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“Public Investment and Golden Rule of Public Finance in an  
Overlapping Generations Model”

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# Public Investment and Golden Rule of Public Finance in an Overlapping Generations Model

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## Abstract

This paper develops an overlapping generations model with debt-financed public investment. The model assumes that the government is subject to the golden rule of public finance and that households are Yaari-Blanchard type. It is shown that the growth-maximizing and utility-maximizing tax rates do not satisfy the Barro tax rule, which is equal to the output elasticity of public capital. Furthermore, we show that both tax rates positively depend on longevity, with an aging population increasing debt per GDP. This result captures a tendency of increasing debt per GDP under population aging in the real world.

*Keywords:* Public capital; Golden rule of public finance; Economic growth

*JEL classification:* H54; H60; O40

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## 1. Introduction

This paper investigates the growth and welfare effects of public investment under the golden rule of public finance using an endogenous growth model with private and public capital. The golden rule and its variations have been legally adopted by some countries between 1985 and 2014 (e.g., Brazil; Costa Rica; Germany; Japan; Luxembourg; Malaysia; United Kingdom).<sup>1</sup> The golden rule incorporates the possibility borrowing to finance productive public investment could pay for itself over the long-term, and a balanced current budget is consistent with a positive steady-state ratio of public debt to GDP if public investment is productive (IMF, 2014, Ch. 3). Debt-financed public investment affects economic growth and intergenerational welfare through different long-term benefits and costs. An increase in longevity increases the benefit to future generation from public investments. Therefore, the central focus of our analysis is to clarify the relationship among public investment, economic growth, and population aging under the specified fiscal rule.

Many studies have investigated the macroeconomic effects of fiscal policy under various fiscal rules, including the golden rule (e.g., Greiner and Semmler 2000; Ghosh and Mourmouras 2004; Greiner 2007, 2010; Minea and Villieu 2009; Groneck 2011; Kamiguchi and Tamai 2012; Tamai 2014, 2016). In particular, the economic impacts of fiscal deficit, fiscal rules, and sustainability of public debt on economic growth have been studied using the extended models of Barro (1990) and Futagami et al. (1993).<sup>2</sup> This interest has been motivated by the productivity slowdown and fiscal deterioration of national budgets that major countries have experienced in recent years; the productivity effects of public investment financed by public debt and the significance of fiscal institutions have been widely recognized since the global financial crisis (IMF, 2014, Ch. 3).

One classical view on the productivity slowdown is rooted in the productivity effects of public capital, as shown by Aschauer (1989). Many subsequent studies have also shown a positive productivity effect of public capital; a decrease in public investment negatively affects economic growth.<sup>3</sup> The debt to GDP ratios in the major countries shown in Figure 1 indicate that the debt to GDP ratio in each country has increased more than 20% since 2007. Furthermore, Figure 1 also show that the indicator has a tendency to increase over time. This fiscal deterioration of national budgets tend to curtail investment expenditure in public capital and therefore will hamper economic growth. Indeed, some empirical studies showed that public debt is negatively associated with economic growth. For instance, Checherita-Westphal and Rother (2012) showed that public debt has a negative impact on economic growth when it accumulates to around 90%–100% of GDP.

[Figure 1]

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<sup>1</sup> See the IMF's Fiscal Rules Dataset.

<sup>2</sup> Numerous studies have examined the economic effects of public investment since the outstanding study by Arrow and Kurz (1970). See Irmen and Kuehnelt (2009) for a survey of this literature.

<sup>3</sup> Aschauer (1989) produced the pioneering study of empirical analysis on the productivity effects of public capital. See Ligthart and Suárez (2011), Pereira and Andrzej (2013), and Bom and Ligthart (2014) for issues with recent empirical studies.

Drawing upon these empirical findings, some theoretical studies have investigated the relationship between economic growth and public debt using an endogenous growth model with public capital and specified fiscal rules. Minea and Villieu (2009) analyzed an economy where the government adopts the golden rule of public finance. Their study shows that fiscal policy under the golden rule of public finance negatively impacts long-run economic growth; however, the possibility exists of improving intertemporal welfare compared with the balanced-budget rules. Groneck (2011) also used a Ramsey-type growth model to analyze the role of the golden rule of public finance on the economic growth rate and welfare. He showed that the government's policy would have positive effects on both long-run growth and welfare if the government spending benefits individuals' utility and raises the marginal product of private capital. He mentioned that positive growth effects are observed only when public consumption expenditures are lowered in the long-run.

On the other hand, the effects of debt-financing public investment on intergenerational welfare has not been sufficiently studied enough to their properties. This reflects the fact that these previous studies assumed that a representative household is infinitely lived. However, public debt serves as a form of intergenerational transfer: if the government issues public debt to invest in public capital, the benefit is provided not only to current individuals but also to future ones. To capture this mechanism clearly, we use the overlapping generations model. Furthermore, our study incorporates the probability of death, thus making it possible to analyze how longevity affects the economic circumstances and government policy under an aging population.

In the empirical literature, Pan and Wang (2012) provided evidence that an increase in the old-age dependency ratio caused an increase in the public debt to GDP ratio in the Euro area. Checherita-Westphal and Rother (2012) also provide evidence of this effect. Figure 2 shows that the ratio of population aged over 65 to the population of all ages has risen in six countries. The public debt to GDP ratio in six countries has also increased, as shown in Figure 1. These evidences and our investigation of the empirical literatures indicate that population aging is important factor influencing the government's debt policy. We show the interaction between population aging and public debt policy using a Yaari-Blanchard type OLG model.

[Figure 2]

Some recent studies have investigated the effects of public investment using the OLG model (Tanaka 2003; Yakita 2008; Tamai 2009; Arai 2011; Teles and Mussolini 2014).<sup>4</sup> However, these studies do not incorporate the golden rule of public finance, and therefore do not focus on the issue of intergenerational effects of debt-financed public investment. As shown by Tamai (2016), the golden rule can actualize the first-best equilibrium. However, the model, which can be compared to the existing, widely used model, needs further development. Therefore, this paper examines the growth

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<sup>4</sup> Tanaka (2003) and Tamai (2009) are extensions of Saint-Paul (1992), while Yakita (2008), Arai (2011), and Teles and Mussolini (2014) are extensions of Bräuning (2005).

and welfare effects of debt-financed public investment under the golden rule and population aging through the former effect.

This paper obtained the following results. First, the Barro tax rule does not hold; the growth-maximizing tax rate is less than the output elasticity of public capital. Second, the growth-maximizing tax rate is not equivalent to the utility-maximizing one, although there are no transitional dynamics. Third, growth- and utility-maximizing tax rates are positively associated with longevity. Based on this relationship, population aging increases the equilibrium tax rate by majority voting and debt to GDP ratio. In contrast to previous studies, our investigation captures the effect of population aging, showing that it affects public investment through the alteration of the tax rate.

The remainder of this paper is organized as follows. The next section explains a basic setup of our model and characterizes the dynamic equilibrium. Section 3 considers the growth and welfare effects of debt-financed public investment. It also characterizes the intergenerational effects of public investment financed by public debt. Section 4 examines the relationship between fiscal policy and longevity. Finally, Section 5 concludes this paper.

