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“A Dynamic Agency Theory of Investment
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A Dynamic Agency Theory of Investment and Managerial Replacement

Abstract

In this paper, we explore a dynamic theory of investment and costly managerial turnover given agency conflicts between the firm manager and investors. We incorporate the possibility of the successive replacement of managers until the firm is finally liquidated, and develop a continuous-time agency model with the q-theory of investment. We derive the dynamic variations of average q, marginal q, and the optimal investment–capital ratio surrounding manager turnover. Furthermore, we also indicate that the firm's optimal replacement/retention decision becomes more permissive with the frequency of the replacement of managers. Our theoretical findings yield empirical implications for the joint dynamics of investment and CEO turnover policy, which are consistent with evidence provided by the existing empirical literature, and provide novel testable hypotheses.

JEL Classification Codes: D86, D92, G31, G32, M12, M51.

Keywords: average q, CEO turnover, continuous-time agency model, investment, marginal q.

1. Introduction

Because of the key economic role played by top corporate managers and the agency conflict with investors, one of the most important decisions for the firm is to decide whether to retain or fire an incumbent CEO following poor stock prices and/or accounting performance. Several empirical studies already find that the performance quality and investment behavior of firms vary markedly around CEO turnover.[1] However, persistent shocks to the firm's profitability occur stochastically over time and thereby affect the firm's performance and stock prices. Furthermore, the empirical estimates suggest that firing a CEO involves a very large CEO turnover cost.[2] Taking into account these factors requires us to develop a dynamic theory of investment and costly managerial turnover in a stochastic environment.

In this paper, we explore a dynamic theory of investment and costly managerial turnover in the context of agency conflicts between investors and managers using the framework of the continuous-time agency model. In the extant literature, the optimal replacement timing of the manager has not been examined. If it has, it has only been in a perfectly stationary environment in the sense that the firm faces the same agency problem with each manager the firm hires (see the literature discussed later in this section). However, in reality, as estimated in Taylor (2010), the cost to the firm's shareholders of replacing the incumbent CEO is large. Furthermore, if the reorganization and/or restructuring of the firm is required with the replacement of the manager, the replacement cost may be larger when the firm's current financial distress worsens.[3] The firm then needs to consider the replacement cost incurred by its shareholders and related to its financially distressed situation. This cost requirement implies that the firm's environment surrounding the replacement of the manager cannot be stationary, even under scale-invariant technology. Nevertheless, none of the existing studies discuss the joint dynamics and inefficiency of the firm's investment and managerial replacement decisions in such a nonstationary environment. Hence, this paper sheds new light on the joint dynamics and inefficiency of the firm's investment and managerial replacement strategies under agency conflicts when a replacement cost arises with

[1] See Huson, Malatesta, and Parrino (2004), Hornstein (2013), and Alderson, Bansal, and Betker (2014).

[2] Taylor (2010) suggests that firing a CEO costs shareholders the equivalent of some 5.9% of firm assets.

[3] If the reorganization and/or restructuring of the firm quickly needs new funds financed by equity, convertible bonds, and/or subordinated bonds, this presumption is more realistic and plausible.

the replacement of each manager that is related to its financially distressed situation.

We examine a continuous-time agency model with the q-theory of investment by incorporating the possibility of the successive replacement of managers prior to the liquidation of the firm. The benchmark is the dynamic agency and investment model in DeMarzo, Fishman, He, and Wang (2012), in which the agent's scaled continuation payoff can be interpreted as a measure of the firm's financial slack. Our primary change to their model is that investors decide at each point of time whether to fire or retain the current manager by considering the replacement cost relating to the firm's financially distressed situation. A second change is that the optimal contract is implemented in a capital structure with a credit line because the financial slack interpretation can be better adapted to the reorganized and/or restructured firm. This second modification enables the replacement cost of the manager to reflect the reorganization and/or restructuring cost of the firm at the replacement. Because this variable replacement cost causes the contract environment with the new manager to differ from that with the old manager, both the scaled value function of investors and the initial value of the new manager's scaled continuation payoff switch at replacement. Thus, we can fully characterize the joint dynamics of the investment and successive managerial turnover strategies in the nonstationary environment.

We summarize our main results as follows. First, the variations in average q and marginal q at the time of manager turnover move in opposite directions if the predecessor's scaled continuation payoff near his replacement time is sufficiently close to his replacement threshold (that is, if the firm's financial slack is relatively low before the replacement of the manager). However, such variations may occur in the same direction if the predecessor's scaled continuation payoff near his replacement time is sufficiently larger than his replacement threshold (that is, if the firm's financial slack is *not* relatively low before the replacement of the manager).

Second, the optimal investment–capital ratio increases following the replacement of each manager if the predecessor's scaled continuation payoff near his replacement time is sufficiently close to his replacement threshold (that is, if the firm's financial slack is relatively low before his replacement). However, this ratio may be lower after the replacement of each manager if the predecessor's scaled continuation payoff near his replacement time is sufficiently larger than his replacement threshold (that is, if the firm's financial slack is *not* relatively low before his replacement).

Third, the threshold of the manager's scaled continuation payoff regarding the replacement of each manager declines with the frequency of the replacement of managers. This implies that the firm's optimal replacement/retention decision becomes more permissive with the frequency of the replacement of managers.

Finally, the main results of our model are unchanged even though contracts are constrained to be renegotiation-proof.

Importantly, very few empirical studies examine patterns in average q, marginal q, and investment behavior in the years surrounding CEO turnover. Our theoretical findings therefore yield useful empirical implications for the dynamics of investment and CEO turnover policy. These theoretical predictions are consistent with the evidence provided by the existing empirical literature and provide novel testable hypotheses.

Our results also provide a new insight into the relation between average q and marginal q. In many empirical studies, average q is used more than marginal q, even though it is often argued that marginal q is a more accurate measure than average q of the firm's investment opportunities. In fact, Caballero and Leahy (1996) suggest that average q can be a better proxy for the firm's investment opportunities with fixed costs of adjustment. Bolton, Chen, and Wang (2011) also argue that average q rather than marginal q can be a more robust predictor of investment for financially constrained firms. However, our results show that the variations in average q and marginal q at the time of manager turnover can be in opposite directions. This implies that average q is not a suitable proxy for marginal q surrounding the turnover of the manager. Furthermore, we suggest that the optimal investment–output ratio can be negatively related to average q around the turnover of the manager. This suggests that if average q is used for the estimation of the investment function, the estimate involves an estimation bias when the manager is replaced. Together, these findings imply that average q is not a better proxy than marginal q for the firm's investment opportunities surrounding the replacement of the manager, in contrast to the suggestions of both Caballero and Leahy (1996) and Bolton, Chen, and Wang (2011).

The work in this paper relates to the expanding literature on continuous-time principal–

agent models using martingale techniques. See, for example, DeMarzo and Sannikov (2006), Biais, Mariotti, Plantin, and Rochet (2007), He (2009), Hoffmann and Pfeil (2010), Piskorski and Tchistyi (2010), Zhu (2013), and Hori and Osano (2013, 2014). For the most part, these studies employ either the cash diversion or hidden-effort choice model, and show that the threat of replacement plays an important role in incentivizing effort when the agent is protected by limited liability.[4] The most important difference between our work and these studies is that we examine the joint dynamics of the firm's investment and successive managerial replacement strategies under agency conflicts. Specifically, studies other than the present analysis do not consider the replacement of the agent, or they only discuss the perfectly stationary environment in the sense that the firm faces the same agency problem with each agent the firm hires. The latter arises partly because of the assumption of the fixed replacement cost of the agent such that neither the scaled value function of investors nor the initial value of the new agent's scaled continuation payoff changes. As a result, these existing studies are essentially reduced to the case in which the replacement of the agent does not take place before the termination of the firm. By contrast, by making the replacement cost relate to the firm's financially distressed situation, we show that at the time of each replacement, the scaled value function of investors switches and the initial value of the new manager's scaled continuation payoff is decreasing. Hence, our model can fully capture the dynamics of the managerial replacement strategy in the sense that both the scaled value function of investors and the initial value of the new manager's scaled continuation payoff change, even under scale-invariant technology, whenever the replacement of managers successively takes place over time.

The dynamic contracting and investment problem given agency conflicts has been examined in Albuquerque and Hopenhayn (2004), Quadrini (2004), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007), and Biais, Mariotti, Rochet, and Villeneuve (2010). In particular, DeMarzo, Fishman, He, and Wang (2012) use the continuous-time agency model detailed above and discuss the q theory of investment under agency conflicts. However, the main difference between our study and theirs is that in our model, the replacement of managers can occur successively over time, whereas in their model, the firm does not

[4] In the risk–averse agent model in Sannikov (2008) and He (2011), as in the discrete model in Spear and Wang (2005) and Wang (2011), a risk–averse agent may be optimally induced to cease to exert effort and then retire once his continuation utility becomes either too high or too low. This is because it is too costly for the firm to incentivize further effort if the agent's continuation utility is too high or too low.

replace the initially hired agent until the firm is liquidated. As a result, our model can fully develop the joint dynamics of the firm's investment and successive managerial replacement strategies under agency conflicts in the nonstationary environment.[5]

This paper also relates to a line of discrete–time research in the dynamic turnover literature in which turnover is derived from variations in managerial productivity. For instance, Garrett and Pavan (2012) develop a dynamic theory of managerial turnover where the quality of the match between a firm and its managers changes stochastically over time. They show that the firm's optimal retention decision becomes more permissive with time. Once again, the main difference between our model and theirs is that in their model the firm's environment is perfectly stationary in the sense that the firm faces the same problem with each manager it hires. This is because they assume that upon separating from the incumbent manager the firm returns to the labor market and is randomly matched with a new manager. Hence, the initial value of the continuation utility is the same for any manager the firm hires. In contrast, in our model, the replacement cost relating to the firm's financially distressed situation dynamically changes both the scaled value function of investors and the initial value of the manager's scaled continuation payoff at replacement. As a result, we can fully analyze the joint dynamics of the firm's investment and managerial replacement strategies given agency conflicts in a nonstationary environment.

The remainder of the paper is organized as follows. Section 2 describes the basic model. Section 3 analyzes the first-best solution. Section 4 derives the optimal contract under agency conflicts. Section 5 examines the model implications. Section 6 discusses the implementation of the optimal contract. Section 7 considers the empirical implications of our theoretical results, and Section 8 concludes. Appendix A provides the more complicated proofs, and Appendix B considers the impact of a renegotiation-proof contract.

2. The Model

A firm produces output by employing capital. The firm's capital stock K_t evolves according to

$$dK_t = (I_t - \delta K_t)dt, \tag{1}$$

[5]Neglecting agency costs, Bolton, Chen, and Wang (2011) provide a model of dynamic investment, financing, and risk management for financially constrained firms.

where I_t is the firm's growth investment rate and $\delta \geq 0$ is the rate of depreciation. Investment entails adjustment costs $G(I_t, K_t)$. We assume that $G(I_t, K_t)$ satisfies $G(0, K_t) = 0$, is increasing and convex in I_t, increasing in K_t, and is homogeneous of degree one in I_t and K_t. Using the homogeneity of $G(I_t, K_t)$, we have

$$I_t + G(I_t, K_t) = c(i_t)K_t, \tag{2}$$

where $i_t = \frac{I_t}{K_t}$ and $c(0) = 0$.

We assume that the incremental gross output over time interval dt is proportional to K_t: $K_t dA_t$, where A_t is the cumulative productivity process. Then, the firm's cumulative cash flow process Y_t is represented by

$$dY_t = K_t[dA_t - c(i_t)dt], \tag{3}$$

where $K_t c(i_t)dt$ is the total cost of investment.

We now consider an agency conflict between the firm's investors and managers. The firm's investors hire a manager to manage the firm and can replace him with a new manager at any time. We assume that all of the managers in the pool of potential applicants are identical, meaning that they have the same preferences and that their productivity is drawn independently from the same distribution and is expected to evolve over time according to the same Brownian motion described below. Hence, the only reason for the investors to replace the incumbent manager is the incentive consideration in that the manager's expected payoff becomes too low so that it is very costly for the investors to give the manager sufficient incentives to work under the limited liability of the manager.

Let the n-th manager denote a manager employed by the firm after the $n-1$-th manager has been replaced. In fact, we assume that the $n-1$-th manager continues to be hired with some probability. Hence, either the incumbent $n-1$-th manager or a new manager hired from the pool of potential applicants becomes the n-th manager.[6] Although the incumbent $n-1$-th manager can continue to be hired with some probability as the n-th manager, we

[6] To compensate the $n-1$-th manager for his continuation value at his replacement, we impose the assumption of stochastic replacement. In the discrete dynamic agency model, Anderson, Bustamante, and Guibaud (2011) exploit stochastic replacement to achieve the same purpose as ours.

refer to the transition from the $n-1$-th manager to the n-th manager as the replacement of the $n-1$-th manager in order to simplify the terminology in the subsequent analysis. Indeed, in our model, all the events where incumbent managers are replaced by other new managers are included in the set of the transition events from the $n-1$-th manager to the n-th manager ($n = 1, \ldots, N$). Thus, by elucidating the features of the transition from the $n-1$-th manager to the n-th manager, we can characterize the joint dynamics of the firm's investment and managerial replacement strategies around the time of the replacement of the incumbent manager by another new manager.

