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“The Role of Expectations in a Specialization-driven Growth Model with  
Endogenous Technology Choice”

by

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# The Role of Expectations in a Specialization-driven Growth Model with Endogenous Technology Choice\*

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## Abstract

Extending the Kim (1989) model of endogenous labor specialization to an overlapping generations model with an endogenous technology choice, we show in this paper that, when the market size and the fixed costs associated with the technologies with labor specialization are small, the growth pattern of this economy depends on worker expectations. In other words, if workers expect low returns of specific human capital, they will not invest in such capital, and the economy will be eventually locked in an underdevelopment trap. On the other hand, if they expect high returns of specific human capital, they invest in such capital, and, as a result, the economy can exhibit long-run growth.

**Key words** : Labor specialization, endogenous choice of technology, endogenous growth, development traps

**JEL Classification** : O11, O14, O41

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# 1 Introduction

In this paper, we analyze the relationship between labor specialization and patterns of growth. This issue has been studied by several researchers, including Yang and Borland (1991) and Becker and Murphy (1992).<sup>1</sup> Yang and Borland (1991) developed a dynamic model of specialization. In their model, as the degree of division of labor increases, each individual becomes more specialized, and therethrough his learning speed rises. As a result, the whole economy can exhibit accelerated growth. Becker and Murphy (1992) developed a multisector model with increasing returns to labor specialization and coordination costs and showed that the degree of specialization of an economy is determined by the market size and coordination costs.<sup>2</sup>

Moreover, Kim (1989) developed an insightful model of labor specialization.<sup>3</sup> In the model, workers can invest in two kinds of human capital. One, general human capital, can be used to reduce the matching cost with a potential employer by expanding the range of the workers' skills, and the other, specific human capital, gives higher productivity on a limited range of specific skill. For a given distribution of each worker's general and specific human capital, a firm determines its organizational structure so as to equate the matching cost and the gain from productivity improvements in narrowing its activity range. Using this model, Kim shows that an increase in the size of the labor market raises specific human capital. Moreover, Kim and Montadi (1992) extend the Kim model to a Ramsey-type growth model and show that the endogenous evolution of labor specialization can yield endogenous growth.

In this paper we extend the Kim model to an overlapping generations model. We consider two kinds of production technologies: those with and without increasing returns due to labor specialization. The former is an advanced type, which is similar to that considered by Kim, and the latter is a primitive type. In this setting, we show that, when the market is significantly large, advanced technologies can be adopted. We also show that workers'

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<sup>1</sup>See Cheng and Yang (2004) and Yang and Ng (1998) for detailed surveys of the literature.

<sup>2</sup>See also Rosen (1983) and Murphy, Shleifer and Vishny (1991).

<sup>3</sup>See also Weitzman (1994) for a similar model.

choice of advanced technologies has strategic complementarities, and, hence, the pattern of growth of the economy may depend on worker expectations. In other words, the economy can exhibit endogenous growth in the case of optimistic expectations but may be captured by a development trap under pessimistic expectations.

The basic mechanism behind these results is as follows. In the case of a large market, the demand for the final good when the advanced technologies are adopted is large and thus the relative return of the advanced technology production is sufficiently high because of increasing returns. If workers have optimistic expectations, all workers will choose to work at firms with advanced technologies, and the economy could exhibit perpetual growth through an increasing degree of labor specialization. In contrast, when the market size is small, because of the high cost of the advanced technologies, no firms will adopt advanced technologies, which would result in a poverty trap.

These results are closely related to those of Murphy, Shleifer, and Vishny (1989), Redding (1996), and Davis (2003, 2005).<sup>4</sup> In the model of Murphy, Shleifer, and Vishny, when the market is not large, no firm could generate sufficient demand to make adoption of an increasing returns technology profitable, even though simultaneous adoption by all firms of the increasing returns technology makes the adoption profitable. Redding (1996) constructs a model with R&D and human capital accumulation and shows that human capital investment and R&D investment are strategic complements, and, thus, multiple equilibria would be possible. In other words, an economy may be captured by a low-skills trap or achieve high-growth equilibrium, depending on expectations. Moreover, Davis constructs several specialization-driven growth models with an underdevelopment trap. In Davis (2003), a trap may occur because of low-quality public institutions or infrastructure, and, in Davis (2005), it could arise because of the low ability of the economy to develop new institutions.<sup>5</sup>

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<sup>4</sup>Rodrik (1996) and Rodriguez-Clare (1996) also derive similar results. However, because their models are static, they cannot analyze the relationship between growth patterns and agents' expectations. See Azariadis and Stachurski (2005) for an excellent survey on the issue of underdevelopment traps.