## 2. The model

This section describes the basic setup of our mathematical model. Following Futagami et al. (1993), final goods are produced using private and public capital. The production function is specified as

$$Y(t) = \phi K(t)^{1-\alpha} G(t)^\alpha, \quad (1)$$

where  $Y(t)$  is total output of final goods,  $K(t)$  is private capital, and  $G(t)$  is public capital. The parameters in equation (1) satisfy  $0 < \alpha < 1$  and  $\phi > 0$ . After-tax factor prices are given by

$$r(t) = (1 - \tau)(1 - \alpha) \frac{Y(t)}{K(t)}, \quad (2a)$$

$$w(t) = (1 - \tau)\alpha Y(t), \quad (2b)$$

where  $\tau$  denotes the tax rate on output.<sup>5</sup>

Regarding the setting of households, we follow Yaari (1965) and Blanchard (1985). Each household faces the probability of death,  $\lambda \geq 0$ , at any moment. Without loss of generality, we assume that the size of a new cohort is also equal to  $\lambda$ . Then, the population size is

$$N(t) = \int_{-\infty}^t \lambda e^{-\lambda(t-v)} dv = 1.$$

We assume that the instantaneous utility function is a logarithmic function of private consumption. Then, the expected lifetime utility of a household born at time  $s$  is

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<sup>5</sup> Tamai (2009) examined how the difference output tax and income tax affect the results of equilibrium analysis using the OLG model presented by Yaari (1965) and Blanchard (1985).

$$EU = E \left[ \int_t^\infty \log c(v, s) e^{-\rho(v-t)} dv \right],$$

where  $c(v, s)$  denotes the private consumption at time  $v$  for a household born at time  $s$  ( $s \leq t$ ). The probability of being alive at time  $v$  is  $e^{-\lambda(v-t)}$ . Therefore, the expected lifetime utility function is rewritten as

$$EU = \int_t^\infty \log c(v, s) e^{-(\rho+\lambda)(v-t)} dv, \quad (3)$$

The budget constraint for a household born at time  $s$  ( $s \leq v$ ) is

$$\frac{da(v, s)}{dv} = (r(v) + \lambda)a(v, s) + w(v) - c(v, s), \quad (4)$$

$$a(s, s) = 0,$$

where  $a(v, s)$  denotes the financial asset at time  $v$  of household born at time  $s$  ( $s \leq t$ ). Households choose their private consumption to maximize equation (3) subject to equation (4). Solving the maximization problem, we obtain

$$\frac{dc(t, s)}{dt} = (r(t) - \rho)c(t, s), \quad (5a)$$

$$\lim_{v \rightarrow \infty} a(v, s) \exp \left[ - \int_t^v \{r(z) + \lambda\} dz \right] = 0. \quad (5b)$$

Using equations (4), (5a), and (5b), the consumption function of generation  $s$  is

$$c(t, s) = (\rho + \lambda)[a(t, s) + h(t)], \quad (6a)$$

where  $h(t)$  is the present value of labor income such as

$$h(t) \equiv \int_t^\infty w(v) \exp \left[ - \int_t^v \{r(z) + \lambda\} dz \right] dv. \quad (6b)$$

*Aggregation.* We now consider aggregate variables and its dynamics. By the definition, the aggregate variables are

$$C(t) \equiv \int_{-\infty}^t c(t, s) \lambda e^{-\lambda(t-s)} ds, \quad (7a)$$

$$A(t) \equiv \int_{-\infty}^t a(t, s) \lambda e^{-\lambda(t-s)} ds, \quad (7b)$$

$$H(t) \equiv \int_{-\infty}^t h(t) \lambda e^{-\lambda(t-s)} ds = h(t). \quad (7c)$$

Using equations (6a)–(7c), we obtain the aggregate consumption function such as

$$C(t) = (\rho + \lambda)[A(t) + H(t)]. \quad (8)$$

Differentiation of equations (7b) and (7c) with respect to  $t$  provides

$$\dot{A}(t) = r(t)A(t) + w(t) - C(t), \quad (9a)$$

$$\dot{H}(t) = [r(t) + \lambda]H(t) - w(t), \quad (9b)$$

Using equations (8)–(9b), we have

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho - (\rho + \lambda)\lambda \frac{A(t)}{C(t)}. \quad (9c)$$

In equation (9c), the final term is based on the *generation replacement effects*. Newcoming households have no financial wealth but existing households have some amount of financial assets. At each moment, some households die and are replaced by newcomers. Therefore, the last term has a negative effect on consumption growth. This term will play important roles in our dynamic analysis of fiscal policy.

The government taxes total sales and issues bonds, and allocates its revenue to interest payments and public investment. Then, the budget constraint for the government is

$$\dot{B}(t) + \tau Y(t) = r(t)B(t) + \dot{G}(t) + C_g(t), \quad (10)$$

where  $B(t)$  represents government bonds and  $C_g(t)$  denotes unproductive government spending that does not affect the utility and production of private sector. Unproductive government spending does not affect the private-sector's utility and production. Then, it is obvious that unproductive government spending has a negative effects on economic growth and welfare. Therefore, we set  $C_g(t) = 0$  in the theoretical part of this paper.

We assume that the government adopts the golden rule of public finance because it is a well-known fiscal rule in public finance and its variations have been adopted in the real world (e.g., Germany; Japan; United Kingdom). The golden rule requires that the government bond issuance is permissible within financing public investment. Therefore, we have  $\dot{B}(t) = \dot{G}(t)$ .<sup>6</sup> To fulfill the government's budget constraint,  $B(0) = G(0)$  is required (See Appendix A.1). Then, we obtain

$$B(t) = G(t). \quad (11)$$

Using equations (10) and (11), we arrive at

$$\tau Y(t) = r(t)B(t).$$

The above equations (1) and (2a) provide

$$b(t) \equiv \frac{B(t)}{K(t)} = \frac{\tau}{(1 - \tau)(1 - \alpha)}, \quad (12a)$$

$$g(t) \equiv \frac{G(t)}{K(t)} = \frac{\tau}{(1 - \tau)(1 - \alpha)}. \quad (12b)$$

We now consider equilibrium conditions. The clearing condition for the financial market is

$$A(t) = K(t) + B(t). \quad (13)$$

Using equations (1)–(2b), (9a), (10), (11), and (13), we arrive at the following resource constraint of

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<sup>6</sup> The golden rule is one special case of deficit-financing such as  $\dot{B}(t) = \mu \dot{G}(t)$  where  $0 < \mu \leq 1$ . If  $0 < \mu < 1$  and  $C_g(t) = \theta Y(t)$ , equation (10) becomes  $\dot{B}(t) = [r(t)B(t) + (\theta - \tau)Y(t)]/(1 - \mu)$ . Under initial condition  $B(0) = \mu G(0)$ ,  $B/G = \mu$  will hold in equilibrium. Using this equation and balanced growth condition, some equations will be modified [e.g., (12a), (12b)]. Then, the dynamic properties of the model will be close to the model under balanced budget regime in the sense of current government budget (i.e., Futamiga et al 1993). Therefore, the effects of fiscal policy in such case will be an intermediate case between Futamiga et al. (1993) and this paper.

