

Consequently, if we do not assume (2), (27) may not be satisfied either. If that is the case, the analysis will be complicated because an equilibrium with randomization could be second-best. Furthermore, if (27) is violated, then the so-called private strategy equilibrium studied by Kandori and Obara (2006) may also work. It is beyond the scope of this paper to consider all those equilibrium possibilities.[17]

Given the above difficulty, one alternative formulation is to replace the assumption (2) with (27). However, this creates a new problem that the period game corresponding to the sharing rule s^* may not be a prisoners' dilemma. It is possible that a partner's best response against S is W, which rather makes the period game a chicken. Since (S, S) is not a period-game equilibrium, we must consider a different kind of trigger strategies. Since Proposition 3 continues to hold, we at least know that the second-best payoff sum is V^*. More complicated is derivation of second-best sharing rules, but we conjecture that results with a similar flavor will obtain. That is, the second-best sharing rule is either a highly asymmetric one inducing only one partner to work, or

[17]The construction by Kandori and Obara (2006) hinges on violation of (27), and does not work under our assumption of (2). Whether other types of private strategy equilibria improve our second-best equilibrium is an open question.

one which favors symmetry and has a similar structure to s^*.

6.4 General Period Games

Our model is very simple, only with two partners, two actions and two signals. If we maintain the assumption of two signals but allow three or more actions and/or partners, the uniform inefficiency result continues to be valid. Some of the results we have presented so far remain true under this generalization. For example, consider an extension to three or more partners with two actions. If the partnership is not productive enough or if the partners are not patient enough, then the static and dynamic second-best outcomes coincide. Under an optimal sharing rule in this case, only a fraction of partners are induced to work, though its exact configuration depends on parameters in a complicated way. The case where repeated play matters is more difficult, because it may be optimal to let only a fraction of partners, different from the one under static implementation, to work in a cooperative stage. We are currently working on this issue.[18]

Extension to the case with three or more signals is quite a different story, because then the folk theorem by Fudenberg, Levine and Maskin (1994) would apply. Hence the efficiency loss which appears in our analysis will vanish if the partners are sufficiently patient. Since the folk theorem is a limit result, however, the equilibrium set depends on the discount factor in a subtle way, which makes derivation of optimal sharing rules very difficult. This extension would be possible, but it would ruin a virtue of our model; it cleanly demonstrates how joint production with a long horizon performs differently from one-shot partnerships, mainly through differences in the sharing rules. We do not deny practical significance of environments where the folk theorem holds. However, we believe that our assumption of small signal space is vital to tractable analysis, and it still represents general cases.

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[18]Another difficulty is that with three or more partners, the folk theorem with communication by Kandori (2003) would work (recall Footnote 10). However, the folk theorem may require a much greater critical discount factor. Furthermore, it depends on mixed strategy equilibria, and if we limit attention to pure strategy equilibria, communication does not help at all.

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