Now, when the n-th manager is hired, his binary action $a_t^n \in \{0, 1\}$ determines the expected rate of output per unit of capital stock:

$$dA_t^n = a_t^n \mu dt + \sigma dZ_t, \tag{4}$$

where μ is the drift of the cash flows, σ is the instantaneous volatility, and $Z = \{Z_t, \mathcal{F}_t; 0 \leq t < \infty\}$ is a standard Brownian motion on the complete probability space $(\Omega, \mathcal{F}, \mathbb{Q})$. When the n-th manager takes the action a_t^n, he enjoys private benefits at the rate $\lambda(1 - a_t^n)\mu dt$ per unit of capital stock. We assume that $0 < \lambda \leq 1$. Thus, we can interpret the action as an effort choice: the n-th manager shirks if $a_t^n = 0$, or works if $a_t^n = 1$.

The firm's investors have unlimited wealth, are risk neutral, and discount the flow of profit at rate $r > 0$. All of the managers are risk neutral, with a negative wage ruled out by limited liability. These managers also discount their consumption at γ $(> r)$. If each manager's savings interest rate is lower than the investors' discount rate and if each manager is risk neutral, DeMarzo and Sannikov (2006) show that there is an optimal contract in which each manager maintains zero savings. Hence, in this model, each manager can be restricted to consuming what the principal pays him at any point in time.

Regarding the information structure, the firm's capital stock K_t and its cash flow Y_t are publicly observable and contractible. Hence, investment I_t and productivity A_t are also contractible. However, the firm's investors cannot observe any managerial action or the flow of any manager's private benefit.

After the $n-1$-th manager has been replaced, the investors sign a contract with the n-th manager that specifies the firm's investment policy I_t^n during his employment, his cumulative compensation U_t^n, and his replacement time τ^n, all of which depend on the history of his performance as described by the productivity process A_t^n. As a negative wage is excluded, U_t^n must be nondecreasing. Let $\Phi^n = \{I^n, U^n, \tau^n\}$ represent the contract with the n-th manager.

For simplicity, we assume that the investors commit to the contract arrangements with each manager. In Appendix B, we show that if the renegotiation cost is the same functional form as the replacement cost, the main results of our model are unchanged, even though the contract is constrained to be renegotiation-proof.

Now, for any contract Φ^n, the n-th manager chooses an action process $\{a_t^n \in \{0, 1\} : \tau^{n-1} \leq t < \tau^n\}$ to solve

$$W(\Phi^n) = \max_{\{a_t^n \in \{0,1\} : \tau^{n-1} \leq t < \tau^n\}} E^a_{\tau^{n-1}} \int_{\tau^{n-1}}^{\tau^n} e^{-\gamma t} \left[dU_t^n + \lambda(1 - a_t^n)\mu K_t dt \right], \tag{5}$$

where $E^a_{\tau^{n-1}}(\cdot)$ is the expectation operator under the probability measure induced by the n-th manager's action process. We assume that if the n-th manager is fired and does not continue to be hired in this firm, he does not return to the managerial labor market and is retired with a value normalized to zero.

In the subsequent analysis, we focus on the case in which it is optimal for the investors to implement the efficient action $a_t^n = 1$ for any $t \in [\tau^{n-1}, \tau^n)$. The sufficient condition for the optimality of implementing this action is provided by Lemma A2 in the proof of Proposition 2 in Appendix A. Henceforth, we use the expectation operator $E_{\tau^{n-1}}(\cdot)$, which is under the measure induced by $\{a_t^n = 1 : \tau^{n-1} \leq t < \tau^n\}$. A contract Φ^n is incentive compatible if it implements the efficient action $a_t^n = 1$ for any $t \in [\tau^{n-1}, \tau^n)$.

At the initial time, the firm holds K_0 in capital. We assume that after the 1st manager is fired and does not continue to be hired by the firm, each manager in the pool of potential applicants prefers to participate in the contract as long as his discounted expected present value is larger than or equal to his outside option normalized to zero. Similarly, we assume that if the incumbent $n-1$-th manager continues to be hired as the n-th manager ($n = 1, \ldots, N$), he still prefers to participate in the contract as long as his discounted expected present value is larger than or equal to zero. This is because each manager can be retired with a value normalized to zero, as has been assumed. However, regarding the 1st manager, he may have to exert additional observable and verifiable effort at time 0 to commence the firm's project. Hence, we assume that the optimal contract must deliver $W(\Phi^1) = W_0$ (> 0) to the 1st manager at time 0.[7] We also suppose that the firm is liquidated when the N-th manager is fired and does not continue to be hired in the firm. In Section 4, we show how N is given to make our analysis meaningful. However, we assume that N is a finite number so that the pool of potential applicants is competitive.

Then, the investors' optimization problem is

$$P(K_0, W_0, \Phi^1, \ldots, \Phi^N) = \max_{\Phi^1,\ldots,\Phi^N} E\sum_{n}{}_1^N \left[\int_{\tau^{n-1}}^{\tau^n} e^{-rt}(dY_t^n - dU_t^n) - e^{-r\tau^n} c_f K_{\tau^n}\right] + e^{-r\tau^N}\ell K_{\tau^N},$$

$$\text{s.t. } (\Phi^1, \ldots, \Phi^N) \text{ is incentive compatible, } W(\Phi^1) = W_0, \text{ and } W(\Phi^n) \geq 0 \text{ for } i = 2, \ldots N, \tag{6}$$

where $\tau^0 = 0$; dY_t^n is determined by (3) when $dA_t = dA_t^n$ and $i_t^n = \frac{I_t^n}{K_t}$; $c_f K_{\tau^n}$ reflects the cost of lost productivity when the n-th manager is replaced with the $n+1$-th manager; and ℓK_{τ^N} is the expected liquidation payoff of the investors when the firm is liquidated. In Section 4, we indicate how ℓ is endogenously determined.

The presence of the replacement cost of the manager is empirically justified by Taylor (2010). By estimating that the required cost of replacing the incumbent CEO is 5.9% of firm value, he discusses why the observed replacement probability of CEOs is low. In Section 4, we show how c_f is endogenously determined. In addition, the investors must incur the cost of lost productivity even though they continue to hire the n-th manager as the $n+1$-th manager. We also discuss the justification for this assumption in Section 4.

3. The First-best Solution

We first determine the optimal allocation without agency problems. With no agency conflicts (that is, $\lambda = 0$ and/or $\sigma = 0$), the firm does not replace the 1st manager employed initially because the firm must incur the replacement cost upon the replacement of the manager and all of the managers in the pool of potential candidates are identical. As in DeMarzo,

[7] Alternatively, we may assume that the 1st incumbent manager only has the ability to commence the firm's project. In this case, the 1st manager's initial expected discounted payoff W_0 will be determined by the relative bargaining power of the investors and the 1st manager when the firm's project is initiated.

Fishman, He, and Wang (2012) and Brunnermeier and Sannikov (2014), our model then reduces to the standard dynamic model of corporate investment (the q theory of investment) à la Hayashi (1982), except that the liquidation value is endogenously determined.

Let q^{FB} denote the average value of capital (average or Tobin's q) in this case. Given the stationarity of the economic environment and the homogeneity of the production technology, we can show that the marginal value of capital (marginal q) equals average q in this situation. Furthermore, these two values are given by

$$q^{FB} = \max_{i} \frac{\mu - c(i)}{r + \delta - i}. \tag{7}$$

To ensure that q^{FB} is well defined, we make the following assumption.

Assumption 1: $(r+\delta)c'(0) < \mu < c(r+\delta)$.

The first inequality of Assumption 1 ensures that the first-best investment is positive and $q^{FB} > 1$,[8] whereas the second inequality of Assumption 1 indicates that the firm cannot profitably grow faster than the discount rate.

Then, the first-order condition for (7) yields the following proposition.

Proposition 1: *The first-best investment is characterized by*

$$c'(i^{FB}) = q^{FB} = \frac{\mu - c(i^{FB})}{r + \delta - i^{FB}}. \tag{8}$$

In addition, the first-best investment is increasing with q.

4. Optimal Contract with Agency

In this section, we characterize the optimal contract with agency concerns. The contracting problem is then to find incentive-compatible contracts with each manager, $(\Phi^1, \ldots, \Phi^N)$, that maximize the expected profit of the investors subject to ensuring each manager's participation in the contract.

For any (Φ^n, τ^{n-1}), the n-th manager's continuation value W_t^n is his future expected

[8] Note that $c(0) = 0$ and $c'(i_t) \geq 1$ for any $i_t \geq 0$.

discounted payoff at time t ($\geq \tau^{n-1}$), given that he will follow $\{a_s^n = 1 : t \leq s < \tau^n\}$:

$$W_t^n = E_t \int_t^{\tau^n} e^{-\gamma(s-t)} dU_s^n ds. \tag{9}$$

The optimal contract is now derived using the technique in DeMarzo and Sannikov (2006) and Sannikov (2008). Specifically, for any (Φ^n, τ^{n-1}), there exists a progressively measurable process $\{\beta_t^n : \tau^{n-1} \leq t < \tau^n\}$ in $\boldsymbol{\mathcal{L}}^*$ such that W_t^n evolves according to[9]

$$dW_t^n = \gamma W_t^n dt - dU_t^n + \beta_t^n K_t \left(dA_t^n - \mu dt\right), \tag{10}$$

under $a_t^n = 1$ for all time periods. The evolution of W_t^n in (10) includes the sensitivity $\beta_t^n(W_t^n)$ of W_t^n to output $K_t dA_t^n$. This suggests that we can characterize the n-th manager's incentive compatibility by considering $\beta_t^n(W_t^n)$.

Indeed, implementing $a_t^n = 1$ is incentive compatible if and only if

$$\beta_t^n(W_t^n) \geq \lambda, \qquad \text{for all } t \in [\tau^{n-1}, \tau^n). \tag{11}$$

Intuitively, if the n-th manager deviates and chooses $a_t^n = 0$, his instantaneous benefit is $\lambda\mu K_t$, whereas his instantaneous cost is the expected reduction of his consumption, $\beta_t^n \mu K_t$. Hence, the n-th manager's incentive compatibility is equivalent to (11).

In this model, the only relevant state variables going forward are the firm's capital stock K_t and the n-th manager's continuation payoff W_t^n. Thus, denote by $P^n(K_t, W_t^n)$ the value function of the investors hiring the n-th manager (the highest expected present value of the profit to the investors, given K_t and W_t^n). Using the scale invariance of the firm's technology arising from the homogeneity assumption, we write $P^n(K_t, W_t^n) = p^n(w_t^n)K_t$. Hence, we can reduce our problem to one with a single state variable $w_t^n = \frac{W_t^n}{K_t}$. For simplicity, we assume that $p^n(w_t^n)$ is concave. The formal proof for the concavity of $p^n(w_t^n)$ is provided by Lemma A1 in the proof of Proposition 2 in Appendix A.

If w_t^n is small, the investors could replace the manager or liquidate the firm. Here, we

[9] A process β^n is in $\boldsymbol{\mathcal{L}}^*$ if $E\left[\int_{\tau^{n-1}}^{\tau^n} Y_s^2 ds\right] < \infty$ for all $t \in [\tau^{n-1}, \tau^n)$.

assume that the firm is not liquidated until the N-th manager is fired and does not continue to be hired in the firm. The integer N will be given below in the replacement boundary to make the analysis meaningful. For $n < N$, the investors can replace the n-th manager with the $n+1$-th manager if w_t^n is small.

Let $\underline{w}^n$ be the replacement boundary for the n-th manager such that the n-th manager is replaced at w_t^n and instead the $n+1$-th manager is hired at $\underline{w}^n$. To compensate the n-th manager for w_t^n at his replacement, the firm needs to exploit stochastic replacement so that the n-th manager continues to be hired as the $n+1$-th manager with some probability at $\underline{w}^n$.

We have also assumed that there is a cost of lost productivity at the replacement of the manager. To simplify the analysis, we impose the following assumption.

Assumption 2: (i) $c_f = \zeta(\underline{w}^n - w_t^n)$ for $\zeta > 0$ and $w_t^n \leq \underline{w}^n$; and (ii) $c_f = w_t^n - \underline{w}^n$ for $w_t^n > \underline{w}^n$.

Assumption 2 implies that the replacement cost is increasing (decreasing) in $\underline{w}^n - w_t^n$ if $w_t^n < \underline{w}^n$ ($w_t^n > \underline{w}^n$).