this economy:

$$\dot{K}(t) = Y(t) - C(t) - C_g(t) - \dot{G}(t) = Y(t) - C(t) - C_g(t) - \dot{B}(t). \quad (14a)$$

Equations (1), (12a), (12b), and (14a) lead to

$$\frac{\dot{K}(t)}{K(t)} = \frac{\phi g^\alpha - z(t)}{1 + g}, \quad (14b)$$

where  $z(t) \equiv C(t)/K(t)$ . Using equations (9c), (12a), (12b), (13), and (14b), we obtain

$$\frac{\dot{z}(t)}{z(t)} = \frac{\dot{C}(t)}{C(t)} - \frac{\dot{K}(t)}{K(t)} = (1 - \tau)(1 - \alpha)\phi g^\alpha - \rho - \frac{(1 + g)(\rho + \lambda)\lambda}{z(t)} - \frac{\phi g^\alpha - z(t)}{1 + g}. \quad (15)$$

We now define the balanced growth equilibrium (BGE) as the equilibrium that satisfies

$$\frac{\dot{C}(t)}{C(t)} = \frac{\dot{K}(t)}{K(t)} = \frac{\dot{G}(t)}{G(t)} = \frac{\dot{B}(t)}{B(t)}.$$

BGE requires  $\dot{z}(t) = 0$ . Equation (15) has the following properties:

$$\begin{aligned} \frac{d}{dz(t)} \left( \frac{\dot{z}(t)}{z(t)} \right) &= \frac{(1 + g)(\rho + \lambda)\lambda}{z(t)^2} + \frac{1}{1 + g} > 0, \\ \lim_{z(t) \rightarrow 0} \left( \frac{\dot{z}(t)}{z(t)} \right) &= -\infty, \quad \lim_{z(t) \rightarrow \infty} \left( \frac{\dot{z}(t)}{z(t)} \right) = \infty. \end{aligned}$$

Therefore, we obtain a unique value of  $z(t)$  that satisfies the BGE condition from equation (15) with  $\dot{z}(t) = 0$ . Define  $\gamma$  as the equilibrium growth rate, which is given as

$$\gamma = (1 - \tau)(1 - \alpha)\phi g^\alpha - \rho - \frac{(1 + g)(\rho + \lambda)\lambda}{z}. \quad (16)$$

### 3. Growth and welfare effects of fiscal policy

#### 3.1 Growth effect of fiscal policy

In this subsection, we examine the growth effects of fiscal policy. We begin our analysis by deriving the effect of fiscal policy on the public to private capital ratio. Differentiating (12b) with respect to  $\tau$ , we obtain

$$\frac{dg}{d\tau} = \frac{1}{(1 - \alpha)(1 - \tau)^2} > 0. \quad (17)$$

A rise in  $\tau$  increases tax revenues and decreases interest payments through after-tax interest rate declines. Then, the government is allowed to issue public bonds to finance public investment. Therefore, a rise in  $\tau$  increases the ratio of public capital to private capital.

Differentiating equation (2a) with respect to  $\tau$  and using equation (17), we have

$$\frac{dr}{d\tau} = -(1-\alpha)\phi g^\alpha + (1-\tau)(1-\alpha)\alpha\phi g^{\alpha-1}\frac{dg}{d\tau} = (1-\alpha)\phi g^\alpha \left[ \frac{\alpha}{\tau} - 1 \right] \geq 0 \Leftrightarrow \tau \leq \alpha. \quad (18)$$

This formula is a well-known result as Barro rule in studies of fiscal policy and economic growth (e.g., Barro 1990; Futagami et al. 1993). However, in the OLG model, growth maximization will be unattainable if the government sets  $\tau$  to  $\alpha$ . This is because the equilibrium growth rate depends on the tax rate not only through the interest rate but also through the specific term of the OLG model.

By differentiation of equation (16) with respect to  $\tau$ , we arrive at the growth effect of fiscal policy:

$$\frac{d\gamma}{d\tau} = \frac{dr}{d\tau} - \frac{d}{d\tau} \left[ \frac{(1+g)(\rho+\lambda)\lambda}{z} \right] = \frac{dr}{d\tau} - (\rho+\lambda)\lambda \frac{d\omega}{d\tau}, \quad (19)$$

where

$$\omega \equiv \frac{1+g}{z} = \frac{A(t)}{C(t)}.$$

The second term in equation (19) captures the effects of income tax through the generation replacement effects. Using equation (15) and the BGE condition, we can derive it as

$$\begin{aligned} (\rho+\lambda)\lambda \frac{d\omega}{d\tau} &= \frac{(\rho+\lambda)\lambda}{(\rho+\lambda)\lambda + \frac{1}{\omega^2}} \left[ \frac{dr}{d\tau} - \frac{d}{d\tau} \left( \frac{\phi g^\alpha}{1+g} \right) \right] \\ &= \frac{(\rho+\lambda)\lambda}{(\rho+\lambda)\lambda + \frac{1}{\omega^2}} \left\{ \frac{dr}{d\tau} - \frac{\phi g^\alpha}{1+g} \left[ \frac{\alpha - \frac{\tau}{1-\tau}}{(1+g)g} \right] \frac{dg}{d\tau} \right\}. \end{aligned} \quad (20a)$$

Note that

$$\left. \frac{d\omega}{d\tau} \right|_{\tau=\alpha} = \frac{\alpha^2}{(1-\alpha) \left[ (\rho+\lambda)\lambda + \frac{1}{\omega^2} \right]} \frac{\phi g^\alpha}{1+g} \frac{dg}{d\tau} \Big|_{\tau=\alpha} > 0, \quad (20b)$$

$$\left. \frac{d\omega}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} = \frac{1}{(\rho+\lambda)\lambda + \frac{1}{\omega^2}} \frac{dr}{d\tau} \Big|_{\tau=\frac{\alpha}{1+\alpha}} > 0. \quad (20c)$$

Without additional parameter restrictions, we cannot identify the sign on equation (19). As described above, previous studies showed that the growth-maximizing tax rate is equal to the output elasticity of public capital (e.g., Barro 1990; Futagami et al. 1993). Therefore, we should evaluate equation (19) at  $\tau = \alpha$ . Inserting  $\tau = \alpha$  into equation (19) and using equation (20b), we obtain

$$\left. \frac{d\gamma}{d\tau} \right|_{\tau=\alpha} = -(\rho+\lambda)\lambda \left. \frac{d\omega}{d\tau} \right|_{\tau=\alpha} = -\frac{(\rho+\lambda)\lambda\omega^2}{1 + (\rho+\lambda)\lambda\omega^2} \frac{\alpha^2}{1-\alpha} \frac{\phi g^{\alpha-1}}{(1+g)^2} \frac{dg}{d\tau} < 0. \quad (21a)$$

Further, using equations (18), (19), and (20c), we have

$$\left. \frac{d\gamma}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} = \left. \frac{dr}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} - (\rho+\lambda)\lambda \left. \frac{d\omega}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} = \frac{1}{1 + (\rho+\lambda)\lambda\omega^2} \left. \frac{dr}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} > 0. \quad (21b)$$

Equations (21a) and (21b) show that further increases in the tax rate from  $\tau = \alpha$  decrease the equilibrium growth rate while further increases in the tax rate from  $\tau = \alpha/(1+\alpha)$  increase the equilibrium growth rate. Therefore, the following proposition holds:

**Proposition 1.** *Under the golden rule of public finance, the growth-maximizing tax rate  $\tau^*$  is less than the output elasticity of public capital  $\alpha$  and satisfies*

$$\frac{\alpha}{1 + \alpha} < \tau^* < \alpha.$$

Barro (1990) and Futagami et al. (1993) analyzed the macroeconomic effects of public investment using endogenous growth models. They showed that the growth-maximizing tax rate is equal to the output elasticity of public capital. This tax rate is well-known as Barro tax rule.<sup>7</sup> Proposition 1 implies that the Barro tax rule does not hold in the OLG model with the golden rule.