<sup>5</sup>Developing a growth model in which the evolution of the division of labor not only drives long-run growth but also undermines the informal institutions, Davis (2006) shows

This paper is organized as follows. The next section contains a description of an overlapping generations model with labor specialization. The steady-state equilibria of the model are characterized in Section 3, and Section 4 is the conclusion.

## 2 The Model

### 2.1 Households

Each generation lives for two periods. The population of each generation is constant and denoted by  $N$ . A generation working in period  $t$  is called generation  $t$ . In the working period, each generation derives utility from the consumption of goods and educational expenditure for their children. In period  $t - 1$ , generation  $t$  is born and educated. In that period, generation  $t$  divides the educational expenditure given by the parents into general and specific education so as to maximize its wage in the next period. A more detailed explanation of this process is given below.

In period  $t$ , the generation  $t$  works, receives wages, and allocates the wages between consumption  $c_t$  and educational expenditure  $e_t$  for the children:

$$w_t = c_t + e_t.$$

The utility function of generation  $t$  is given by

$$U_t = (1 - \alpha) \ln c_t + \alpha \ln e_t, \quad 0 < \alpha < 1.$$

This setting yields the following optimal consumption and education expenditure for the child:

$$c_t = (1 - \alpha)w_t, \quad \text{and} \quad e_t = \alpha w_t. \quad (1)$$

### 2.2 Production

There is one final good, which is produced by the use of labor. The price of the final good is normalized as 1. At each time, there is a continuum of

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that over-specialization occurs when individuals regard the quality of informal institutions as being independent of their specialization decisions.

workers, and the total population of the workers is  $N$ . Each worker has its own characteristics concerning its suitability in specific production, and the workers are uniformly distributed on a unit circle. A worker located at point  $u$  on the circle is indexed as  $u$ , and the index represents its production ability at the point. If the worker works at a firm located somewhere other than point  $u$ , some matching costs are required.

Given the educational expenditure determined by the parent, each child accumulates two kinds of human capital, extensive (denoted by  $g$ ) and intensive (denoted by  $b$ ), and, when old, the worker supplies one unit of labor and receives a wage. Extensive human capital presents the generality of a worker's skill, whereas intensive human capital stands for the depth of his skill; in other words, the accumulation of extensive human capital reduces the matching costs with potential producers, while that of intensive human capital improves the workers' productivity.

In this economy, there are many production technologies, and they are classified into two types.<sup>6</sup> One is a "primitive technology," which is a variant of the standard constant-returns-to-scale neoclassical production technology. The other is an advanced type, which consists of a continuum of similar production technologies. Our specification of the advanced technologies basically follows Kim (1989). Each advanced technology has a different characteristic, and the characteristic is represented by its location on the same unit circle as workers. This type of technology exhibits increasing returns to labor specialization.

Each firm chooses one of the two types of technology.<sup>7</sup> The measure of workers employed by the firms with advanced technologies is denoted by  $n$ . Because workers are assumed to be uniformly distributed on a unit circle,  $n$  also represents the density of the distribution.

When young, each child determines  $g$  and  $b$  under rational expectations about the behavior of other children in the current period and the technology choice in the next period. When old, he chooses a firm to work for. Once a worker chooses a firm with one type of technology, the worker cannot work at firms with the other type of technology.

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<sup>6</sup>Iwaisako (2002) employs a similar setting.

<sup>7</sup>Once a firm selects one technology, the firm cannot switch it to the other type.

## 2.3 Primitive Technology

Each worker can increase his productivity at a firm with the primitive technology by accumulating general human capital; in other words, when worker  $j$ 's extensive human capital is  $g_j$ , the worker can produce  $a(g_j)$  unit of the final good at a firm with the primitive technology, where

$$a(g_j) = \delta g_j^\eta, \quad \delta > 0 \quad \text{and} \quad \eta > 0. \quad (2)$$

The aggregate production function of the firms with the primitive technology,  $Y^P$ , is represented by

$$Y^P = \int_{j \in S^P} a(g_j) dj, \quad (3)$$

where  $S^P$  is the set of workers employed by the primitive firms. Aggregate employment by these firms (a measure of  $S^P$ ) is denoted by  $x$ .<sup>8</sup>

In a symmetric equilibrium, we have  $a(g_j) = \delta g^\eta$  for  $\forall j$ , and, thus, (3) is reduced to

$$Y^P = A(g)x,$$

where  $A(g) \equiv \delta g^\eta$ . Because of free entry the equilibrium profit must be zero, and, thus, we have

$$w_p = A(g) = \delta g^\eta, \quad \eta < \frac{1}{2}, \quad (4)$$

which gives the wage offered by the primitive firms. The restriction  $\eta < 1/2$  implies that general skill accumulation generates a moderate improvement of productivity. As is shown below, the condition is necessary for our model to have a stable equilibrium.