We first justify Assumption 2(i). When $w_t^n \leq \underline{w}^n$, the replacement cost can be caused by the reorganization and/or restructuring of the firm. If the n-th manager is replaced at w_t^n ($\leq \underline{w}^n$) and instead the $n+1$-th manager is hired at $\underline{w}^n$, the firm can launch its reorganization and/or restructuring of the firm, which requires a larger cost per unit of capital stock when the firm's current and future situation worsens. Indeed, the less favorable present and future prospects of the firm are represented by the lower w_t^n. Hence, when the firm replaces the n-th manager at w_t^n and hires the $n+1$-th manager at $\underline{w}^n$ ($\geq w_t^n$), it needs to spend a larger reorganization and/or restructuring cost per unit of capital stock if $\underline{w}^n - w_t^n$ is larger. This assumption can also be justified in another way by the implementation result derived in Section 6. This is because (30) enables us to rewrite Assumption 2(i) by $c_f = \zeta\lambda(m_t^n - \underline{m}^n)$ for $m_t^n \geq \underline{m}^n$, where m_t^n is the credit line balance per unit of capital stock when the n-th manager is hired; and $\underline{m}^n$ is the replacement threshold regarding m_t^n.[10] Thus, this can be interpreted such that the replacement cost of the n-th manager per unit of capital stock

[10] In Proposition 6 in Section 6, $\underline{m}^n$ is given by $\frac{\overline{w}^n - \underline{w}^n}{\lambda}$.

is higher, the more constrained the firm's liquidity and financial slack position.[11]

These justifications can be applied not only to the case in which the n-th manager is fired and another new manager is hired from the pool of potential applicants as the $n+1$-th manager, but also to the the case in which the n-th manager continues to be employed as the $n+1$-th manager at $\underline{w}^n$. This is because both the reorganization and/or restructuring cost and the new equity issuing cost are needed, even in the latter case.

We next justify Assumption 2(ii). In fact, when $w_t^n > \underline{w}^n$, the stochastic replacement cannot ensure w_t^n for the n-th manager at his replacement because w_t^n must reduce to $\underline{w}^n$. Thus, in this case, the firm is forced to continue to employ the n-th manager as the $n+1$-th manager with probability 1 by giving him $w_t^n - \underline{w}^n$ as an immediate payment. This implies that the replacement cost is represented by Assumption 2(ii) in this case.

However, as argued below in the payment boundary, it is not optimal for the investors to reduce w_t^n by making the immediate payment to the n-th manager before w_t^n hits $\overline{w}^n$. Hence, without loss of generality, we can neglect the possibility of the replacement of the manager when $w_t^n > \underline{w}^n$, and focus on the possibility of the replacement of the manager when $w_t^n \leq \underline{w}^n$ so that the n-th manager is replaced at w_t^n $(\leq \underline{w}^n)$ and the $n+1$-th manager is hired at $\underline{w}^n$.

The conditions to be satisfied by $\underline{w}^n$ are now discussed as follows. The argument above ensures that the replacement of the n-th manager occurs for $w_t^n \leq \underline{w}^n$, but does not occur for $w_t^n > \underline{w}^n$. As the investors' scaled value function changes from $p^n(w_t^n)$ to $p^{n+1}(w_t^{n+1})$ when they replace the n-th manager with the $n+1$-th manager, the optimality requires

$$p^n(w_t^n) \quad \leq \quad p^{n+1}(\underline{w}^n) - \zeta(\underline{w}^n - w_t^n), \qquad \text{for } w_t^n \leq \underline{w}^n, \tag{12a}$$

$$p^n(w_t^n) \quad > \quad p^{n+1}(\underline{w}^n) - (w_t^n - \underline{w}^n), \qquad \text{for } w_t^n > \underline{w}^n. \tag{12b}$$

Here, the left-hand side of (12) is the investors' scaled value when they continue to employ the n-th manager, whereas the right-hand side of (12) is the investors' scaled value minus the

[11] Furthermore, to reduce the credit line balance m_t^n to $\underline{m}^n$ at the replacement of the n-th manager, our implementation requires that the firm issues additional new equity to the initial shareholders. If this new equity issue must be made promptly, and if the prompt equity issue obliges the firm to incur additional costs that are proportional to the new amount of equity, the replacement cost of c_f can also include such issuing costs that are proportional to $m_t^n - \underline{m}^n$. Bolton, Chen, and Wang (2011) also model the frictional financing costs a firm incurs when it chooses to issue external equity.

replacement cost when they replace the n-th manager. Inequality (12a) shows that $p^n(w_t^n) \leq p^{n+1}(\underline{w}^n)$ for $w_t^n \leq \underline{w}^n$. This implies that $p^n(\underline{w}^n) \leq p^{n+1}(\underline{w}^n)$. However, at the replacement boundary $\underline{w}^n$, we must have

$$p^n(\underline{w}^n) = p^{n+1}(\underline{w}^n). \tag{13}$$

This is because if $p^n(\underline{w}^n) < p^{n+1}(\underline{w}^n)$, (12b) could not be satisfied if w_t^n is sufficiently close to $\underline{w}^n$. Furthermore, it follows from (12a) and (13) that

$$\begin{aligned} p^n(w_t^n) - p^n(\underline{w}^n) &\leq p^{n+1}(\underline{w}^n) - p^n(\underline{w}^n) - \zeta(\underline{w}^n - w_t^n) \\ &\leq \zeta(w_t^n - \underline{w}^n), \qquad \text{for } w_t^n \leq \underline{w}^n. \end{aligned} \tag{14}$$

If $w_t^n - \underline{w}^n \to 0$, the optimality also requires that the replacement boundary for the n-th manager, $\underline{w}^n$, must be the largest scaled continuation payoff of the n-th manager that satisfies

$$p^{n\prime}(\underline{w}^n) = \zeta. \tag{15}$$

The reason is that if $p^{n\prime}(\underline{w}^n) < \zeta$, (14) must be satisfied with strict inequality for $w_t^n = \underline{w}^n$. Given (13), this finding implies that (12a) must be satisfied with strict inequality for $w_t^n = \underline{w}^n$. However, this contradicts (13). Note that (15) also satisfies (12b) because (12b) only results in $p^{n\prime}(\underline{w}^n) > -1$. In the end, for $w_t^n < \underline{w}^n$, the investors immediately replace the n-th manager at w_t^n and newly hire the $n+1$-th manager at $\underline{w}^n$, which can be summarized by $p^n(w_t^n) = p^{n+1}(\underline{w}^n) - \zeta(\underline{w}^n - w_t^n)$.

However, we suppose that the investors do not hire any new manager from the pool of potential applicants yet as $n = N$. Thus, it needs to be optimal for the investors to fire the N-th manager with zero value and liquidate the firm.[12] This implies that

$$p^{N\prime}(0) = \zeta, \tag{15$'$}$$

[12] Alternatively, we may suppose that the investors must always liquidate the firm after the N-th manager is replaced. Although the liquidation value needs to be considered exogenous in this case, the lower boundary condition in the case of the N-th manager is then given by $p^N(0) = \ell$. Indeed, in this situation, the HJB equation and all the boundary conditions are the same as those of DeMarzo, Fishman, He, and Wang (2012), when the N-th manager is hired. Hence, the value function, $p^N(w^N)$, is equivalent to that of DeMarzo, Fishman, He, and Wang (2012) for all $w^N \in [0, \overline{w}^N]$. Even so, however, all of the results in our paper still continue to hold, except that $p^N(w^N)$ is equivalent to the value function of DeMarzo, Fishman, He, and Wang (2012).

$$p^N(0) = \ell. \tag{16}$$

Equation (15′) shows that the N-th manager is replaced at $w^N = 0$, whereas (16) means that the firm is liquidated at $w^N = 0$.[13] Note that the liquidation value ℓ is determined endogenously so that the firm is liquidated at $w^N = 0$.

To make our analysis meaningful, we focus on the case in which $\underline{w}^{N-1} > 0$ and $w_0 = \frac{W_0}{K_0} > \underline{w}^1$. Then, Proposition 3 derived in the next section ensures that $0 < \underline{w}^{n-1} < w_0$ for $n = 1, \ldots, N-1$. We also focus on the case in which N is set so that $p^1(w_0)$ is larger than the maximum value attainable by the investors when the 1st manager is never replaced until the liquidation of the firm. This assumption can be justified if N and the replacement cost parameter ζ are sufficiently small.

To complete the discussion of the replacement boundary, we need to determine the probability of the n-th manager being replaced at $\underline{w}^n$ when $n < N$. To compensate the n-th manager for w_t^n $(< \underline{w}^n)$, we suppose that the investors replace the n-th manager with a new manager from the pool of potential applicants with probability $1 - \frac{w_t^n}{\underline{w}^n}$, whereas they retain the n-th manager and increase w_t^n by $\underline{w}^n$ with probability $\frac{w_t^n}{\underline{w}^n}$. As the n-th manager is retired with value 0 when he is fired and does not continue to be hired in the firm, this replacement strategy can compensate the n-th manager for w_t^n at his replacement.

For the upper boundary of w^n, note that there is a benefit from deferring the n-th manager's compensation because early compensation for a small w_t^n increases the possibility of the inefficient replacement of managers or the inefficient liquidation of the firm. On the other hand, there is a cost in deferring the n-th manager's compensation because he has a higher discount rate than the investors. This trade-off means that there is a compensation level for the n-th manager, $\overline{w}^n$, such that it is optimal to pay the n-th manager with cash if $w_t^n \geq \overline{w}^n$ and to defer compensation otherwise. Let $du_t^n = \frac{dU_t^n}{K_t}$. Thus, $du_t^n = \max(w_t^n - \overline{w}^n, 0)$, which implies that $p^n(w_t^n) = p^n(\overline{w}^n) - (w_t^n - \overline{w}^n)$, for $w_t^n > \overline{w}^n$. Hence, as shown in DeMarzo, Fishman, He, and Wang (2012), the payment boundary for the n-th manager, $\overline{w}^n$, is the smallest scaled continuation payoff of the n-th manager that satisfies

$$p^{n\prime}(\overline{w}^n) = -1. \tag{17}$$

[13] These conditions can be interpreted as an extension of the lower boundary condition in the fixed replacement cost model in He (2012, Section 3.1) into the variable replacement cost model.

For $w_t^n \in [\underline{w}^n, \overline{w}^n]$ ($w_t^N \in [0, \overline{w}^N]$), the n-th manager is not paid anything, $du_t^n = 0$, when $n < N$ ($n = N$). Thus, the evolution of w_t^n follows directly from the evolution of W_t^n from (10) and K_t from (1). Under the incentive–compatible contract ($\beta_t^n \geq \lambda$), it follows from (1), (4), and (10) with $a_t^n = 1$ and $du_t^n = 0$ that

$$dw_t^n = [\gamma - (i_t^n - \delta)]\, w_t^n dt + \beta_t^n \left(dA_t^n - \mu dt\right) = [\gamma - (i_t^n - \delta)]\, w_t^n dt + \beta_t^n \sigma dZ_t. \tag{18}$$

Equation (18) implies that the promised payoff of the n-th manager grows on average at his discount rate γ less the net growth rate $(i_t^n - \delta)$ of the firm.

Now, using (18) and Ito's lemma, we can obtain the following Hamilton–Jacobi–Bellman (HJB) equation to characterize $p^n(w^n)$ for $w^n \in [\underline{w}^n, \overline{w}^n]$ when $n < N$ ($w^N \in [0, \overline{w}^N]$ when $n = N$):

$$rp^n(w^n) = \sup_{i^n \geq 0,\, \beta^n \geq \lambda} [\mu - c(i^n)] + (i^n - \delta)p^n(w^n) + [\gamma - (i^n - \delta)]\, w^n p^{n\prime}(w^n) + \frac{1}{2}\left(\beta^n\right)^2 \sigma^2 p^{n\prime\prime}(w^n), \tag{19}$$

with $p^n(w_t^n) = p^{n+1}(\underline{w}^n) - \zeta(\underline{w}^n - w_t^n)$ for $w_t^n < \underline{w}^n$; and $p^n(w_t^n) = p^n(\overline{w}^n) - (w_t^n - \overline{w}^n)$ for $w_t^n > \overline{w}^n$. The first–term on the right-hand side of (19) expresses the instantaneous expected cash flows, the second term is the expected change in the value of the firm from capital accumulation, and the third and fourth terms are the expected change in the scaled value of the firm because of the drift and volatility in the n-th manager's scaled continuation payoff w^n.

As $p^{n\prime\prime}(w^n) < 0$, the outside investors dislike volatility in w^n and optimally choose the sensitivity of w^n to output. That is, $\beta^n = \lambda$ in (19). Furthermore, we derive the optimal investment–capital ratio i_t^n that satisfies the following Euler equation.

$$c'(i^n(w)) = p^n(w^n) - w^n p^{n\prime}(w^n). \tag{20}$$

Equation (20) shows that the investors' marginal cost of investing equals their marginal value of investing. The marginal value of investing is represented by $p^n(w^n)$ plus the marginal effect of decreasing w^n as the firm grows.

The investors' scaled value function $p^n(w^n)$ is now jointly determined by (18) and (19) in the range of $w^n \in [\underline{w}^n, \overline{w}^n]$ when $n < N$ ($w^N \in [0, \overline{w}^N]$ when $n = N$). We also have the boundary conditions (13) and (15) for the replacement boundaries $\underline{w}^n$ when $n = 1, \ldots, N-1$, and the "smooth pasting" conditions (17) for the payout boundaries when $n = 1, \ldots, N$. Only when $n = N$ is the firm liquidated. Thus, (15′) and (16) must be satisfied for $n = N$. To complete our characterization, we need the "super contract" conditions in order to determine the optimal levels of $\overline{w}^n$ for $n = 1, \ldots, N$:

$$p^{n\prime\prime}(\overline{w}^n) = 0. \tag{21}$$

Then, using (17) and (19), we show that (21) is equivalent to

$$p^n(\overline{w}^n) + \overline{w}^n = \sup_{i^n \geq 0} \frac{\mu - c(i^n) - (\gamma - r)\overline{w}^n}{r + \delta - i^n}, \qquad n = 1, \ldots, N. \tag{22}$$

In (22), the left-hand side is the total firm value at $\overline{w}^n$, while the right-hand side is the perpetuity value of the firm's cash flows given the cost of maintaining w^n at $\overline{w}^n$ because there is a cost to deterring the n-th manager's compensation when $\gamma > r$. Again, this implies that postponing payment is optimal until (22) is satisfied.