Using the OLG model, Tamai (2009) showed that the Barro tax rule holds in the case of a balanced budget with the output tax. However, this is not true in the OLG model with the golden rule of public finance. When  $\lambda = 0$ , the Barro tax rule is true under the golden rule. However, in the OLG model with  $\lambda > 0$ , public investments financed by public bonds have intergenerational redistributive effects and generation replacement effects. Through these effects, a rise in the tax rate has additional negative effects on economic growth in the OLG model with the golden rule.

Some previous studies showed that the difference in fiscal rules creates differences in equilibrium growth rates (e.g., Minea and Villieu 2009; Greiner 2010; Groneck 2011). However, the OLG model has the generation replacement effects and implies the golden rule's different effects of on economic growth. Through these effects, fiscal policy under the golden rule has different welfare effects compared with those demonstrated in previous studies. We will examine this point in the next section.

### 3.2 Welfare effects of fiscal policy

This subsection analyzes welfare effects of fiscal policy. The indirect utility functions for individuals born at different times take different values. In particular, using equations (3) and (6a), that of newcomers (born at time  $t$ ) and others born at time  $s$  ( $s > t$ ) are

$$V(t, t) = \frac{r - \rho}{(\rho + \lambda)^2} + \frac{\log(\rho + \lambda)}{\rho + \lambda} + \frac{\log h(t)}{\rho + \lambda}, \quad (22a)$$

$$V(t, s) = \frac{r - \rho}{(\rho + \lambda)^2} + \frac{\log(\rho + \lambda)}{\rho + \lambda} + \frac{\log[a(t, s) + h(t)]}{\rho + \lambda}. \quad (22b)$$

The indirect utility depends on weighted terms of two effects: the growth effect, represented by first

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<sup>7</sup> Furthermore, Futagami et al. (1993) demonstrated that the welfare-maximizing effect does not coincide with the growth-maximizing effect in the model with public capital accumulation, although Barro (1990) found compatibility between growth and welfare maximization. This property holds if and only if the production function has constant elasticities of output with respect to inputs. Misch et al. (2013) examined this issue using the CES production function.

terms of equations (22a) and (22b), and the consumption effect, represented by second and third terms in those equations. The consumption effect can be decomposed into the effect on marginal propensity to consume (second terms) and wealth holding by households (third terms). Therefore, fiscal policy affects indirect utility level through these effects.

We now consider the wealth effect of fiscal policy. Fiscal policy does not affect stock level of financial assets although it affects asset portfolios (i.e.,  $da(t, s)/d\tau = 0$ ).<sup>8</sup> Then, we have

$$\frac{dK(t)}{d\tau} = -\frac{K(t)}{1+g} \frac{dg}{d\tau} < 0. \quad (23)$$

Equation (23) implies that a rise in the tax rate brings induces a crowding-out effect on private capital. A rise in the tax rate enables the government to issue public bonds. However, it simultaneously means that private capital is crowded out.

Total wealth holdings of households is composed of financial wealth and human wealth. Therefore, fiscal policy affects private consumption through the effect on human wealth. In equilibrium, human wealth becomes

$$H(t) = h(t) = \frac{w(t)}{r + \lambda - \gamma} = \frac{\alpha r K(t)}{(1 - \alpha)(\rho + \lambda)(1 + \lambda\omega)}. \quad (24)$$

Using equations (23) and (24), the effect of fiscal policy on human wealth is given by

$$\frac{1}{H(t)} \frac{dH(t)}{d\tau} = \frac{1}{r} \frac{dr}{d\tau} - \frac{1}{1+g} \frac{dg}{d\tau} - \frac{\lambda}{1 + \lambda\omega} \frac{d\omega}{d\tau}. \quad (25a)$$

The crowding-out effect in equation (23) has a negative effect on labor income. On the other hand, a rise in the tax rate increases labor income for low tax rates through a productivity effect on public capital. Furthermore, a rise in the tax rate affects the discount rate through a change in the interest rate. Equation (25a) is composed of these effects and therefore the effects of a tax rate rise on human wealth are mixed. However, evaluation of equation (25a) at some tax rates using equations (20b) and (20c) give

$$\left. \frac{1}{H(t)} \frac{dH(t)}{d\tau} \right|_{\tau=\alpha} = -\frac{1}{1+g} \frac{dg}{d\tau} - \frac{\lambda}{1 + \lambda\omega} \frac{d\omega}{d\tau} \Big|_{\tau=\alpha} < 0. \quad (25b)$$

$$\left. \frac{1}{H(t)} \frac{dH(t)}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} = -1 - \alpha - \frac{\lambda}{1 + \lambda\omega} \frac{d\omega}{d\tau} \Big|_{\tau=\frac{\alpha}{1+\alpha}} < 0. \quad (25c)$$

Using equations (6a), (9c), (10), and (25a), we can derive the consumption effect of fiscal policy as follows:

$$\frac{dc(t)}{d\tau} = (\rho + \lambda) \left[ \frac{da(t, s)}{d\tau} + \frac{dh(t)}{d\tau} \right] = (\rho + \lambda) \frac{dh(t)}{d\tau}, \quad (26a)$$

<sup>8</sup> Differentiating  $A(t)$  w.r.t. the income tax rate, we obtain

$$0 = \frac{dA(t)}{d\tau} = \frac{dK(t)}{d\tau} + \frac{dB(t)}{d\tau} = (1+g) \frac{dK(t)}{d\tau} + K(t) \frac{dg}{d\tau}.$$

Using this equation, we obtain equation (24).

$$\frac{dC(t)}{d\tau} = (\rho + \lambda) \left[ \frac{dA(t)}{d\tau} + \frac{dH(t)}{d\tau} \right] = (\rho + \lambda) \frac{dH(t)}{d\tau}. \quad (26b)$$

A rise in the tax rate affects only human wealth, and therefore the intuition of equations (26a) and (26b) are directly explained by that of equation (25a).