## 2.4 Advanced Technologies

The advanced technologies exhibit increasing returns to labor specialization and have their own characteristic indices, which are distributed on the same circle on which workers' characteristics are distributed. Thus, each firm

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<sup>8</sup>Denote the density of worker  $i$  by  $x_i$ . Then we have

$$x \equiv \int_{i \in S^P} x_i di.$$

is indexed by skill index  $i$ , which denotes a point on the unit circle. If a worker's skill characteristics precisely match the firm's characteristics, the worker could start to work at the firm without any cost. However, if it differs from the firm's characteristics, costly training of the worker, "job requirement," would be necessary, and the cost would be paid by the firm. Since it is assumed that worker  $u$ 's productivity at firm  $i$  depends on the distance between the worker and the firm,  $s \equiv |i - u|$ , the worker's productivity at the firm could be denoted as a function of  $s$ , which is represented by  $x(s)$ . Firm  $i$ 's production is increased by the worker's specific human capital  $b$  and decreased by the distance between the firm and the worker. Specifically, the worker's productivity at the firm,  $x(s)$ , is represented by

$$x(s) = 1 + b - \frac{s}{g}.$$

The advanced technologies require an identical minimum scale  $M$ , and due to the Romer (1986) - Lucas (1988) type externalities, the productivity of a firm with the advanced type depends on the average level of general human capital of workers at the firm. Formally, we specify the productivity function of firm  $i$ , which adopts the advanced type, as

$$y_i = \begin{cases} 0 & x_i < M \\ A_i(x_i - M) & x_i \geq M, \end{cases} \quad (5)$$

where  $x_i$  and  $A_i$  are defined as

$$x_i \equiv \int_{s \in S_i} x(s) ds, \quad A_i \equiv \int_{s \in S_i} a(g_s) dF(s). \quad (6)$$

In (6),  $y_i$ ,  $S_i$ ,  $A_i$ , and  $F(s)$  represent firm  $i$ 's output, the set of workers employed by the firm, the Arrow-Romer type external effect, and the distribution function of  $a(g_s)$ , respectively. Thus,  $x_i$  and  $A_i$ , respectively, represent the total labor input and the productivity of firm  $i$ .

Following Kim (1989), let us restrict our analysis to symmetric equilibrium.<sup>9</sup> Then, since every worker chooses the same values of  $g$  and  $b$ , we have  $A_i = A(g)$ . Thus, (5) is reduced to

$$y_i = \begin{cases} 0 & x_i < M \\ A(g)(x_i - M) & x_i \geq M. \end{cases}$$

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<sup>9</sup>In the Kim model, there exist asymmetric equilibria along with the symmetric equilibria. See Kim (1990) for this point.

Under the assumption of a zero-profit symmetric bargaining equilibrium, the wage rate offered by the firms with the advanced technologies is determined by

$$w_A = A(g) \left[ 1 + b - \left( \frac{M}{gn} \right)^{\frac{1}{2}} \right], \quad (7)$$

where  $n$  is the number of workers employed in these firms. Appendix A provides a formal proof of (7). Each worker correctly anticipates this wage function before investing in the human capital.

## 2.5 Technology Choice

In this subsection we analyze the process of technology choice. Children select their composition of skill to maximize their income (wage in the old period). For this purpose, they rationally expect wages offered in both the primitive and advanced firms. As is shown below, the wage rates depend on the technology choice, and, thus, their decision on skill composition depends on the expectation on technology choice.

The labor market equilibrium condition is

$$N = n + x,$$

where  $N$  is the total population of workers. The equilibrium wage  $w^*$  satisfies

$$w^* = \max(w_p, w_A)$$

because workers can freely choose the firm to work for. This relationship can be rewritten as follows:

$$w_A \begin{matrix} \geq \\ < \end{matrix} w_p \iff \begin{cases} (n, x) = (N, 0) \\ \text{interior solution} \\ (n, x) = (0, N). \end{cases} \quad (8)$$

Since a slight perturbation changes an interior solution into a corner solution,  $(N, 0)$  or  $(0, N)$ , we focus our analysis on the corner solutions.

### 2.5.1 Human Capital Investment

When investing in two kinds of human capital, each worker rationally anticipates the wage rate at each type of firm. After the human capital investment, the worker is employed by a firm offering the highest wage.