The following proposition summarizes the optimal contract. To simplify the analysis, we assume the case of quadratic investment adjustment costs:

$$c(i) = i + \frac{1}{2}\theta(i)^2, \tag{23}$$

where $\theta > 0$. Then, (20) implies that the optimal investment–capital ratio i_t^n is given by

$$i^n(w) = \frac{p^n(w^n) - w^n p^{n\prime}(w^n) - 1}{\theta}. \tag{24}$$

We also provide a formal verification argument for the optimal policy in the proof of Proposition 2 in Appendix A.

Proposition 2: *(i) The investors' value function after the n-th manager is hired is represented by $P^n(K, W^n) = p^n(w^n)K$, where $p^n(w^n)$ is the investors' scaled value function. The investors hire N managers cumulatively before the firm is liquidated.*

(ii) Suppose that $n < N$. For $w^n \in [\underline{w}^n, \overline{w}^n]$, $p^n(w^n)$ is strictly concave and uniquely solves the HJB equation (19) with boundary conditions (13), (15), (17), and (21). For $w^n < \underline{w}^n$, $p^n(w^n) = p^n(\underline{w}^n) - \zeta(\underline{w}^n - w^n)$. For $\overline{w}^n < w^n$, $p^n(w^n) = p^n(\overline{w}^n) - (w^n - \overline{w}^n)$.
(iii) Suppose that $n = N$. For $w^N \in [0, \overline{w}^N]$, $p^N(w^N)$ is strictly concave and uniquely solves the HJB equation (19) with boundary conditions (15′), (16), (17), and (21). For $\overline{w}^N < w^N$, $p^N(w^N) = p^N(\overline{w}^N) - (w^N - \overline{w}^N)$.
(iv) The manager's scaled continuation payoff w^n evolves according to (18) for $w^n \in [\underline{w}^n, \overline{w}^n]$ when $n < N$, and for $w^N \in [0, \overline{w}^N]$ when $n = N$. Cash payments du_t^n are equal to zero for $w_t^n \in [\underline{w}^n, \overline{w}^n)$ and reflect w_t^n back to $\overline{w}^n$ when $n < N$; whereas du_t^N is equal to zero for $w_t^N \in [0, \overline{w}^N)$ and reflects w_t^N back to $\overline{w}^N$. The n-th manager is replaced when $w_t^n < \underline{w}^n$ for $n < N$, which causes w_t^{n+1} to start at $\underline{w}^n$.[14] However, the contract is terminated at time τ^N when w_t^N reaches 0. Optimal investment is given by $I_t^n = i^n(w_t^n)K_t$, where $i^n(w_t^n)$ is determined by (20) ((24)). The sensitivity of w_t^n to output is given by $\beta_t^n = \lambda$.

Proof: See Appendix A. ■

The concavity of $p^n(w^n)$ reveals the "investors' induced aversion" to fluctuations in w^n because the variations in w^n increase the risks of the inefficient replacement of the manager and the inefficient liquidation of the firm. According to DeMarzo, Fishman, He, and Wang (2012), the investors' induced aversion to fluctuations in w^n implies that the investors behave in a risk-averse manner toward idiosyncratic risk because of the agent's friction, even though they are risk neutral.

Figure 1 depicts an example of the investors' scaled value function $p^n(w^n)$ and $p^{n+1}(w^{n+1})$. In this figure, we do not illustrate the detailed properties of $p^n(w^n)$ or $p^{n+1}(w^{n+1})$ in the range of $w^j > \widehat{w}^j$, where $\widehat{w}^j = \arg\max_{w^j} p^j(w^j)$ for $j = n, n+1$. This is because in that range, the graphs of $p^n(w^n)$ and $p^{n+1}(w^{n+1})$ are similar to the investors' scaled value function illustrated in DeMarzo, Fishman, He, and Wang (2012).

[14] Note that the n-th manager is replaced with a new manager with probability $1 - \frac{w_t^n}{\underline{w}^n}$, while he is retained as the $n+1$-th manager with probability $\frac{w_t^n}{\underline{w}^n}$.

5. Model Implications

In this section, we examine the properties of the replacement boundaries and analyze the model's predictions for the investors' scaled value, average q, marginal q, and the investment–capital ratio when considering the firm's successive decisions concerning manager turnover. Although the incumbent n-th manager continues to be hired as the $n+1$-th manager with some probability, all the replacements of the incumbent managers by the other new managers are characterized by the replacement boundaries in our model. Thus, this investigation enables us to analyze the joint dynamics of the firm's investment and managerial turnover around the time of the replacement of the incumbent manager by another new manager.

We begin by proving the following proposition about the replacement boundaries.

Proposition 3: (i) $0 < \underline{w}^{n+1} < \underline{w}^n$ *for* $n = 1, \ldots, N-2$.
(ii) $\underline{w}^n$ *is uniquely determined for* $n = 1, \ldots, N-1$.

Proof: It follows from the definition of $\underline{w}^n$ that if $\underline{w}^n \leq \underline{w}^{n+1}$, the investors must immediately replace the $n+1$-th manager and incur the replacement cost after the $n+1$-th manager is employed. Hence, we can exclude this possibility without loss of generality,[15] and verify that $\underline{w}^{n+1} < \underline{w}^n$. Note that $\underline{w}^n > 0$ for $n < N$ because we focus on the case $\underline{w}^{N-1} > 0$. Finally, it is immediate from the strict concavity of $p^n(w^n)$ and $p^{n\prime}(\underline{w}^n) = \zeta$ that $\underline{w}^n$ is uniquely determined. ■

Proposition 3 suggests that the replacement boundary declines with the frequency of the replacement of managers, even though the incumbent manager can be retained with some probability at or below the replacement boundary. This implies that the firm's optimal replacement/retention decision becomes more permissive with time. Although this result may be analogous to that of Garrett and Pavan (2012), their result suggests that the productivity level that the firm requires for retention declines with the length of the manager's tenure. Thus, their result is applied to the one-shot replacement strategy of the firm facing a perfectly stationary environment with no turnover cost in the sense that the firm is confronted with the same agency problem with each manager it hires. By contrast, our result holds for the successive replacement/retention strategy of the firm that employs and replaces managers

[15] If there is a sufficiently small administration cost incurred by the investors and the manager at the contract termination, this procedure can also be justified at $\underline{w}^n = \underline{w}^{n+1}$.

by incurring variable replacement costs related to the firm's financial distress. Intuitively, the firm would have to immediately replace the $n+1$-th manager and incur the additional replacement cost if it replaced the n-th manager when $\underline{w}^n \leq \underline{w}^{n+1}$. Hence, the firm desires to delay the replacement of the $n+1$-th manager by w^{n+1} that is sufficiently smaller than $\underline{w}^n$.

We next derive the implications of the properties of $p^n(w)$. We first obtain the following lemma.

Lemma 1: *(i) $p^n(w) < p^{n+1}(w)$ for any $w \in [0,\ \underline{w}^n)$ and $n = 1, \ldots, N-1$.*
(ii) $p^n(w) = p^{n+1}(w)$ for $w = \underline{w}^n$ and $n = 1, \ldots, N-1$.
(iii) $p^n(w) > p^{n+1}(w)$ for $n = 1, \ldots, N-1$, if w is not sufficiently larger than $\underline{w}^n$.

Proof: Suppose that $n < N-1$. Using the concavity of $p^{n+1}(w)$, $p^{n\prime}(\underline{w}^n) = p^{n+1\prime}(\underline{w}^{n+1}) = \zeta$, $\underline{w}^n > \underline{w}^{n+1}$ from Proposition 3, and $p^n(w) = p^{n+1}(\underline{w}^n) - \zeta(\underline{w}^n - w)$ for any $w \leq \underline{w}^n$, we verify all the statements of this lemma. Suppose that $n = N-1$. Then, it follows from the concavity of $p^N(w)$, $p^{N-1\prime}(\underline{w}^{N-1}) = p^{N\prime}(0) = \zeta$, and $p^{N-1}(w) = p^N(\underline{w}^{N-1}) - \zeta(\underline{w}^{N-1} - w)$ for any $w \leq \underline{w}^{N-1}$ that all the statements of this lemma hold. ■

The intuition behind Lemma 1 is that the firm incurs the replacement cost as a function of its financially distressed situation if it replaces the n-th manager with the $n+1$-th manager. Thus, when the n-th manager is replaced for $w \in [0,\ \underline{w}^n)$ and $n < N$, $p^{n+1}(w)$ must be higher than $p^n(w)$ to compensate the replacement cost in this range of w. However, after the $n+1$-th manager is hired when $n < N-1$, he is not replaced until w falls below $\underline{w}^{n+1}$, which is lower than $\underline{w}^n$ (see Proposition 3). Similarly, after the N-th manager is hired, he is not replaced until w falls to 0, which is lower than $\underline{w}^{N-1}$. As discussed below in Lemma 2(ii), this causes $p^{n\prime}(w) > p^{n+1\prime}(w)$ in the range of $w \in (\underline{w}^{n+1},\ \underline{w}^n]$ when $n < N-1$, and $p^{N-1\prime}(w) > p^{N\prime}(w)$ in the range of $w \in (0,\ \underline{w}^{N-1}]$. Hence, because of $p^n(\underline{w}^n) = p^{n+1}(\underline{w}^n)$, this feature makes $p^n(w)$ higher than $p^{n+1}(w)$ for $n = 1, \ldots, N-1$, if w is not sufficiently larger than $\underline{w}^n$.

We further show the following lemma.

Lemma 2: *(i) $p^{n\prime}(w) = p^{n+1\prime}(w) = \zeta$ for any $w \in [0,\ \underline{w}^{n+1}]$ and $n = 1, \ldots, N-2$.*
(ii) $p^{n\prime}(w) > p^{n+1\prime}(w)$ for any $w \in (\underline{w}^{n+1},\ \underline{w}^n]$ and $n = 1, \ldots, N-2$; and $p^{N-1\prime}(w) > p^{N\prime}(w)$ for any $w \in (0,\ \underline{w}^{N-1}]$.

(iii) In addition, if w is sufficiently close to $\underline{w}^n$ and if $w > \underline{w}^n$, then $p^{n\prime}(w) > p^{n+1\prime}(w)$ for $n = 1, \ldots, N-1$.

Proof: It is evident from Proposition 3 and (15) that $p^{n\prime}(w) = p^{n+1\prime}(w) = \zeta$ for any $w \in [0,\ \underline{w}^{n+1}]$ and $n = 1, \ldots, N-2$. It also follows from Proposition 3, (15), and the concavity of $p^{n+1}(w)$ that $p^{n\prime}(w) = \zeta > p^{n+1\prime}(w)$ for any $w \in (\underline{w}^{n+1},\ \underline{w}^n]$ and $n = 1, \ldots, N-2$. In addition, using the concavity of $p^N(w)$ with $p^{N\prime}(0) = \zeta$ and $p^{N-1}(w) = p^N(\underline{w}^{N-1}) - \zeta(\underline{w}^{N-1} - w)$ for $w \leq \underline{w}^{N-1}$, we must have $p^{N-1\prime}(w) > p^{N\prime}(w)$ for any $w \in (0,\ \underline{w}^{N-1}]$. Finally, as $p^{n\prime}(\underline{w}^n) > p^{n+1\prime}(\underline{w}^n)$, we show that if w is sufficiently close to $\underline{w}^n$ and $w > \underline{w}^n$, then $p^{n\prime}(w) > p^{n+1\prime}(w)$ for such w and $n = 1, \ldots, N-1$. ■

The intuition underlying Lemma 2 is as follows. In the range of $w \in [0,\ \underline{w}^{n+1}]$ for $n = 1, \ldots, N-2$, both the n-th and the $n+1$-th managers are replaced. Hence, both the sensitivities of $p^n(w)$ and $p^{n+1}(w)$ with respect to w are equal to the replacement cost per unit of variation in w, that is, ζ. In the range of $w \in (\underline{w}^{n+1},\ \underline{w}^n]$ for $n = 1, \ldots, N-2$ ($w \in (0,\ \underline{w}^{N-1}]$), Proposition 3 shows that the n-th ($N-1$-th) manager is replaced, whereas the $n+1$-th (N-th) manager is not replaced. Hence, the firm delays the replacement of the $n+1$-th manager (the liquidation of the firm) by $\underline{w}^{n+1}$ (by 0). Under the investors' induced aversion to fluctuations in w, the delay of the replacement of the $n+1$-th manager (the liquidation of the firm) reduces the marginal value of his continuation payoff for the investors, thus decreasing $p^{n+1\prime}(w)$ ($p^{N\prime}(w)$). This causes the sensitivity of $p^{n+1}(w)$ ($p^N(w)$) to be smaller than that of $p^n(w)$ ($p^{N-1}(w)$) in this range. If w ($> \underline{w}^n$) is sufficiently close to $\underline{w}^n$, the tendency of $p^{n\prime}(w) > p^{n+1\prime}(w)$ continues for such w in which the n-th manager is not replaced.