Finally, we consider that a rise in the tax rate affects household welfare through the dynamic effects such as those given by equations (25a)–(26b). Taking into account equations (25a)–(26b), we arrive at the following formulas (see Appendix A.2):

$$\begin{aligned} \frac{\partial V(t, s)}{\partial \tau} &= \frac{1}{(\rho + \lambda)^2} \frac{\partial r}{\partial \tau} + \frac{1}{(\rho + \lambda)[a(t, s) + h(t)]} \frac{\partial h(t)}{\partial \tau} \\ &= \frac{1}{(\rho + \lambda)\tau} \left[ \frac{r}{\rho + \lambda r} \frac{\tau \partial r}{\partial \tau} + \beta(t, s) \frac{\tau}{h(t)} \frac{\partial h(t)}{\partial \tau} \right], \end{aligned} \quad (27a)$$

$$\frac{\partial V(t, t)}{\partial \tau} = \frac{1}{(\rho + \lambda)^2} \frac{\partial r}{\partial \tau} + \frac{1}{(\rho + \lambda)h(t)} \frac{\partial h(t)}{\partial \tau} = \frac{1}{(\rho + \lambda)\tau} \left[ \frac{r}{\rho + \lambda r} \frac{\tau \partial r}{\partial \tau} + \frac{\tau}{h(t)} \frac{\partial h(t)}{\partial \tau} \right], \quad (27b)$$

where

$$\beta(t, s) \equiv \frac{h(t)}{a(t, s) + h(t)}.$$

Equation (27a) implies that the utility-maximizing tax rate is equal to the output elasticity of public capital for the generation with  $\beta = 0$ ;  $\tau_{\beta=0}^* = \alpha > \tau^*$ . Furthermore, equation (27b) can be derived from (27a) when  $\beta = 1$ .<sup>9</sup>

We focus on the relationship between the growth-maximizing and utility-maximizing fiscal policy. Inserting  $\tau = \alpha$  into (27a) and (27b) yields

$$\left. \frac{\partial V(t, s)}{\partial \tau} \right|_{\tau=\alpha} = \frac{1}{(\rho + \lambda)[a(t, s) + h(t)]} \left. \frac{\partial h(t)}{\partial \tau} \right|_{\tau=\alpha} < 0, \quad (28a)$$

$$\left. \frac{\partial V(t, t)}{\partial \tau} \right|_{\tau=\alpha} = \frac{1}{(\rho + \lambda)h(t)} \left. \frac{\partial h(t)}{\partial \tau} \right|_{\tau=\alpha} < 0. \quad (28b)$$

Equations (28a) and (28b) imply that the utility-maximizing tax rates are less than the output elasticity of public capital. However, the signs on equations (27a) and (27b) evaluated at the growth-maximizing tax rate are ambiguous. Alternatively, we check the sign on equation (27b) evaluated at  $\tau = \alpha/(1 + \alpha)$  (Appendix A.2 provides the derivation of the following equation):

$$\left. \frac{\partial V(t, t)}{\partial \tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} = \frac{1}{(\rho + \lambda)} \left[ -1 - \alpha + \frac{\frac{1}{(\rho + \lambda)\omega^2} + \frac{\lambda^2\omega}{1 + \lambda\omega}}{(\rho + \lambda)\lambda + \frac{1}{\omega^2}} (1 - \alpha)\alpha\phi g^\alpha \right]. \quad (29a)$$

If  $\lambda$  and  $\rho$  are sufficiently small, then  $\lambda\rho$  and  $\lambda^2$  are close to zero. Using this relation, equation (29a) becomes

<sup>9</sup> At time  $t$ , the generation born at time  $t$  has no financial assets;  $a(t, t) = 0$ . Thus,  $\beta(t, t) = \beta = 1$ .

$$\begin{aligned} \left. \frac{\partial V(t, t)}{\partial \tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} &= \frac{1}{(\rho + \lambda)} \left[ -1 - \alpha + \frac{(1 - \alpha)^{1-\alpha} \alpha^{1+\alpha} \phi}{\rho + \lambda} \right] \stackrel{\geq}{\leq} 0 \\ &\Leftrightarrow \phi \stackrel{\geq}{\leq} \left( \frac{1 + \alpha}{\alpha} \right) \frac{(\rho + \lambda)}{(1 - \alpha)^{1-\alpha} \alpha^\alpha}. \end{aligned} \quad (29b)$$

A large (small)  $\phi$  implies that the positive effect of a tax-rate rise on marginal productivity of private capital is large (small). Then, its positive effect on utility is also large and the magnitude correlation between the growth- and welfare-maximizing tax rates might be reversible.

These results are summarized as the following proposition:

**Proposition 2.** (i) Suppose that  $\lambda$  and  $\rho$  are small enough to satisfy  $\lambda\rho \approx 0$  and  $\lambda^2 \approx 0$ . (i) If  $\phi$  is sufficiently small, then the relationship between growth-maximizing and utility-maximizing tax rates under the golden rule of public finance satisfies

$$\tau_{\beta=0}^* < \frac{\alpha}{1 + \alpha} < \tau^* < \alpha = \tau_{\beta=1}^*.$$

(ii) In contrast, if  $\phi$  is sufficiently large, then the relationship between growth-maximizing and utility-maximizing tax rates becomes

$$\frac{\alpha}{1 + \alpha} < \min[\tau_{\beta=0}^*, \tau^*] < \max[\tau_{\beta=0}^*, \tau^*] < \alpha = \tau_{\beta=1}^*.$$

This proposition shows that the share of financial assets to total assets is key to deriving the utility-maximizing tax rate and the relationship between growth-maximizing and utility-maximizing tax rates. By intuition, we can relate  $\beta$  to life expectancy. If true, Proposition 2 also implies that a relationship exists between growth-maximizing tax rate, utility-maximizing tax rate, and life expectancy. Therefore, we investigate this relationship in the next section.

#### 4. Longevity and fiscal policy

Before analyzing the relationship between longevity and fiscal policy, we investigate the effects of an increase in  $\lambda$ . Note that life expectancy is equal to  $1/\lambda$ . Total differentiation of equations (12a), (12b), (15), and (16) lead to

$$\frac{\partial g}{\partial \lambda} = \frac{\partial b}{\partial \lambda} = 0, \quad (30a)$$

$$\frac{\partial \omega}{\partial \lambda} = -\frac{(\rho + 2\lambda)\omega^3}{1 + (\rho + \lambda)\lambda\omega^2} < 0, \quad (30b)$$

$$\frac{\partial \gamma}{\partial \lambda} = -(\rho + 2\lambda)\omega - (\rho + \lambda)\lambda \frac{\partial \omega}{\partial \lambda} = -\frac{(\rho + 2\lambda)\omega}{1 + (\rho + \lambda)\lambda\omega^2} < 0. \quad (30c)$$

Equation (30a) shows that a change in  $\lambda$  does not affect the ratio of public to private capital or that of debt to private capital. The golden rule of public finance requires that interest payments are equal to tax revenue. Then, the ratio of public to private capital and that of debt to private capital are independent of the ratio of private consumption to private capital, which is affected by  $\lambda$ . The result of equation (30b) is standard. An increase in  $\lambda$  raises the marginal propensity to consume; accordingly, private consumption increases and the growth rate of private capital declines. Therefore, a rise in  $\lambda$  increases the ratio of private consumption to private capital. Finally, equation (30c) shows that an increase in  $\lambda$  decreases the equilibrium growth rate.