The human capital investment process is analyzed here. The cost function of human capital accumulation is

$$e = G(b, g) = (1 + b)^\beta g^\gamma, \quad \beta > 1, \gamma > 1, \quad (9)$$

where  $e$  is the expenditure for human capital investment and this value is exogenous to the worker because it is determined by the worker's parents.  $\beta$  and  $\gamma$  respectively denote the cost parameters of specialized and generalized skill acquisition. The multiplicative formulation of the cost function makes it possible to obtain an analytical solution and the restrictions on the parameter values ( $\beta > 1$  and  $\gamma > 1$ ) imply that the return from educational expenditure in human capital investment exhibits decreasing returns to scale.

A worker's equilibrium allocation of the expenditure between  $b$  and  $g$  depends on the worker's expectation of other workers' behavior. When the worker expects  $(n, x) = (0, N)$ , setting  $b = 0$  and maximizing only with respect to  $g$  is optimal since  $b$  is not utilized in the production with the primitive technology. Therefore, (4) and (9) yield the following equilibrium combination of human capitals:

$$(b, g) = (0, e^{\frac{1}{\gamma}}). \quad (10)$$

When the worker expects  $(n, x) = (N, 0)$ , the wage rate in firms with advanced technologies  $w_A$  is given by

$$w_A \equiv w_A(n; e) = A(g) \left( \frac{M}{n} \right)^{\frac{-\gamma}{\beta-2\gamma}} e^{\frac{1}{\beta-2\gamma}} \left( \frac{2\gamma}{\beta} \right)^{\frac{\beta}{\beta-2\gamma}} \left( \frac{\beta-2\gamma}{2\gamma} \right), \quad (11)$$

as is derived in Appendix B. Here, we assume  $\beta > 2\gamma$  in order to assure that  $w_A$  will be positive, which means that the cost of specific human capital acquisition is relatively higher than that of general human capital acquisition. If this inequality does not hold, no firms adopt the advanced technologies.

As is shown below, a major determinant of technological choice in our model is the expectation of workers on the two wage rates. In analyzing this point, it is useful to introduce technological-level-adjusted wage rates

$$\hat{w}_A \equiv w_A/A(g) \text{ and } \hat{w}_p \equiv w_p/A(g) = 1. \quad (12)$$

Comparing these two rates, we can analyze the type of technology that will be adopted in equilibrium.

### 2.5.2 Equilibrium Choice of Technology

Given the common factor  $A(g)$ , it follows from (11) and (12) that a reduction in  $n$  decreases  $\hat{w}_A$  and, from (4) and (12), that  $\hat{w}_p$  is independent of  $n$ . Thus, we can depict the technological-level-adjusted wage rates as functions of  $n$  (see Figure 1). In order to see the property of the  $\hat{w}_A$  schedule let us differentiate  $\hat{w}_A$  with respect to  $\beta$  and  $\gamma$  (the  $\hat{w}_p$  schedule is trivially independent of  $\beta$  and  $\gamma$ ). Then, we have

$$\frac{\partial \ln \hat{w}_A}{\partial \beta} < 0, \quad \text{and} \quad \frac{\partial \ln \hat{w}_A}{\partial \gamma} > 0$$

under the condition that the population size  $n$  is large, the fixed cost  $M$  is low and educational expenditure  $e$  is large. In this case, a reduction in the cost of specific skill accumulation or a rise in the cost of general skill accumulation shifts the  $\hat{w}_A$  schedule upward but has no effect on the  $\hat{w}_p$  schedule.

Using Figure 1, we next examine how the equilibrium technology choice is determined. Equations (11) and (12) show the following inequality for sufficiently small values of  $n$ :

$$1 = \hat{w}_p > \hat{w}_A(n; e) = \left(\frac{M}{n}\right)^{\frac{-\gamma}{\beta-2\gamma}} e^{\frac{1}{\beta-2\gamma}} \left(\frac{2\gamma}{\beta}\right)^{\frac{\beta}{\beta-2\gamma}} \left(\frac{\beta-2\gamma}{2\gamma}\right), \quad (13)$$

as is depicted in Figure 1. Thus, if workers expect that no workers will be employed by firms with advanced technologies, then all workers will always choose firms with the primitive technology. In contrast, if

$$\hat{w}_A(n; e) > 1 \quad (14)$$

holds, firms with advanced technologies are chosen by workers.<sup>10</sup> Using (4) and (11), we can express (14) as

$$\left(\frac{M}{n}\right)^{\frac{-\gamma}{\beta-2\gamma}} e^{\frac{1}{\beta-2\gamma}} \left(\frac{2\gamma}{\beta}\right)^{\frac{\beta}{\beta-2\gamma}} \left(\frac{\beta-2\gamma}{2\gamma}\right) > 1.$$

This inequality may hold if  $n$  is sufficiently large (see Figure 1).