Average q is defined as the ratio between firm value and capital stock. As total firm value includes the claim held by the manager and is equal to $P(K, W) + W$, average q when the n-th manager is hired is represented by

$$q_a^n(w) = \frac{P^n(K, W) + W}{K} = p^n(w) + w, \qquad n = 1, \ldots, N. \tag{25}$$

On the other hand, marginal q measures the incremental impact of a unit of capital on firm value. Thus, marginal q when the n-th manager is hired is given by

$$q_m^n(w) = \frac{\partial\left[P^n(K,W) + W\right]}{\partial K} = P_K^n(K,W) = p^n(w) - wp^{n\prime}(w), \qquad n = 1, \dots, N. \tag{26}$$

As in DeMarzo, Fishman, He, and Wang (2012), using (7), (25), and (26) with $p^{n\prime}(w) \geq -1$, note that the following inequality is attained:

$$q^{FB} > q_a^n(w) \geq q_m^n(w), \qquad \text{for any } w \geq \underline{w}^n \text{ and } n < N; \text{ and for any } w \geq 0 \text{ and } n = N. \tag{27}$$

Hence, it follows from (8), (20), (26), (27), and the convexity of $c(\cdot)$ that

$$i^n(w) < i^{FB}, \qquad \text{for any } w \geq \underline{w}^n \text{ and } n < N; \text{ and for any } w \geq 0 \text{ and } n = N. \tag{28}$$

Because the replacement of the n-th manager causes w^n to end at $\underline{w}^n$, we focus on the case of $w^n \geq \underline{w}^n$ when considering investment under the employment of the n-th manager. In addition, when the n-th manager is hired, he is not replaced as long as $w > \underline{w}^n$. However, if $w \leq \underline{w}^n$, the n-th manager is replaced and the $n+1$-th manager starts his management at $w = \underline{w}^n$. This implies that, when examining the effect of the replacement of the manager on investment, we should evaluate average q, marginal q, and the investment–output ratio at $w > \underline{w}^n$ when the n-th manager is hired, while evaluating these variables at $w = \underline{w}^n$ when the n-th manager is replaced with the $n+1$-th manager.

Let $\underline{\underline{w}}^n$ be the smallest scaled continuation payoff of the n-th manager that satisfies $p^n(\underline{w}^n) = p^{n+1}(w^n)$ for $\underline{w}^n < w^n$. The following proposition about average and marginal q is obtained.

Proposition 4: *(i) For average q, $q_a^{n+1}(\underline{w}^n) < q_a^n(w)$ for any $w \in (\underline{w}^n,\ w^{n+})$ and $n = 1, \dots, N-1$, where $w^{n+} = \min(\underline{\underline{w}}^n, \overline{w}^n)$.*

(ii) For marginal q,

(a) $q_m^{n+1}(\underline{w}^n) > q_m^n(w)$ for $n = 1, \dots, N-1$ if w $(> \underline{w}^n)$ is sufficiently close to $\underline{w}^n$. However, if w $(> \underline{w}^n)$ is not sufficiently close to $\underline{w}^n$, it is possible that $q_m^{n+1}(\underline{w}^n) \leq q_m^n(w)$ for $n = 1, \dots, N-1$.

(b) $q_m^n(w)$ is increasing in w. In particular, $q_m^n(\underline{w}^n) < q_m^n(w)$ for $\underline{w}^n < w$ and $n = 1, \dots, N$.

Proof: (i) It follows from the concavity of $p^n(w)$, $p^{n\prime}(\underline{w}^n) = \zeta > 0$ and $p^n(\underline{w}^n) = p^{n+1}(\underline{w}^n)$

$= p^{n+1}(\underline{\underline{w}}^n)$ with (25) that this statement is verified.

(ii) For $n = 1, \ldots, N-1$, let $\psi^{n,n+1}(w, \underline{w}^n) = q_m^n(w) - q_m^{n+1}(\underline{w}^n) = p^n(w) - p^{n+1}(\underline{w}^n) - wp^{n\prime}(w) + \underline{w}^n p^{n+1\prime}(\underline{w}^n)$. Then, $\psi^{n,n+1}(\underline{w}^n, \underline{w}^n) < 0$ and $\frac{\partial \psi^{n,n+1}(w,\underline{w}^n)}{\partial w} = -wp^{n\prime\prime}(w) > 0$ because of Lemmas 1(ii) and 2(ii) and the concavity $p^n(w)$. If w $(> \underline{w}^n)$ is sufficiently close to $\underline{w}^n$, then $q_m^n(w) < q_m^{n+1}(\underline{w}^n)$ still holds for such w. However, if w $(> \underline{w}^n)$ is not sufficiently close to $\underline{w}^n$, it is possible that $q_m^n(w) \geq q_m^{n+1}(\underline{w}^n)$ for such w. Thus, the statement of (a) is verified. Furthermore, for $n = 1, \ldots, N$, note that $\frac{\partial q_m^n(w)}{\partial w} = -wp^{n\prime\prime}(w) > 0$ because of the concavity $p^{n+1}(w)$. Then, we verify the statement of (b). ■

The intuition behind the results of Proposition 4 is as follows. It follows from the definition of w^{n+} that $p^{n\prime}(\underline{w}^n) = \zeta > 0$ ensures $p^n(w) \geq p^n(\underline{w}^n)$ $(= p^{n+1}(\underline{w}^n))$ for $w \in [\underline{w}^n,\ w^{n+}]$ when $n = 1, \ldots, N-1$. Then, given the definition of $q_a(w)$ from (25), the result of Proposition 4(i) is immediately obtained.

For the results of Proposition 4(ii), notice that $p^n(\underline{w}^n) = p^{n+1}(\underline{w}^n)$ and $p^{n\prime}(\underline{w}^n) = \zeta > p^{n+1\prime}(\underline{w}^n)$ for $n = 1, \ldots, N-1$, given Lemmas 1(ii) and 2(ii). The first relation is evident from the definition of $\underline{w}^n$, whereas the second relation is derived from (15) and the delay of the replacement of the $n+1$-th manager in view of Proposition 3 under the investors' induced aversion to fluctuations in w. Then, using the definition of $q_m(w)$ from (26), this implies that $q_m^n(\underline{w}^n) < q_m^{n+1}(\underline{w}^n)$. If w $(> \underline{w}^n)$ is sufficiently close to $\underline{w}^n$, the relation of $q_m^n(w) < q_m^{n+1}(\underline{w}^n)$ continues to hold for such w. However, the investors' induced aversion to fluctuations in w implies that $q_m^n(w) - q_m^{n+1}(\underline{w}^n)$ is increasing in w. Hence, if w $(> \underline{w}^n)$ is not sufficiently close to $\underline{w}^n$, it is possible that $q_m^n(w) \geq q_m^{n+1}(\underline{w}^n)$ for such w. Thus, these results lead to Proposition 4(ii)(a). In addition, given that $q_m^n(w)$ is increasing in w, Proposition 4(ii)(b) is evident.

Proposition 4 can be interpreted as follows. Proposition 4(i) indicates that average q becomes lower after the replacement of each manager, except when the predecessor's scaled continuation value is not considerably larger. Proposition 4(ii)(a) suggests that marginal q becomes higher after the replacement of each manager if the predecessor's scaled continuation value is sufficiently close to his replacement threshold. However, it is possible that marginal q becomes lower after the replacement of the manager if his scaled continuation value is sufficiently larger than his replacement threshold. Proposition 4(ii)(b) shows that marginal

q is increasing in the manager's scaled continuation value.

Several remarks about Proposition 4 are in order. DeMarzo, Fishman, He, and Wang (2012) suggest that agency costs cause marginal q to differ from average q even though their model features homogeneity properties as in Hayashi (1982). In our model, if we further consider the possibility that the investors can successively replace any hired manager by incurring the variable replacement cost related to the firm's financially distressed situation, Propositions 4(i) and 4(ii)(a) imply that the variations of average q and marginal q around the replacement time of each manager move in opposite directions if the predecessor's scaled continuation value before his replacement is sufficiently close to his replacement threshold. However, these statements imply that the variations of average q and marginal q around the replacement time of each agent may move in the same direction if the predecessor's scaled continuation value is sufficiently larger than his replacement threshold before his replacement.

Thus, we summarize these discussions in the following corollary.

Corollary to Proposition 4: *The variations of average q and marginal q around the replacement time of each manager are in opposite directions if the predecessor's scaled continuation value near his replacement time is sufficiently close to his replacement threshold, but may be in the same direction if his scaled continuation value near his replacement is sufficiently larger than his replacement threshold.*

We now turn to the model's prediction of the investment–capital ratio. Given (20) ((24)) and the convexity of $c(\cdot)$, we show the following proposition.

Proposition 5: *(i) $i^{n+1}(\underline{w}^n) > i^n(w)$ for $n = 1, \ldots, N-1$ if w $(> \underline{w}^n)$ is sufficiently close to $\underline{w}^n$. However, if w $(> \underline{w}^n)$ is not sufficiently close to $\underline{w}^n$, it is possible that $i^{n+1}(\underline{w}^n) \leq i^n(w)$ for $n = 1, \ldots, N-1$.*
(ii) $i^n(w)$ is increasing in w. In particular, $i^n(\underline{w}^n) < i^n(w)$ for $\underline{w}^n < w$ and $n = 1, \ldots, N$.

Proof: These statements are evident from Proposition 4(ii) with $c'' > 0$ and (20). ■

Proposition 5(i) indicates that around the replacement time of each manager, the optimal investment–capital ratio is higher under the successor than under the predecessor if the predecessor's scaled continuation value before his replacement is sufficiently close to his replacement threshold. However, the optimal investment–capital ratio may be lower under the successor than under the predecessor if the predecessor's scaled continuation value before his replacement is sufficiently larger than his replacement threshold. Proposition 5(ii) shows that the optimal investment–capital ratio is increasing in the manager's scaled continuation value.

These discussions are summarized as follows.

Corollary to Proposition 5: *(i) The optimal investment–capital ratio is higher after the replacement of each manager if the predecessor's scaled continuation value near his replacement time is sufficiently close to his replacement threshold, but may be lower after the replacement of each manager if his scaled continuation value near his replacement time is sufficiently larger than his replacement threshold.*
(ii) The optimal investment–capital ratio under each manager is increasing in his scaled continuation value.

6. Implementation of the Optimal Contract

We consider the implementation of the optimal contract à la DeMarzo and Sannikov (2006) in terms of securities that include equity, long-term debt, and a credit line.[16] These securities are held by widely dispersed investors or intermediaries. As the optimal contract is conditional on w, the implementation result is unaffected, regardless of whether the manager designs the capital structure and investment policies to maximize his own payoff or the investors design these policies to maximize the value of the firm.

However, the securities used in the implementation are quite different from those in DeMarzo and Sannikov (2006), although both of the models use equity, long-term debt, and a credit line. More specifically, the firm issues not only initial equity to raise initial capital K_0 at time 0, but also additional new equity to the initial shareholders to finance additional funds required at the replacement of each manager. Besides, when the initial equity is issued at time 0 (the new equity is issued at the replacement of the $n-1$-th manager), the firm grants the 1st manager (n-th manager) a fraction α^1 (α^n) of the firm's equity as restricted stock. In fact, if any manager is fired, he must return all of his holdings of this

[16] An alternative implementation is given in DeMarzo, Fishman, He, and Wang (2012), in which the firm retains a cash reserve rather than a credit line.

restricted stock to the firm. The remaining fraction of the firm's equity is held by the initial shareholders at any time. Equity holders receive dividend payments paid from the firm's available cash or credit. However, no managers can receive part of the liquidation payoff. In this sense, the manager's equity is inside equity with the provision that it is worthless in the event of his replacement or the firm's termination.

The firm is also provided financial slack with long-term debt and a credit line. According to the rule determined at time 0, the firm issues or buys back long-term debt with face value b_t^n per unit of capital stock at any time when the n-th manager is hired. The long-term debt issued when the n-th manager is hired is a consol bond that pays continuous coupons at rate x_t^n per unit of capital stock.[17] We let the coupon rate be r, so that $b_t^n = \frac{x_t^n}{r}$. If the firm defaults on a coupon payment, debt holders force termination of the project. A revolving credit line opened after hiring the n-th manager provides the firm with available credit up to a limit c^{Ln} per unit of capital stock. Balances on the credit line per unit of capital stock are m_t^n, and are charged a fixed interest rate r^c. The firm borrows and repays funds on the line of credit at the discretion of the manager. If m_t^n exceeds c^{Ln}, the firm defaults and the project is terminated.