Denote  $\tau_{s=t}^*$  and  $\tau_{s<t}^*$  by the tax rate that maximizes the utility from being born at  $s = t$  and  $s > t$ , respectively. Using equations (19), (27a), and (27b), we obtain the following results (Appendix A.3):

$$\frac{d\tau^*}{d\lambda} < 0, \frac{d\tau_{s=t}^*}{d\lambda} < 0, \frac{d\tau_{s<t}^*}{d\lambda} < 0. \quad (31)$$

By equation (31), the following proposition is true.

**Proposition 3.** *Suppose the second-order conditions for growth-maximizing and utility-maximizing fiscal policy. An increase in life expectancy raises the growth-maximizing tax rate and the utility-maximizing tax rates of each generation's utility.*

The interpretation of Proposition 3 is as follows. The equations for assets and the ratio of human wealth to total wealth are as follows (Appendix A.4):

$$a(t, s) = \frac{\psi K(s)}{r - \rho - \gamma} [1 - e^{-(r-\rho-\gamma)(t-s)}] e^{(r-\rho)(t-s)}, \quad (32a)$$

$$\beta(t, s) = \frac{r - \rho - \gamma}{[e^{(r-\rho-\gamma)(t-s)} - 1] + r - \rho - \gamma}, \quad (32b)$$

where

$$\psi \equiv \frac{(r - \rho)(1 - \tau)\alpha\phi g^\alpha}{r + \lambda}.$$

Equations (32a) and (32b) imply that older generations have a greater amount of financial assets than younger generations, and older generations also have a higher ratio of financial wealth to total wealth than younger generations. When life expectancy increases, all existing generations benefit from economic growth. Then, the generation replacement effect decreases. Therefore, all generations desire a higher tax rate to maximize their utilities, in contrast with the previous situation.

According to Proposition 3, we can consider the relationship between longevity and tax rate

determined through majority voting. In the model, the median's voter age is  $\log 2 / \lambda$ . Therefore, a rise in life expectancy increases the median age. In turn, this gives individuals an incentive to vote for a higher tax rate than the current one. A rise in the tax rate increases the debt per GDP through the fiscal rule. By Proposition 3 and this consideration, we obtain the following result:

**Corollary.** *When the tax rate is determined through majority voting, population aging raises the equilibrium tax rate and also increases the ratio of debt to GDP.*

## 5. Conclusion

This paper examined the growth and welfare effects of debt-financed public investment under the golden rule of public finance using the Yaari-Blanchard model. This paper showed that Barro tax rule does not hold, and that the growth-maximizing and utility-maximizing tax rates increase along with life expectancy. Population aging seriously affects fiscal policy and economic performance; it increases the tax rate and debt per GDP in the voting equilibrium.

In contrast to previous theoretical studies, we showed that population aging is an important factor in the government's debt-financing public investment, because it alters the median age and raises the tax rate in an economy. Finally, our results show that theoretically, population aging increases the ratio of public debt to GDP, and this finding is consistent with the result of empirical studies.

Finally, we will discuss future directions of study. In this paper, we simplified the supply of labor, therefore omitting the possibility of labor retirement. Labor retirement is an important when considering intergenerational transfers and the distribution of utility. Furthermore, this paper assumed that the government taxes only factor payments at a constant flat rate. However, this assumption should be relaxed to enable differentiated tax rates and variations over time. This relaxation allows us to analyze the dynamic equilibrium numerically. These extensions will be fruitful to considering policy implication as applied to more realistic situations.

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## Appendix

### A.1. No Ponzi condition for the government's budget

Equation (10) leads to

$$\dot{B}(t) + \tau Y(t) = r(t)B(t) + \dot{G}(t) \Rightarrow \dot{B}(t) = r(t)B(t) + \dot{G}(t) - \tau Y(t). \quad (\text{A1})$$

Multiplying  $e^{-\int_0^t r(v)dv}$  to equation (A1) and integrating it with respect to  $t$ , we have

$$\begin{aligned} \int_0^T \dot{B}(t)e^{-\int_0^t r(v)dv} dt &= \int_0^T r(t)B(t)e^{-\int_0^t r(v)dv} dt \\ &+ \int_0^T \dot{G}(t)e^{-\int_0^t r(v)dv} dt - \tau \int_0^T Y(t)e^{-\int_0^t r(v)dv} dt. \end{aligned} \quad (\text{A2})$$

Equation (A2) is reduced to

$$\left[ B(t)e^{-\int_0^t r(v)dv} \right]_0^T = \left[ G(t)e^{-\int_0^t r(v)dv} \right]_0^T + \int_0^T r(t)G(t)e^{-\int_0^t r(v)dv} dt - \tau \int_0^T Y(t)e^{-\int_0^t r(v)dv} dt.$$

When  $t \rightarrow \infty$ , the above equation becomes

$$\begin{aligned} \lim_{t \rightarrow \infty} B(t)e^{-\int_0^t r(v)dv} - \lim_{t \rightarrow \infty} G(t)e^{-\int_0^t r(v)dv} \\ = \int_0^\infty r(t)G(t)e^{-\int_0^t r(v)dv} dt - \tau \int_0^T Y(t)e^{-\int_0^t r(v)dv} dt. \end{aligned} \quad (\text{A3})$$

The golden rule leads to

$$B(t) - G(t) = B(0) - G(0). \quad (\text{A4})$$

Inserting equation (A4) into  $G(t)$  of equation (A3),

$$\tau \int_0^T Y(t)e^{-\int_0^t r(v)dv} dt = \int_0^\infty r(t)B(t)e^{-\int_0^t r(v)dv} dt + [G(0) - B(0)] \int_0^\infty r(t)e^{-\int_0^t r(v)dv} dt. \quad (\text{A5})$$

Using equation (11) and  $\tau Y(t) = r(t)B(t)$ , equation (A5) is

$$[G(0) - B(0)] \int_0^\infty r(t)e^{-\int_0^t r(v)dv} dt = 0.$$

Therefore, we need  $G(0) = B(0)$  to satisfy the balanced budget with equation (11).

### A.2. Derivation of equation (29a)

$$\begin{aligned} \left. \frac{\partial V(t, t)}{\partial \tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} &= \frac{1}{(\rho + \lambda)} \left[ \left. \frac{1}{\rho + \lambda} \frac{dr}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} - 1 - \alpha - \frac{\lambda}{1 + \lambda\omega} \left. \frac{d\omega}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} \right] \\ &= \frac{1}{(\rho + \lambda)} \left[ \left. \frac{1}{\rho + \lambda} \frac{dr}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} - 1 - \alpha - \frac{\lambda}{1 + \lambda\omega} \frac{1}{(\rho + \lambda)\lambda + \frac{1}{\omega^2}} \left. \frac{dr}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{(\rho + \lambda)} \left[ -1 - \alpha + \frac{\lambda + \frac{1}{(\rho + \lambda)\omega^2} - \frac{\lambda}{1 + \lambda\omega}}{(\rho + \lambda)\lambda + \frac{1}{\omega^2}} \frac{dr}{d\tau} \Big|_{\tau = \frac{\alpha}{1 + \alpha}} \right] \\
&= \frac{1}{(\rho + \lambda)} \left[ -1 - \alpha + \frac{\frac{1}{(\rho + \lambda)\omega^2} + \frac{\lambda^2\omega}{1 + \lambda\omega}}{(\rho + \lambda)\lambda + \frac{1}{\omega^2}} (1 - \alpha)\alpha\phi g^\alpha \right].
\end{aligned}$$