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<sup>10</sup>If  $w_A = w_p$ , each worker is indifferent to both types of firms.

Furthermore, because  $\hat{w}_A(n; e)$  is a monotonic function of  $e$ , we can express these results in terms of  $e$ . Define a new function:

$$\bar{e}(n) \equiv \left( \frac{\beta}{\beta - 2\gamma} \right)^{\beta - 2\gamma} \left( \frac{\beta}{2\gamma} \right)^{2\gamma} \left( \frac{M}{n} \right)^\gamma.$$

Then, if

$$e > \bar{e}(n), \tag{15}$$

each worker selects to work at an advanced firm. On the other hand, if

$$e < \bar{e}(n), \tag{16}$$

each worker chooses to work at a firm with primitive technology. This implies that, if educational expenditure is sufficiently high and/or the fixed cost is sufficiently low, all workers can be employed by firms with advanced technologies. However, it should be noted here that whether or not this technology selection is realized depends on worker expectations. As (15) shows, for a given value of  $n$ , if the level of educational expenditure is sufficiently large, i.e., if  $e > \bar{e}(n)$ , all firms decide to adopt the advanced type and, as a result,  $n$  increases. In this case, because all firms adopt the advanced technologies, we have  $n = N$  in equilibrium. In contrast, if workers expect  $n = 0$ ,  $e$  becomes smaller than  $\lim_{n \rightarrow 0} \bar{e}(n)$  because  $\lim_{n \rightarrow 0} \bar{e}(n) = \infty$ . Thus, all workers choose to work at primitive firms; in other words,  $n = 0$  always constitutes an equilibrium.

These arguments are summarized in Figure 1. As is depicted in Figure 1, when the curve which represents  $\hat{w}_A(n; e)$  intersects with the  $\hat{w}_p$  line, there are multiple equilibria, while, when the curve is located below the  $\hat{w}_p$  line, there is only an equilibrium without labor specialization.

### 3 Steady States

#### 3.1 Long-run Growth with Labor Specialization

When  $n = N$ , the equilibrium wage rate is determined by  $w_{At} = w_A(N, e_{t-1})$ , as shown by (11). Combining (1), (4), (11),  $n = N$ , and the equilibrium level of  $g_t$  (given by (B6) in Appendix B), we obtain the following equilibrium

wage dynamics:

$$w_{t+1} = \delta \left( \frac{\beta}{2\gamma} \right)^{\frac{\beta(1-2\eta)}{\beta-2\gamma}} \left( \frac{M}{N} \right)^{\frac{\beta\eta-\gamma}{\beta-2\gamma}} \left( \frac{\beta-2\gamma}{2\gamma} \right) (\alpha w_t)^{\frac{1-2\eta}{\beta-2\gamma}}. \quad (17)$$

As is apparent from (17), the property of equilibrium dynamics depends on the values of several parameters. It is easy to show that, when  $\frac{1-2\eta}{\beta-2\gamma} > 1$ , that is, when  $1 > 2\eta + \beta - 2\gamma$ ,  $w_t$  diverges as time goes to infinity, whereas when  $2\eta + \beta - 2\gamma > 1$ ,  $w_t$  converges to

$$w^* = \left[ \alpha^{1-2\eta} \delta^{\beta-2\gamma} \left( \frac{\beta}{2\gamma} \right)^{\beta(1-2\eta)} \left( \frac{M}{N} \right)^{\beta\eta-\gamma} \left( \frac{\beta-2\gamma}{2\gamma} \right)^{\beta-2\gamma} \right]^{\frac{1}{\beta-2\gamma+2\eta-1}}.$$

Because our main concern lies in analyzing the relationship between labor specialization and the long-run growth rate, we impose the following restriction:

*Balanced Growth Restriction*

$$2\eta + \beta - 2\gamma = 1.$$

Under this restriction, the wage rate of the economy  $w_{At}$  grows at a constant rate:

$$\hat{w} \equiv \frac{w_{t+1} - w_t}{w_t} = \alpha \delta \left( \frac{M}{N} \right)^{-\frac{\beta-1}{2}} \left( \frac{\beta}{\beta+2\eta-1} \right)^{\beta} \frac{1-2\eta}{\beta+2\eta-1} - 1. \quad (18)$$