Our implementation need not specify the priority between long-term debt and the credit line. However, if long-term debt is risky, the firm must trade at a discount because the coupon rate is equal to r. One way of dealing with this problem is to assume that as in DeMarzo and Sannikov (2006), lenders provide both long-term debt and the credit line so that the firm pays an amount that exactly offsets the discount on long-term debt because of credit risk. Note that lenders can receive the high interest rate r^C on the credit line if $r^C - r$ is sufficiently large.[18] Another way is to assume that when the n-th manager is hired, the liquidation value of the firm per unit of capital stock is larger than b_t^n and that the long-term debt is senior to the credit line, as in Hennessy and Whited (2005), DeMarzo and Sannikov (2006), and Hennessy, Levy, and Whited (2007). The long-term debt is then risk free.

The next proposition shows that the optimal contract can be implemented with a capital

[17] If $b_t^n < 0$, long-term debt is interpreted as a compensating balance, as in DeMarzo and Sannikov (2006).

[18] The credit line is almost always provided by banks or financing companies. In a sample of 11,758 credit lines obtained by 4011 public firms between 1996 and 2003 in Loan Pricing Corporation's DealScan, the median commitment fee is 25 basis points and the median interest rate on drawn funds is 150 basis points above LIBOR (see Sufi (2009)). Thus, the assumption that $r^C - r$ is sufficiently large can be justified.

structure based on the securities introduced above.[19] The proof of this proposition is provided in Appendix A.

Proposition 6: *There exists a capital structure that implements the optimal contract and has the following features:*

$$\alpha^n = \lambda, \qquad n = 1, \ldots, N, \tag{29a}$$

$$b_t^n = \frac{\mu}{r} - \frac{\gamma - i_t^n + \delta}{r} c^{Ln}, \qquad n = 1, \ldots, N, \tag{29b}$$

$$c^{Ln} = \frac{\overline{w}^n}{\lambda}, \qquad n = 1, \ldots, N. \tag{29c}$$

The line of credit has interest rate $r^C = \gamma$. For the balance $m_t^n \geq 0$, the n-th manager's scaled continuation payoff w_t^n is determined by the current draw m_t^n on the line of credit:

$$w_t^n = \lambda(c^{Ln} - m_t^n), \qquad n = 1, \ldots, N, \tag{30}$$

which coincides with the scaled continuation payoff of the n-th manager in Proposition 2. Then, it is optimal for the n-th manager to choose effort $a_t^n = 1$ and the investment–capital ratio i_t^n given by (24). The dividends are not paid until the credit line is fully repaid ($m_t^n = 0$). However, if m_t^n hits 0, all excess cash flows are issued as dividends. If $n < N$, the n-th manager is replaced when m_t^n is above $\frac{\overline{w}^n - \underline{w}^n}{\lambda}$. Then, new equity is issued to the initial shareholders, until the credit line returns to $\frac{\overline{w}^n - \underline{w}^n}{\lambda}$ and the replacement cost is covered. In addition, the n-th manager's equity is returned to the firm, but the $n+1$-th manager is granted new equity. If $n = N$, the firm is liquidated when m_t^N hits c^{LN}.

Several remarks on the theoretical implications of Proposition 6 are in order. First, equation (29a) shows that to eliminate the managers' incentive to divert cash, the investors need to provide each manager with a fraction of equity λ.

Second, equation (30) ensures that no manager pays dividends prematurely by drawing down the line of credit per unit of capital stock $c^{Ln} - m_t^n$ immediately and then defaulting. This is because (30) implies that the n-th manager's immediate payoff (his share of the firm's profit) when he follows this deviation—the right-hand side of (30)—would be equal to w_t^n,

[19] In this capital structure implementation, the manager can choose when to draw on or repay the credit line, how much to pay in dividends, and whether to accumulate cash balances within the firm.

which he can receive by committing to the rule of our capital structure implementation.

Third, (29b) shows that long-term debt per unit of capital stock b_t^n is decreasing in m_t^n because Proposition 5(ii) with (30) indicates that i_t^n is decreasing in m_t^n. This means that long-term debt is a substitute for the credit lines. Furthermore, if γ is close to r, (29b) suggests that the total debt capacity of the firm is represented by

$$b_t^n + c^{Ln} \simeq \frac{\mu}{r} + \frac{i_t^n - \delta}{r} c^{Ln}.$$

Thus, the total debt capacity of the firm per unit of capital stock is also decreasing in m_t^n.

Finally, the n-th manager has the incentive to pay dividends only when $m_t^n = 0$. However, all excess cash flows are paid as dividends once $m_t^n = 0$. On the other hand, the n-th manager for $n < N$ is replaced as long as m_t^n hits $\frac{\overline{w}^n - w^n}{\lambda}$. Given (29c), this implies that when $n < N$, the n-th manager is replaced even though the credit line balance is not fully exhausted.

7. Empirical Implications

As shown in Proposition 6, when the manager's scaled continuation payoff increases, this can be interpreted as greater financial slack (see equation (30)). Furthermore, Proposition 4(ii)(b) also shows that marginal q is increasing in the manager's scaled continuation payoff. These results imply that the higher the manager's scaled continuation payoff, the higher the financial slack and the higher the marginal q. Given these findings, we propose the following empirical implications for average q, marginal q, and the investment–capital ratio, using the corollaries to Propositions 4 and 5.

(A) The variations of average q and marginal q around the replacement time of each manager move in opposite directions if the firm's financial slack (marginal q) is relatively low before the replacement of the manager, but may move in the same direction if the firm's financial slack (marginal q) is not relatively low before the replacement of the manager.

(B)(i) The optimal investment–capital ratio (marginal q) is higher after the replacement of each manager if the firm's financial slack (marginal q) is relatively low before the replacement of the manager, but may be lower after the replacement of each manager if the firm's financial slack (marginal q) is not relatively low before the replacement of the manager.

(ii) The optimal investment–capital ratio under each manager is increasing as the firm's financial slack (marginal q) becomes higher.

Very few studies examine the patterns in capital budgeting and investment policy in the years surrounding CEO turnover. Huson, Malatesta, and Parrino (2004), using US firm data from 1971 to 1994, report that capital expenditure intensity increases around forced CEO turnovers. On the other hand, using a sample of US firm data from 1990 to 2007, Fee, Hadlock, and Pierce (2013) provide weak evidence that capital expenditure intensity decreases around overtly forced CEO turnovers. However, if they use the larger sample of suspected forced CEO turnovers, they obtain stronger evidence that the capital expenditure intensity increases around suspected forced CEO turnovers. These findings are broadly consistent with our implication (B) if the firm's financial slack or marginal q in their samples is relatively low before the replacement of the manager.

Using US firm data from 1989 to 2005, Hornstein (2013) shows that the estimated marginal q rises substantially as the firm nears the turnover year, peaks shortly after turnover, and then slowly stabilizes. This finding is again consistent with our implication (B) if the firm's financial slack is relatively low before the replacement of the manager and is stable after the turnover time elapses. Furthermore, Hornstein (2013) indicates that the impact of CEO turnover is asymmetric for under- and overinvesting firms.[20] Specifically, the regressed coefficient of the estimated marginal q upon average q is positively correlated and highly significant for underinvesting firms around CEO turnover. By contrast, this same variable is negative for overinvesting firms around CEO turnover, although it is insignificant. These findings are also consistent with our implication (A) because marginal q is relatively high (low) for under- (over)investing firms.

Our implications (A) and (B) also provide a new insight into the relation between average q and marginal q. The neoclassical q model of investment with adjustment costs shows that the investment–capital ratio is positively related to marginal q. In fact, in many empirical studies, average q is used as a proxy for the firm's investment opportunities. This is because (i) average q is easily observable, (ii) Hayashi (1982) provides conditions under which average q is equal to marginal q, and (iii) average q is positively related to marginal q in most of the

[20] According to her definition, under– (over)investing firms are defined as firms whose marginal q is larger (smaller) than 1.0 or 0.78.

investment models, although average q is not necessarily equal to marginal q.

However, several studies show that there is no longer a monotonic relation between investment and marginal q, and that average q can actually be a more robust indicator for investment. For example, Caballero and Leahy (1996) suggest that average q is a better proxy for the firm's investment opportunities with fixed costs of adjustment. Bolton, Chen, and Wang (2011) also argue that average q rather than marginal q can be a more robust predictor of investment for financially constrained firms.

Our implication (A) shows that the variations in average q and marginal q at the time of manager turnover can be in opposite directions. This implies that average q is not a suitable proxy for marginal q surrounding the turnover of the manager. In addition, combining our implications (A) and (B), we argue that the optimal investment–output ratio can be negatively related to average q at the time of manager turnover. This suggests that if average q is used for the estimation of the investment function, the estimation result involves an estimation bias when the manager is replaced. These findings imply that average q is not a better proxy than marginal q for the firm's investment opportunities surrounding the replacement of the manager, in contrast to the suggestions of both Caballero and Leahy (1996) and Bolton, Chen, and Wang (2011).

Our theory also derives new implications for the likelihood of CEO turnover, using Proposition 3.

(C) The more frequent the replacement of previous CEOs, the less likely the replacement of the succeeding CEO for the firm's poor financial slack,.

This statement provides new empirical implications for the likelihood of CEO turnover. Specifically, CEO retention is more likely, despite the firm's poor performance, the more frequent the replacement of previous CEOs.

8. Conclusion

We explore the dynamic theory of investment and costly managerial turnover under agency conflicts between the investors and the manager. To this end, we develop the continuous-time agency model with the q-theory of investment by incorporating the possibility of the successive replacement of managers before the firm is finally liquidated. The model enables us to discuss how the investors decide at each point of time whether to fire or retain the current manager by considering incurring a replacement cost related to the firm's financially distressed situation.

We show that the variations of average q and marginal q around the turnover are in opposite directions if the predecessor's scaled continuation payoff near his replacement time is sufficiently close to his replacement threshold (that is, if the firm's financial slack is relatively low before the replacement of the manager). However, such variations may occur in the same direction if his scaled continuation payoff near his replacement time is sufficiently larger than his replacement threshold (that is, if the firm's financial slack is not relatively low before the replacement of the manager). Our results also imply that the optimal investment–capital ratio increases after the replacement of each manager if the predecessor's scaled continuation payoff near his replacement time is sufficiently close to his replacement threshold (that is, if the firm's financial slack is relatively low before the replacement of the manager), but this ratio may be lower after the replacement of each manager if his scaled continuation payoff near his replacement time is sufficiently larger than his replacement threshold (that is, if the firm's financial slack is not relatively low before the replacement of the manager). Furthermore, we show that the firm's optimal replacement/retention decision becomes more permissive with the frequency of the replacement of managers. These main results are unaffected even when contracts are constrained to be renegotiation-proof. Finally, our theoretical findings yield several empirical implications for the dynamics of investment and CEO turnover policy that are consistent with evidence given in the existing empirical literature and provide novel testable hypotheses. In addition, our empirical implications suggest that if we need to consider the possibility of the replacement of the manager, average q is not necessarily a better proxy than marginal q for the firm's investment opportunities, in contrast to the suggestions of several theoretical studies such as Caballero and Leahy (1996) and Bolton, Chen, and Wang (2011).

Appendix A

Proof of Proposition 2: As required by DeMarzo, Fishman, He, and Wang (2012), we impose the following conditions. First, we assume the usual regularity condition on the payment policies

$$E\sum_{n}{}_1^N \left(\int_{\tau^{n-1}}^{\tau^n} e^{-\gamma t} dU_t^n\right)^2 < \infty. \tag{A1}$$

We also put the regularity conditions on the investment policies

$$E\left[\int_0^T \left(e^{-rt}K_t\right)^2 dt\right] < \infty \text{ for all } T > 0, \tag{A2}$$

and

$$\lim_{T\to\infty} E\left(e^{-rT}K_T\right) = 0. \tag{A3}$$

Now, we prove the following lemma.

Lemma A1: *When $n < N$, the function $p^n(w^n)$ is strictly concave on $(\underline{w}^n, \overline{w}^n)$. When $n = N$, the function $p^N(w^N)$ is also strictly concave on $[0, \overline{w}^N)$.*

Proof: We focus on the case of $n < N$ because the proof in the case of $n = N$ is similar.

Substituting the optimal value of $i^n(w)$ given by (24) into (19) and rearranging it with $\beta^n = \lambda$, (20), and (23), we obtain

$$(r+\delta)p^n(w^n) = \mu + \frac{\left[p^n(w^n) - w^n p^{n\prime}(w^n) - 1\right]^2}{2\theta} + (\gamma+\delta)w^n p^{n\prime}(w^n) + \frac{\lambda^2\sigma^2}{2}p^{n\prime\prime}(w^n). \tag{A4}$$

Differentiating (A4) with respect to w^n yields

$$(r+\delta)p^{n\prime}(w^n) = -\frac{\left[p^n(w^n) - w^n p^{n\prime}(w^n) - 1\right] w^n p^{n\prime\prime}(w^n)}{\theta} + (\gamma+\delta)p^{n\prime}(w^n)$$

$$+(\gamma+\delta)w^n p^{n\prime\prime}(w^n) + \frac{\lambda^2\sigma^2}{2}p^{n\prime\prime\prime}(w^n). \tag{A5}$$

Evaluating (A5) at $\overline{w}^n$, and using $p^{n\prime}(\overline{w}^n) = -1$ and $p^{n\prime\prime}(\overline{w}^n) = 0$, we have

$$\frac{\lambda^2\sigma^2}{2}p^{n\prime\prime\prime}(\overline{w}^n) = \gamma - r > 0.$$

Hence, $p^{n\prime\prime}(\overline{w}^n - \epsilon^n) < 0$ for sufficiently small $\epsilon^n > 0$.