### A.3. Derivation of equation (31)

Total differentiation of equation (19) when  $\partial\gamma/\partial\tau = 0$  is

$$\frac{\partial^2\gamma}{\partial\tau^2}d\tau + \frac{\partial^2\gamma}{\partial\lambda\partial\tau}d\lambda = 0 \Rightarrow \frac{d\tau^*}{d\lambda} = -\frac{\partial^2\gamma}{\partial\lambda\partial\tau}/\frac{\partial^2\gamma}{\partial\tau^2}. \quad (\text{A6})$$

Assume that the denominator of equation (A6) is negative (by the second-order condition). We then have

$$\frac{\partial^2\gamma}{\partial\lambda\partial\tau} = \frac{\partial}{\partial\lambda} \left[ \frac{\partial r}{\partial\tau} - (\rho + \lambda)\lambda \frac{\partial\omega}{\partial\tau} \right] = -(\rho + 2\lambda)\lambda \frac{\partial\omega}{\partial\tau} - (\rho + \lambda)\lambda \frac{\partial}{\partial\lambda} \left( \frac{\partial\omega}{\partial\tau} \right) < 0, \quad (\text{A7})$$

where

$$\frac{\partial}{\partial\lambda} \left( \frac{\partial\omega}{\partial\tau} \right) = \frac{1}{1 + (\rho + \lambda)\lambda\omega^2} \left[ \frac{(\rho + 2\lambda)}{(\rho + \lambda)\lambda} - \frac{2}{\omega} \frac{\partial\omega}{\partial\lambda} \right] \frac{\partial\omega}{\partial\tau} > 0 \text{ for } \tau = \tau^*.$$

Equations (A6) and (A7) lead to the left inequality in equation (31).

Assume that the second-order conditions are satisfied. Derivation of the middle and right inequalities in equation (31) are also derived in a similar manner. Total differentiation of equation (27b) when  $\partial V(t, t)/\partial\tau = 0$  is

$$\frac{\partial^2 V(t, t)}{\partial\tau^2}d\tau + \frac{\partial^2 V(t, t)}{\partial\lambda\partial\tau}d\lambda = 0 \Rightarrow \frac{d\tau_{\beta=0}^*}{d\lambda} = -\frac{\partial^2 V(t, t)}{\partial\lambda\partial\tau} / \frac{\partial^2 V(t, t)}{\partial\tau^2}, \quad (\text{A8})$$

We can calculate the numerator of (A8) as

$$\frac{\partial^2 V(t, t)}{\partial\lambda\partial\tau} = -\frac{1}{\rho + \lambda} \left[ \frac{1}{(\rho + \lambda)^2} \frac{\partial r}{\partial\tau} - \frac{\partial}{\partial\lambda} \left( \frac{1}{h(t)} \frac{\partial h(t)}{\partial\tau} \right) \right] < 0, \quad (\text{A9})$$

where

$$\begin{aligned}
\frac{\partial}{\partial\lambda} \left( \frac{1}{h(t)} \frac{\partial h(t)}{\partial\tau} \right) &= -\frac{\partial}{\partial\lambda} \left( \frac{\lambda}{1 + \lambda\omega} \frac{\partial\omega}{\partial\tau} \right) = -\frac{\left[ \frac{\partial\omega}{\partial\tau} + \lambda \frac{\partial}{\partial\lambda} \left( \frac{\partial\omega}{\partial\tau} \right) \right] (1 + \lambda\omega) - \lambda \frac{\partial\omega}{\partial\tau} \left( \omega + \lambda \frac{\partial\omega}{\partial\lambda} \right)}{(1 + \lambda\omega)^2} \\
&= -\frac{\frac{\partial\omega}{\partial\tau} + (1 + \lambda\omega)\lambda \frac{\partial}{\partial\lambda} \left( \frac{\partial\omega}{\partial\tau} \right) - \lambda^2 \frac{\partial\omega}{\partial\tau} \frac{\partial\omega}{\partial\lambda}}{(1 + \lambda\omega)^2} < 0 \text{ for } \tau = \tau_{\beta \geq 0}^*.
\end{aligned}$$

Using equations (A8) and (A9), we obtain the middle inequality in equation (31). In case of the right inequality in equation (31), we have

$$\frac{\partial^2 V(t, s)}{\partial \tau^2} d\tau + \frac{\partial^2 V(t, s)}{\partial \lambda \partial \tau} d\lambda = 0 \Rightarrow \frac{d\tau_{\beta > 0}^*}{d\lambda} = -\frac{\partial^2 V(t, s)}{\partial \lambda \partial \tau} / \frac{\partial^2 V(t, s)}{\partial \tau^2}, \quad (\text{A10})$$

$$\frac{\partial^2 V(t, s)}{\partial \lambda \partial \tau} = -\frac{1}{\rho + \lambda} \left[ \frac{1}{(\rho + \lambda)^2} \frac{\partial r}{\partial \tau} - \frac{\partial \beta}{\partial \lambda} \frac{1}{h(t)} \frac{\partial h(t)}{\partial \tau} - \beta \frac{\partial}{\partial \lambda} \left( \frac{1}{h(t)} \frac{\partial h(t)}{\partial \tau} \right) \right] < 0, \quad (\text{A11})$$

where

$$\frac{\partial h(t)}{\partial \lambda} = -\left[ \frac{1}{\rho + \lambda} + \frac{\omega}{1 + \lambda\omega} \left( 1 + \frac{\lambda}{\omega} \frac{\partial \omega}{\partial \lambda} \right) \right] h(t) < 0,$$

$$\frac{\partial \beta}{\partial \lambda} = \frac{\beta(1 - \beta)}{h(t)} \frac{\partial h(t)}{\partial \lambda} < 0.$$

Therefore, equations (A10) and (A11) provide the right inequality in equation (31).