Intuitively speaking, under (18), the sum of positive effects from increasing returns to specialization and positive external effects in general human capital accumulation offset increasing costs in human capital accumulation, and therefore the economy can exhibit perpetual balanced growth.<sup>11</sup>

As (18) indicates, the balanced growth rate depends on several parameters in the model. In order to see the dependence of the growth rate on the parameters we differentiate (18) with respect to  $\alpha$ ,  $M$  and  $N$  and obtain

$$\frac{\partial \hat{w}}{\partial \alpha} > 0, \quad \frac{\partial \hat{w}}{\partial N} > 0 \quad \text{and} \quad \frac{\partial \hat{w}}{\partial M} < 0.$$

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<sup>11</sup>A similar condition is derived by Davis (2004).

These results show that educational expenditure, which is determined by  $\alpha$ , and the scale of the market, which may be captured by the size of the population in this study, are positively related to the long-run growth rate, and the fixed cost is negatively related to the growth rate.<sup>12</sup>

### 3.2 No-growth Equilibrium without Labor Specialization

In the case of  $(n, x) = (0, N)$ , all firms adopt the primitive technology and no firms employ the advanced type. The wage dynamics are described by

$$w_{t+1} = \delta(\alpha w_t)^{\frac{\eta}{\gamma}},$$

which is derived from (1), (4), and (10). Because  $\eta < 1 < \gamma$ , this difference equation is stable, and the economy always converges to the following steady state:

$$\bar{w}_p = \delta^{\frac{\gamma}{\gamma-\eta}} \alpha^{\frac{\eta}{\gamma-\eta}}. \quad (19)$$

In other words, this equilibrium exhibits no long-run growth. Because this equilibrium always exists even if there is another equilibrium with labor specialization, our economy has the potential for underdevelopment traps.

### 3.3 Multiple Equilibria and Underdevelopment Traps

Let us derive a condition under which our economy has multiple equilibria. As is apparent from our analysis above, when  $w_A(N) > w_p$ , there are multiple equilibria. From (1) and (19), we obtain the following steady-state educational expenditure in a no-growth equilibrium:

$$\bar{e}_p = (\delta\alpha)^{\frac{\gamma}{\gamma-\eta}}. \quad (20)$$

From (20), (15), and  $n = N$ , it can easily be seen that, if

$$N > M \left( \frac{\beta}{2\gamma} \right)^2 \left( \frac{\beta}{\beta - 2\gamma} \right)^{\frac{\beta-2\gamma}{\gamma}} (\delta\alpha)^{\frac{-1}{\gamma-\eta}} \quad (21)$$

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<sup>12</sup>It is noteworthy that, as Davis (2004) shows, in many models with division of labor, transaction costs endogenously determine the market size, and the size is independent of the population. In this sense, it may be more adequate to regard  $M$  as a crucial determinant of the market size. However, even in this interpretation, a reduction in  $M$  would lead to a larger market and raise the long-run growth rate, and hence the market size is positively related to the growth rate in a balanced growth equilibrium.

holds, then there is an endogenous growth equilibrium along with the no-growth equilibrium. It is obvious that there are parameter sets that satisfy the above inequality. For example, a low fixed cost and a large size of population yield the possibility of multiple equilibria. In this case, if we can change worker expectations, the economy can escape from the underdevelopment trap.

In summary, the degree of labor specialization in our economy critically depends on the market size and expectations of agents. If the size of the market is sufficiently large, the demand for the final good when the advanced technologies are adopted is large enough to reduce the relative production cost under the advanced technologies. As a result, if agents have optimistic expectations, the economy adopts the advanced technologies and can exhibit perpetual growth through an increasing degree of labor specialization. In contrast, if the market is small and/or the production cost by the advanced technologies is high, the advanced technologies are not employed by any firms; in other words, the economy adopts only the primitive technology, and, thus, the economy exhibits only exogenous growth. An economy in which (21) does not hold inevitably falls into a no-growth equilibrium. In this case, some policies that affect  $\alpha$ ,  $M$ ,  $N$ , and  $\delta$  are necessary to escape from the trap.

## 4 Conclusion

This study extended the Kim (1989) model of endogenous labor specialization to an overlapping generations model with an endogenous technology choice and obtained the following results. The adoption of the increasing returns technology with labor specialization is possible if the size of the market is sufficiently large and/or the fixed cost is sufficiently small, while the adoption of the primitive technology, which is the constant returns technology, is always possible. Therefore, there can be multiple equilibria in this economy, and its pattern of growth may depend on worker expectations.