Next, let $q^n(w^n) \equiv p^n(w^n) - w^n p^{n\prime}(w^n)$. Then, it follows from (A4) that

$$(r+\delta)q^n(w^n) = \mu + \frac{[q^n(w^n)-1]^2}{2\theta} + (\gamma - r)w^n p^{n\prime}(w^n) + \frac{\lambda^2\sigma^2}{2}p^{n\prime\prime}(w^n). \tag{A6}$$

Suppose that there exists some $w^{n\circ} < \overline{w}^n$ such that $p^{n\prime\prime}(w^{n\circ}) = 0$. Choose the largest $w^{n\circ}$ such that $p^{n\prime\prime}(w^{n\circ} + \epsilon^n) < 0$. Then, for any $w^n \in (w^{n\circ}, \overline{w}^n)$, we must have $p^{n\prime\prime}(w^n) < 0$ because $p^{n\prime\prime}(\overline{w}^n - \epsilon^n) < 0$ for sufficiently small $\epsilon^n > 0$. Evaluating (A6) at $w^{n\circ}$, we show

$$(r+\delta)q^n(w^{n\circ}) = \mu + \frac{[q^n(w^{n\circ})-1]^2}{2\theta} + (\gamma - r)w^{n\circ} p^{n\prime}(w^{n\circ}).$$

Using $p^{n\prime}(\overline{w}^n) = -1$ and $p^{n\prime\prime}(w^n) \leq 0$ for any $w^n \in [w^{n\circ}, \overline{w}^n]$, it follows that $q^n(w^{n\circ}) \equiv p^n(w^{n\circ}) - w^{n\circ}p^{n\prime}(w^{n\circ}) < p^n(w^{n\circ}) + w^{n\circ} < p^{FB}(w^{n\circ}) + w^{n\circ} \equiv q^{FB}$.[21] In addition, it is found from (8), (23), and (24) that $(r+\delta)q^{FB} = \mu + \frac{[q^{FB}-1]^2}{2\theta}$. Given that $\psi(q) \equiv (r+\delta)q - \mu - \frac{(q-1)^2}{2\theta}$ is increasing in q for any $q < 1 + \theta(r+\delta)$ and that $1 + \theta(r+\delta) > q^{FB} > q^n(w^{n\circ})$,[22] we obtain $p^{n\prime}(w^{n\circ}) < 0$. Now, evaluating (A5) at $w^{n\circ}$, we have

$$(r+\delta)p^{n\prime}(w^{n\circ}) = (\gamma+\delta)p^{n\prime}(w^{n\circ}) + \frac{\lambda^2\sigma^2}{2}p^{n\prime\prime\prime}(w^{n\circ}),$$

which implies that $p^{n\prime\prime\prime}(w^{n\circ}) = \frac{2(r-\gamma)}{\lambda^2\sigma^2}p^{n\prime}(w^{n\circ}) > 0$ because of $p^{n\prime}(w^{n\circ}) < 0$ and $\gamma > r$. However, this is inconsistent with the choice of $w^{n\circ}$, where $p^{n\prime\prime}(w^{n\circ}) = 0$, but $p^{n\prime\prime}(w^n + \epsilon^n) < 0$. Therefore, $p^n(w^n)$ is strictly concave over the whole domain $w^n \in (\underline{w}^n, \overline{w}^n)$. ‖

Next, let us consider any incentive-compatible contract Φ^n and any replacement time of the $n-1$-th manager τ^{n-1}. For any $t \in [\tau^{n-1}, \tau^n]$, define the auxiliary gain process J^n as

$$\begin{aligned} J^n(\Phi^n, t) &= \int_{\tau^{n-1}}^t e^{-r(s-\tau^{n-1})}(dY_s^n - dU_s^n) + e^{-r(t-\tau^{n-1})}P^n(K_t, W_t^n) \\ &= \int_{\tau^{n-1}}^t e^{-r(s-\tau^{n-1})}\left[K_s dA_s^n - I_s^n ds - \frac{\theta\,(I_s^n)^2}{2K_s}ds - dU_s^n\right] + e^{-r(t-\tau^{n-1})}P^n(K_t, W_t^n), \end{aligned} \tag{A7}$$

where W_t^n evolves according to (10). Note that the process J^n is such that J^n is $\mathcal{F}_t$-

[21] Because $P^{FB}(K, W^n) = q^{FB}K - W^n$, we have $p^{FB}(w^n) = q^{FB} - w^n$.

[22] Note that $q^{FB} = 1 + \theta i^{FB}$, where $i^{FB} = r + \delta - \sqrt{(r+\delta)^2 - 2\frac{\mu-(r+\delta)}{\theta}}$.

measurable. For any $t \leq \tau^n$, any arbitrary incentive-compatible contract Φ^n, and any replacement time of the $n-1$-th manager τ^{n-1}, it follows from $w_t^n = \frac{W_t^n}{K_t}$, $i_t^n = \frac{I_t^n}{K_t}$, $P^n(K_t, W_t^n)$ $= K_t p^n(w_t^n)$, and Ito's lemma that

$$e^{r(t-\tau^{n-1})} dJ^n(\Phi^n, t) = K_t \left\{ \left[\mu - i_t^n - \frac{\theta}{2} (i_t^n)^2 - (\delta + r - i_t^n) p^n(w_t^n) + [\gamma - (i_t^n - \delta)] w_t^n p^{n\prime}(w_t^n) \right. \right.$$
$$\left. \left. + \frac{1}{2} (\beta_t^n)^2 \sigma^2 p^{n\prime\prime}(w_t^n) \right] dt - [1 + p^{n\prime}(w_t^n)] du_t^n + [1 + \beta_t^n p^{n\prime}(w_t^n)] \sigma dZ_t \right\}. \tag{A8}$$

Given (19) and (23), the first piece in the large bracket in the right-hand side of (A8) always stays at zero under the optimal investment policy (24) and the optimal incentive policy β_t^n $= \lambda$, whereas this term is nonpositive under the other investment and incentive policies. The second element in the large bracket captures the optimality of the cash payment policy. This part is nonpositive because Lemma A1 with $p^{n\prime}(\overline{w}^n) = -1$ shows $p^{n\prime}(w_t^n) \geq -1$ for all $w_t^n \in$ $[\underline{w}^n, \overline{w}^n]$, whereas it is equal to zero under the optimal payment policy $du_t^n(w_t^n) = 0$ for all $w_t^n \in [\underline{w}^n, \overline{w}^n]$.

Define $\mu_{Jt}^n \equiv e^{-r(t-\tau^{n-1})} K_t \times$ (first element in the large bracket in the right-hand side of (A8)). Then, the auxiliary gain process can be summarized as

$$dJ^n(\Phi^n, t) = \mu_{Jt}^n dt - e^{-r(t-\tau^{n-1})} K_t [1 + p^{n\prime}(w_t^n)] du_t^n + e^{-r(t-\tau^{n-1})} K_t [1 + \beta_t^n p^{n\prime}(w_t^n)] \sigma dZ_t,$$

where $\mu_{Jt}^n dt - e^{-r(t-\tau^{n-1})} K_t [1 + p^{n\prime}(w_t^n)] du_t^n \leq 0$ for all $w_t^n \in [\underline{w}^n, \overline{w}^n]$. Let $\varphi_t^n = e^{-r(t-\tau^{n-1})} K_t [1$ $+ \beta_t^n p^{n\prime}(w_t^n)] \sigma dZ_t$. Conditions (A1) and (A2) imply that $E\left[\int_0^T \varphi_t^n dZ_t\right] = 0$ for all $T > 0$ (note that $p^{n\prime}(w_t^n)$ is bounded). Thus, the process J^n is an $\mathcal{F}_t$-supermartingale up to time $t = \tau^n$. Furthermore, the process J^n is an $\mathcal{F}_t$-martingale under the contract satisfying the conditions of this proposition up to time $t = \tau^n$.

Under any Φ^n and τ^{n-1}, the investors' expected payoff is

$$\widetilde{J}^n(\Phi^n) = E_{\tau^{n-1}} \left[\int_{\tau^{n-1}}^{\tau^n} e^{-r(s-\tau^{n-1})} (dY_s^n - dU_s^n) - e^{-r(\tau^n - \tau^{n-1})} c_f K_{\tau^n} \right.$$
$$\left. + e^{-r(\tau^n - \tau^{n-1})} P^{n+1}(K_{\tau^n}, W_{\tau^n}^n) \right], \qquad \text{for } n < N, \tag{A9a}$$

$$\widetilde{J}^N(\Phi^N) = E_{\tau^{N-1}}\left[\int_{\tau^{N-1}}^{\tau^N} e^{-r(s-\tau^{N-1})}(dY_s^N - dU_s^N) + e^{-r(\tau^N-\tau^{N-1})}\ell K_{\tau^N}\right]. \tag{A9b}$$

Then, it follows from (A7) and (A9) with $P^n(K_{\tau^n}, W_{\tau^n}^n) = K_{\tau^n}p^n(w_{\tau^n}^n) = K_{\tau^n}p^{n+1}(w_{\tau^n}^n) = P^{n+1}(K_{\tau^n}, W_{\tau^n}^n)$ and $P^N(K_{\tau^N}, W_{\tau^N}^N) = \ell K_{\tau^N}$ that for any $t \in [\tau^{n-1}, \infty)$,

$$\begin{aligned}
\widetilde{J}^n(\Phi^n) &= E_{\tau^{n-1}}\left[J^n(\Phi^n, \tau^n) - e^{-r(\tau^n-\tau^{n-1})}c_f K_{\tau^n}\right] \\
&= E_{\tau^{n-1}}\Big\{J^n(\Phi^n, t\wedge\tau^n) + e^{-r(t-\tau^{n-1})}\mathbf{1}_{t\le\tau^n}\Big[\int_t^{\tau^n} e^{-r(s-t)}(dY_s^n - dU_s^n) \\
&\quad + e^{-r(\tau^n-t)}P^{n+1}(K_{\tau^n}, W_{\tau^n}^n) - P^n(K_t, W_t^n)\Big] - e^{-r(t-\tau^{n-1})-r(\tau^n-t)}c_f K_{\tau^n}\Big\} \\
&\le P^n(K_{\tau^{n-1}}, W_{\tau^{n-1}}^n) + (q^{FB}-\ell)E_t e^{-r(t-\tau^{n-1})}K_t, \qquad \text{for } n < N;
\end{aligned} \tag{A10a}$$

and for any $t \in [\tau^{N-1}, \infty)$,

$$\begin{aligned}
\widetilde{J}^N(\Phi^n) &= E_{\tau^{N-1}}\Big\{J^N(\Phi^N, t\wedge\tau^N) + e^{-r(t-\tau^{N-1})}\mathbf{1}_{t\le\tau^N}\Big[\int_t^{\tau^N} e^{-r(s-t)}(dY_s^N - dU_s^N) \\
&\quad + e^{-r(\tau^N-t)}\ell K_{\tau^N} - P^N(K_t, W_t^N)\Big]\Big\} \\
&\le P^N(K_{\tau^{N-1}}, W_{\tau^{N-1}}^N) + (q^{FB}-\ell)E_t e^{-r(t-\tau^{N-1})}K_t.
\end{aligned} \tag{A10b}$$

The first-terms in the right-hand sides of the inequalities of (A10a) and (A10b) follow from the facts that $J^n(\Phi^n, t\wedge\tau^n)$ for $n < N$ and $J^N(\Phi^N, t\wedge\tau^N)$ are a supermartingale and that $J^n(\Phi^n, \tau^{n-1}) = P^n(K_{\tau^{n-1}}, W_{\tau^{n-1}}^n)$ for $n < N$ and $J^N(\Phi^N, \tau^{N-1}) = P^N(K_{\tau^{N-1}}, W_{\tau^{N-1}}^N)$. In addition, to derive the second terms in the right-hand sides of the inequalities of (A10a) and (A10b), let us notice that

$$E_{\tau^{n-1}}\left\{\int_t^{\tau^n} e^{-r(s-t)}(dY_s^n - dU_s^n) + e^{-r(\tau^n-t)}\left[P^{n+1}(K_{\tau^n}, W_{\tau^n}^n) - c_f K_{\tau^n}\right]\right\} \le \left(q^{FB} - w_t^n\right)K_t,$$

and

$$E_{\tau^{N-1}}\left\{\int_t^{\tau^N} e^{-r(s-t)}(dY_s^N - dU_s^N) + e^{-r(\tau^N-t)}\ell K_{\tau^N}\right\} \le \left(q^{FB} - w_t^N\right)K_t.$$

This is because the right-hand side of these inequalities is the upper bound on the principal's expected profit under the first-best contract. Using the facts that $p^n(w^n) \ge p^N(0) = \ell$ for any $w^n \ge \underline{w}^n$ and $n = 1, \ldots, N-1$ and that $w^N + p^N(w^N)$ is increasing in w^N because of

$p^{N\prime}(w^N) \geq -1$ for any $w^N \geq 0$, we have

$$\left(q^{FB} - w_t^n\right) K_t - P^n(K_t, W_t^n) \leq \left(q^{FB} - \ell\right) K_t, \qquad \text{for } n = 1, \ldots, N.$$

Now, it follows from (A3) that taking $t \to \infty$ yields $\widetilde{J}^n(\Phi^n) \leq P^n(K_{\tau^{n-1}}, W^n_{\tau^{n-1}})$ for all incentive-compatible contracts and for all $n = 1, \ldots, N$. On the other hand, under the optimal contract that satisfies the conditions of the proposition, the investors' payoff $\widetilde{J}^n(\Phi^n)$ achieves $P^n(K_{\tau^{n-1}}, W^n_{\tau^{n-1}})$ because the above inequality holds in equality when $t \to \infty$.