#### A.4. Derivation of equations (32a) and (32b)

Equations (1), (2b), (4), (6a), (12b), and (21) lead to

$$\frac{da(v, s)}{dv} = (r - \rho)a(v, s) + \psi K(s)e^{\gamma v}. \quad (\text{A12})$$

The complementary solution to equation (A12) is

$$a(t, s) = X e^{(r - \rho)(t - s)}. \quad (\text{A13})$$

In equation (A13),  $X$  will be depend on  $t$ . Differentiating equation (A13) with respect to  $t$ , we obtain

$$\frac{da(t, s)}{dt} = \frac{dX}{dt} e^{(r - \rho)(t - s)} + X(r - \rho)e^{(r - \rho)(t - s)} = \frac{dX}{dt} e^{(r - \rho)(t - s)} + (r - \rho)a(t, s). \quad (\text{A14})$$

Using equations (A12)–(A14), we have

$$\frac{dX}{dt} = \psi K(s) e^{-(r - \rho - \gamma)(t - s)}. \quad (\text{A15})$$

Solving equation (A15) with respect to  $t$ , we derive

$$X = Z - \frac{\psi K(s) e^{-(r - \rho - \gamma)(t - s)}}{r - \rho - \gamma}. \quad (\text{A16})$$

Inserting equation (A16) into equation (A13) yields

$$a(t, s) = \left[ Z e^{(r - \rho)(t - s)} - \frac{\psi K(s) e^{\gamma(t - s)}}{r - \rho - \gamma} \right]. \quad (\text{A17})$$

Since we have  $a(t, t) = 0$ , equation (A17) must satisfy

$$a(t, t) = Z - \frac{\psi K(s)}{r - \rho - \gamma} = 0 \Rightarrow Z = \frac{\psi K(s)}{r - \rho - \gamma}. \quad (\text{A18})$$

Using equations (A17) and (A18), we arrive at

$$a(t, s) = \frac{\psi K(s)}{r - \rho - \gamma} [e^{(r-\rho)(t-s)} - e^{\gamma(t-s)}] = \frac{\psi K(s)}{r - \rho - \gamma} [1 - e^{-(r-\rho-\gamma)(t-s)}] e^{(r-\rho)(t-s)}.$$

The above equation corresponds to equation (32a). Using equation (32a) and the definition of  $\beta$ , we obtain

$$\begin{aligned} \beta(t, s) &= \frac{(r - \rho - \gamma)K(t)}{[1 - e^{-(r-\rho-\gamma)(t-s)}]e^{(r-\rho)(t-s)}K(s) + (r - \rho - \gamma)K(t)} \\ &= \frac{r - \rho - \gamma}{[1 - e^{-(r-\rho-\gamma)(t-s)}]e^{(r-\rho)(t-s)}e^{\gamma(s-t)} + r - \rho - \gamma} \\ &= \frac{r - \rho - \gamma}{[e^{(r-\rho-\gamma)(t-s)} - 1] + r - \rho - \gamma}. \end{aligned}$$

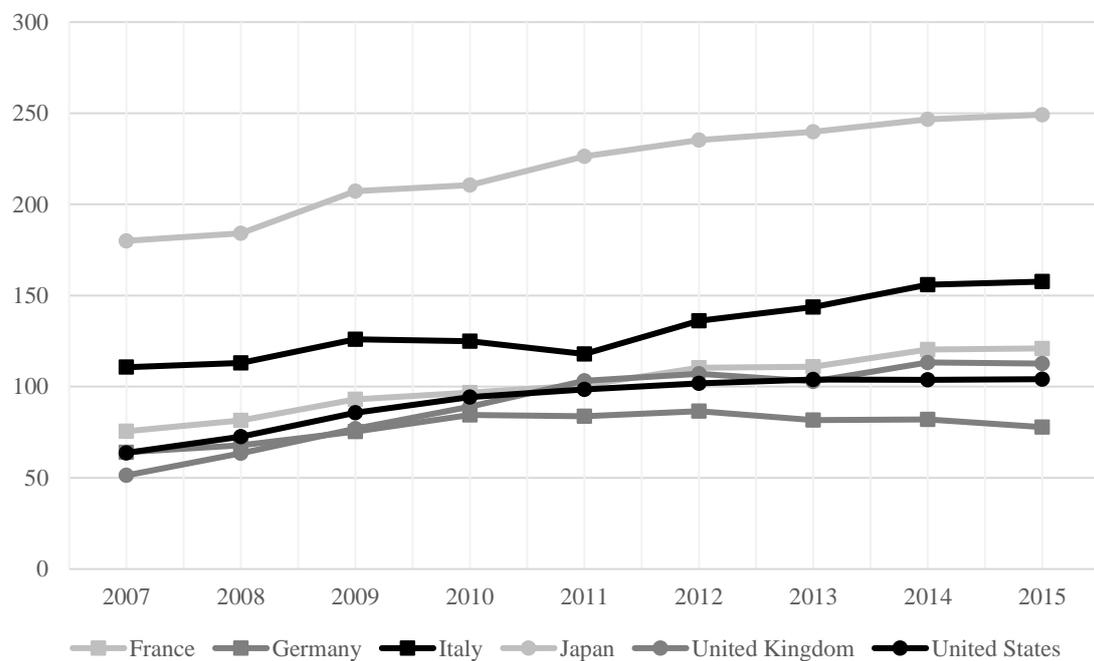
Therefore, equation (32b) is derived.

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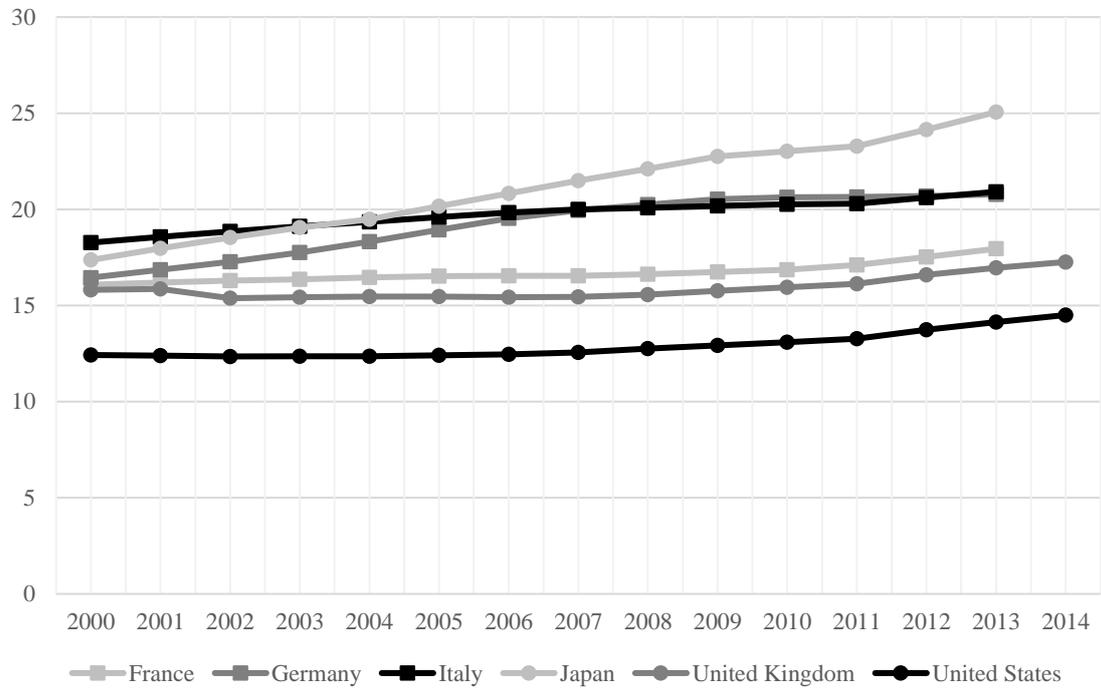
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## Figure



**Figure 1. Ratio of debt to GDP (%)**

Data: OECD stat



**Figure 2. Population aged over 65 /Population of all ages (%)**

Data: OECD stat