# Appendix

## A. Wage Offered by Advanced Firms

This appendix derives the equilibrium wage offered by advanced firms. An advanced firm hires workers if the marginal value product of the worker at the firm exceeds the firm's training cost, and a worker selects a firm offering the highest net wage if the net wage is not smaller than the his reservation wage  $\tilde{w}$ . The profit of a typical advanced firm is given as

$$\Pi(H) = A(g) \left\{ 2n \int_0^H \left( 1 + b(s) - \frac{s}{g(s)} \right) ds - M \right\} - 2n \int_0^H w(s) ds, \quad (\text{A1})$$

where  $H$  is the maximum acceptable skill difference of the firm. Following Kim (1989), we limit our analysis to the zero-profit symmetric bargaining equilibrium; in other words, we assume that (i) all workers choose the same skill combination  $(b, g)$ , (ii) the advanced production functions are symmetric for all firms and the wages offered by these firms are identical, and (iii) firms are equally spaced on the circle. Under these assumptions, an  $i$ th firm with an advanced technology employs workers distributed on the interval  $(i - H, i + H)$ , and, therefore, the extent of workers employed by the firm is  $2H$  and the number of firms in the market,  $n$ , is equal to  $1/(2H)$ .

The equilibrium wage is determined by Nash bargaining between the worker and the firm, that is, the wage is determined so as to maximize the Nash product:

$$NP = \left( \underbrace{x(s) - w(s)}_{\substack{\text{accept} \\ \text{(firm)}}} - \underbrace{0}_{\substack{\text{refuse} \\ \text{(firm)}}} \right) \left( \underbrace{w(s)}_{\substack{\text{accept} \\ \text{(worker)}}} - \underbrace{\tilde{w}}_{\substack{\text{refuse} \\ \text{(worker)}}} \right),$$

where  $\tilde{w}$  is the worker's reservation wage. We assume that the reservation wage is the highest possible wage in the negotiation with other firms on the unit circle because we assume that, once a worker selects a firm with advanced technology, the worker cannot work at firms with primitive technology, although the worker can work at other firms with advanced technologies. Because the distance between two firms is  $2H$  and symmetric equilibrium gives  $b = b(s)$  and  $g = g(s)$ , we have

$$\tilde{w}(s) = A(g) \left( 1 + b - \frac{2H - s}{g} \right).$$

Thus, the wage can be computed as

$$w(s) = A(g) \frac{\left[1 + b - \frac{s}{g}\right] + \left[1 + b - \frac{2H-s}{g}\right]}{2} = A(g) \left(1 + b - \frac{H}{g}\right), \quad (\text{A2})$$

where  $A(g) = \delta g^n$  from (6). It should be noted that this wage function is independent of  $s$  and, thus, a symmetric equilibrium is possible.

From the zero profit condition in (A1), we have

$$H = \left(\frac{gM}{n}\right)^{\frac{1}{2}}. \quad (\text{A3})$$

Substituting (A3) into (A2), we obtain (7) in the main text.

## B. Selection of Technology

The wage rate is determined by condition (8), and the cost function is specified as (9). In this setting, the optimal allocation of human capitals is derived as follows. When a worker expects  $n = 0$ , the worker maximizes its general skill, and, thus

$$(\bar{g}, \bar{b}) = (e^{\frac{1}{\gamma}}, 0). \quad (\text{B1})$$

When the expectation of workers is given by  $n$ , the Lagrangian function is defined as

$$\begin{aligned} \mathcal{L} &= w^*(b, g) + \lambda[e - G(b, g)] \\ &= A \left[1 + b - \left(\frac{M}{gn}\right)^{\frac{1}{2}}\right] + \lambda [e - (1 + b)^\beta g^\gamma]. \end{aligned}$$

From this we obtain the following first-order conditions<sup>13</sup>:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial b} &= \frac{\partial w^*}{\partial b} - \lambda \frac{\partial G}{\partial b} \\ &= A + \lambda [-\beta(1 + b)^{\beta-1} g^\gamma] = 0, \end{aligned} \quad (\text{B2})$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial g} &= \frac{\partial w^*}{\partial g} - \lambda \frac{\partial G}{\partial g} \\ &= \frac{A}{2} \left(\frac{M}{n}\right)^{\frac{1}{2}} g^{-\frac{3}{2}} + \lambda [-\gamma(1 + b)^\beta g^{\gamma-1}] = 0. \end{aligned} \quad (\text{B3})$$

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<sup>13</sup>It can be easily shown that the second order condition is satisfied under our setting.

Combining (B2) and (B3) gives the optimal condition for human capital allocation as follows:

$$\frac{\frac{\partial}{\partial b} w^*(b, g)}{\frac{\partial}{\partial g} w^*(b, g)} = \frac{\frac{\partial}{\partial b} G(b, g)}{\frac{\partial}{\partial g} G(b, g)}. \quad (\text{B4})$$

This equation implies that the marginal rate of substitution of skills is equal to the marginal rate of transformation of skills. This relationship between  $b_t$  and  $g_t$  is solved as follows.

$$b = \frac{1}{2} \frac{\beta}{\gamma} \left( \frac{M}{n} \right)^{\frac{1}{2}} g^{-\frac{1}{2}} - 1. \quad (\text{B5})$$

Substituting (B5) into the cost function yields

$$g = \left( \frac{\beta}{2\gamma} \right)^{\frac{2\beta}{\beta-2\gamma}} \left( \frac{M}{n} \right)^{\frac{\beta}{\beta-2\gamma}} e^{\frac{-2}{\beta-2\gamma}}. \quad (\text{B6})$$

Combining (B6) and (B5), we obtain the level of intensive human capital in an endogenous growth equilibrium as a function of educational expenditure:

$$b = \left( \frac{\beta}{2\gamma} \right)^{\frac{-2\gamma}{\beta-2\gamma}} \left( \frac{M}{n} \right)^{\frac{-\gamma}{\beta-2\gamma}} e^{\frac{1}{\beta-2\gamma}} - 1. \quad (\text{B7})$$

Substituting (B6) and (B7) into  $w_A$ , the wage offered by advanced firms is obtained as

$$w_A = A \left( \frac{M}{n} \right)^{\frac{-\gamma}{\beta-2\gamma}} e^{\frac{1}{\beta-2\gamma}} \left( \frac{2\gamma}{\beta} \right)^{\frac{\beta}{\beta-2\gamma}} \left( \frac{\beta - 2\gamma}{2\gamma} \right). \quad (\text{B8})$$

It should be noted here that  $w_A$  in (B8) is always non-negative under the assumption of  $\beta > 2\gamma$ . As is shown in the main text, in an endogenous growth equilibrium, the wage rate continues to grow, and  $e$  is proportional to the wage rate (see [1]). Hence, in a balanced-growth equilibrium,  $w_A$  is positive if the initial value of  $w_A$  is positive. This case can be easily supported by choosing adequate parameter values, and, thus, we assume this case in this paper.

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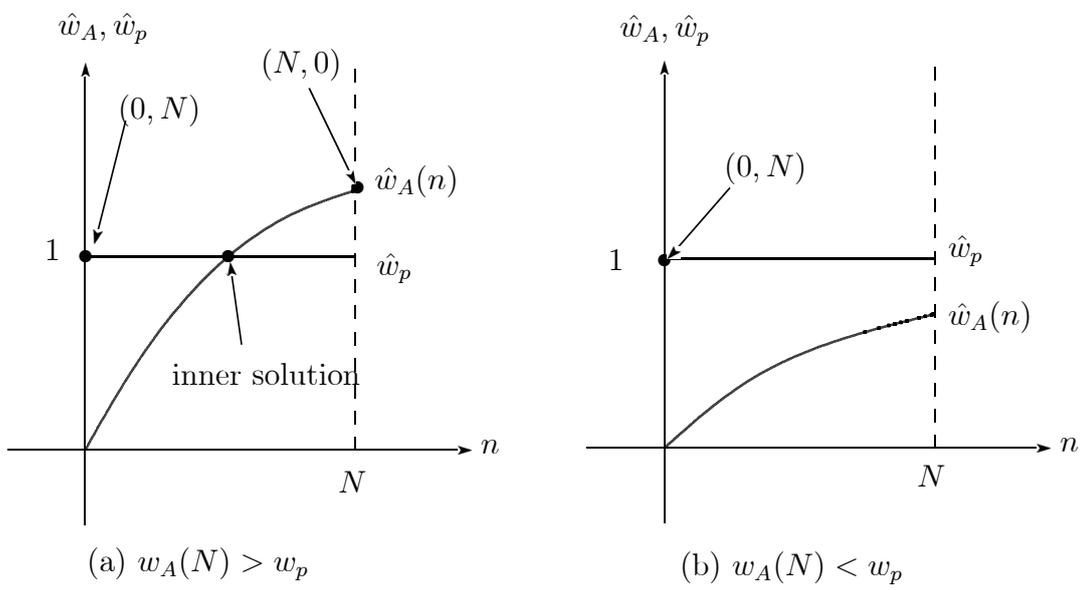


Figure 1: Selection of Technology