The remaining problem is to derive a sufficient condition for the optimality of implementing $\{a_t^n = 1 : \tau^{n-1} \leq t < \tau^n\}$ for $n = 1, \ldots, N$.

Lemma A2: *The sufficient condition for the optimality of implementing* $\{a_t^n = 1 : \tau^{n-1} \leq t < \tau^n\}$ *all the time for* $n = 1, \ldots, N$ *is*

$$(r+\delta)p^n\left(\frac{\lambda\mu}{\gamma+\delta}\right) - (\gamma - r)\left[p^n(\widehat{w}^n) - p^n\left(\frac{\lambda\mu}{\gamma+\delta}\right)\right] \geq \frac{\left[p^n(\overline{w}^n) + \overline{w}^n - 1\right]^2}{2\theta},$$

where $\widehat{w}^n = \arg\max_{w^n} p^n(w^n)$.

Proof: When the n-th manager is induced to shirk, he enjoys a private benefit $\lambda\mu dt$ per unit of capital stock. Because the n-th manager's payoff would not need to depend on cash flows when he is induced to shirk, his promised payoff would evolve according to

$$dW_t^n = \begin{cases} \gamma W_t^n dt - dU_t^n + \lambda K_t(dA_t^n - \mu dt), & \text{if } a_t^n = 1, \\ \gamma W_t^n dt - dU_t^n - \lambda\mu K_t dt, & \text{if } a_t^n = 0. \end{cases} \tag{A11}$$

Using $w_t^n = \frac{W_t^n}{K_t}$, (A11) is written by

$$dw_t^n = \begin{cases} \left[\gamma - (i_t^n - \delta)\right] w_t^n dt - du_t^n + \lambda(dA_t^n - \mu dt), & \text{if } a_t^n = 1, \\ \left[\gamma - (i_t^n - \delta)\right] w_t^n dt - du_t^n - \lambda\mu dt, & \text{if } a_t^n = 0. \end{cases}$$

Because $p^n(w^n)$ is concave, it could be beneficial for the investors to reduce the volatility of w_t^n by inducing the n-th manager to shirk. For that not to be the case and for $a_t^n = 1$ to remain optimal, it must be that for all $w^n \in [\underline{w}^n, \overline{w}^n]$, the investors' payoff rate per unit of capital stock under our existing contract would satisfy

$$rp^n(w^n) \geq \sup_{i^n \geq 0} -c(i^n) + (i^n - \delta)p^n(w^n) + \{[\gamma - (i^n - \delta)]\, w^n - \lambda\mu\}\, p^{n\prime}(w^n). \tag{A12}$$

Note that $du_t^n = 0$ under our existing contract. Using (20), (23), and (24), we rewrite (A12) so that for all $w^n \in [\underline{w}^n, \overline{w}^n]$,

$$(r+\delta)p^n(w^n) - [(\gamma+\delta)w^n - \lambda\mu]\, p^{n\prime}(w^n) \geq \frac{[p^n(w^n) - w^n p^{n\prime}(w^n) - 1]^2}{2\theta}. \tag{A13}$$

As (23) implies that $c'(0) > 1$, it follows from (20) and $c''(i) > 0$ that $p^n(w^n) - w^n p^{n\prime}(w^n) > 1$ for all $w^n \in [\underline{w}^n, \overline{w}^n]$. In addition, $\frac{\partial[p^n(w^n) - w^n p^{n\prime}(w^n)]}{\partial w^n} > 0$ for all $w^n \in [\underline{w}^n, \overline{w}^n]$. Hence, using (17), the right-hand side of (A13) is smaller than $\frac{[p^n(\overline{w}^n) + \overline{w}^n - 1]^2}{2\theta}$. As a result, if the left-hand side of (A13) is larger than $(r+\delta)p^n(\frac{\lambda\mu}{\gamma+\delta}) - (\gamma - r)\left[p^n(\widehat{w}^n) - p^n(\frac{\lambda\mu}{\gamma+\delta})\right]$, a sufficient condition for ensuring (A12) is

$$(r+\delta)p^n\left(\frac{\lambda\mu}{\gamma+\delta}\right) - (\gamma - r)\left[p^n(\widehat{w}^n) - p^n\left(\frac{\lambda\mu}{\gamma+\delta}\right)\right] \geq \frac{[p^n(\overline{w}^n) + \overline{w}^n - 1]^2}{2\theta}.$$

The remaining problem is to show that

$$\begin{aligned} & (r+\delta)p^n(w^n) - [(\gamma+\delta)w^n - \lambda\mu]\, p^{n\prime}(w^n) \\ \geq\ & (r+\delta)p^n\left(\frac{\lambda\mu}{\gamma+\delta}\right) - (\gamma - r)\left[p^n(\widehat{w}^n) - p^n\left(\frac{\lambda\mu}{\gamma+\delta}\right)\right]. \end{aligned} \tag{A14}$$

As $p^n(w^n)$ is concave, it follows that if $\left(\frac{\lambda\mu}{\gamma+\delta} - w^n\right) p^{n\prime}(w^n) \geq 0$, then

$$\begin{aligned} p^n\left(\frac{\lambda\mu}{\gamma+\delta}\right) &\leq p^n(w^n) + \left(\frac{\lambda\mu}{\gamma+\delta} - w^n\right) p^{n\prime}(w^n) \\ &\leq p^n(w^n) + \frac{\gamma+\delta}{r+\delta}\left(\frac{\lambda\mu}{\gamma+\delta} - w^n\right) p^{n\prime}(w^n). \end{aligned} \tag{A15}$$

Because the definition of $\widehat{w}^n$ implies $p^n(\widehat{w}^n) \geq p^n\left(\frac{\lambda\mu}{\gamma+\delta}\right)$, rearranging (A15) ensures (A14). On the other hand, if $\left(\frac{\lambda\mu}{\gamma+\delta} - w^n\right) p^{n\prime}(w^n) < 0$, it also follows from the concavity of $p^n(w^n)$

that

$$\begin{aligned}
& (r+\delta)p^n\left(\frac{\lambda\mu}{\gamma+\delta}\right)-(\gamma-r)\left[p^n(\widehat{w}^n)-p^n\left(\frac{\lambda\mu}{\gamma+\delta}\right)\right] \\
\leq \;& (r+\delta)p^n\left(\frac{\lambda\mu}{\gamma+\delta}\right)-(\gamma-r)\left[p^n(w^n)-p^n\left(\frac{\lambda\mu}{\gamma+\delta}\right)\right] \\
= \;& (r+\delta)p^n\left(w^n\right)+(\gamma+\delta)\left[p^n\left(\frac{\lambda\mu}{\gamma+\delta}\right)-p^n(w^n)\right] \\
\leq \;& (r+\delta)p^n(w^n)+(\gamma+\delta)\left(\frac{\lambda\mu}{\gamma+\delta}-w^n\right)p^{n\prime}(w^n), \qquad \text{(A16)}
\end{aligned}$$

which again yields (A14). ‖

Lemma A2 provides the sufficient condition for implementing $a_t^n = 1$ all the time. ■

Proof of Proposition 6: Let div_t^n denote an increasing process that represents the cumulative dividends per unit of capital stock when the n-th manager is hired. Then, the credit line balance $K_t m_t^n$ evolves according to

$$d(K_t m_t^n) = \gamma K_t m_t^n dt + K_t x_t^n dt + K_t d\left(div_t^n\right) - K_t dA_t^n, \qquad \text{(A17)}$$

where we can assume that $d\left(div_t^n\right)$ and dA_t^n are such that $d(K_t m_t^n) \geq 0$. It follows from (29b) and $b_t^n = \frac{x_t^n}{r}$ that

$$\lambda x_t^n dt = \lambda\left[\mu - (\gamma - i_t^n + \delta)c^{Ln}\right] dt. \qquad \text{(A18)}$$

It also follows from (30) with (A17) and (A18) that

$$\begin{aligned}
dw_t^n &= -\lambda dm_t^n = -\lambda\left[\gamma m_t^n dt + x_t^n dt + d\left(div_t^n\right) - dA_t^n\right] + \lambda(i_t^n - \delta)m_t^n dt \\
&= (\gamma - i_t^n + \delta)w_t^n dt - \lambda d\left(div_t^n\right) + \lambda(dA_t^n - \mu dt). \qquad \text{(A19)}
\end{aligned}$$

Let $du_t^n = \lambda d\left(div_t^n\right)$. Given that $a_t^n = 1$ under the incentive–compatible contract ($\beta_t^n = \lambda$) and $d\left(div_t^n\right) = 0$ for $m_t^n > 0$ (that is, $w_t^n < \overline{w}^n$), it follows from Proposition 2 that the capital structure given by this proposition is optimal for the manager.

Under the capital structure proposed by this proposition and the optimal action chosen by each manager $a_t^n = 1$, the principal's expected utility equals

$$E\left[\sum_{n}{}_1^N \int_{\tau^{n-1}}^{\tau^n} e^{-rs}(\mu - du_s^n)K_t - \sum_{n}{}_1^{N-1} e^{-r\tau^n} c_f K_{\tau^n} + e^{-r\tau^N} \ell K_{\tau^N}\right],$$

where $\tau^0 = 0$, $\tau^n = \inf\{t \mid w_t^n = \underline{w}^n\}$ when $n < N$, and $\tau^N = \inf\{t \mid w_t^N = 0\}$. Note that in this case, the manager's scaled continuation utility w_t^n evolves according to (A19) (that is, equation (18) for $du_t^n = 0$), as in the optimal contract. In addition, it follows from (29c) and (30) that $m_t^n = \frac{\overline{w}^n - \underline{w}^n}{\lambda}$ and $m_t^n = 0$ imply $w_t^n = \underline{w}^n$ and $w_t^n = \overline{w}^n$, respectively. Hence, the capital structure given by this proposition is also optimal for the principal. We therefore conclude that the proposed capital structure implements the optimal contract. ■

Appendix B

In DeMarzo, Fishman, He, and Wang (2012), both the principal and the agent may achieve an ex post Pareto-improving allocation by renegotiating the contract as long as the principal's scaled value function has a positive slope. Similarly, our contract may not be renegotiation-proof either. However, in our model, the firm incurs a cost of lost productivity that is proportional to $\underline{w}^n - w^n$ for $n < N$, when an incumbent manager is replaced. Hence, unlike DeMarzo, Fishman, He, and Wang (2012), the investors' scaled value function $p_{RP}^n(w^n)$ that is renegotiation-proof need not be weakly decreasing in w^n for $n < N$. For simplicity, we assume that the investors incur the renegotiation cost under renegotiation and that the renegotiation cost is the same form as the replacement cost.

Let $\widetilde{w}_{RP}^n$ be a renegotiation boundary and $\underline{w}_{RP}^n$ a replacement boundary. We characterize the two boundaries by dividing the analysis into the two cases: $n < N$ and $n = N$.

We begin with the case of $n < N$. Then, at $w^n \leq \widetilde{w}_{RP}^n$ for $n < N$, the investors need to design a lottery or stochastic replacement in order to prevent the n-th manager's deviation: the investors increase w^n by $\widetilde{w}_{RP}^n$ if they continue to hire the n-th manager as the $n+1$-th manager or increase w^n by $\underline{w}_{RP}^n$ if they fire the n-th manager and hire a new manager from the pool of potential applicants as the $n+1$-th manager. However, because the investors under renegotiation incur the renegotiation cost that has the same form as the replacement cost, this stochastic replacement is the same as that discussed in the absence of renegotiation in the text. Hence, we must have $\widetilde{w}_{RP}^n = \underline{w}_{RP}^n$.

We next discuss the case of $n = N$. At $w^N \leq \widetilde{w}^N_{RP}$, the investors under renegotiation would have to design a lottery or stochastic liquidation to prevent the N-th manager's deviation: the investors increase w^N by $\widetilde{w}^N_{RP}$ if they continue to hire the N-th manager or set w^N on 0 if they liquidate the firm. However, because of the concavity of $p^N(w^N)$ and $p^{N\prime}_{RP}(0) = \zeta$, the investors find it unprofitable to undertake such stochastic liquidation under the renegotiation cost in the range of $w^N \geq 0$. Thus, renegotiation does not occur in the case of $n = N$.

These discussions are summarized by the following proposition.

Proposition B1: *In the sense that the HJB equation and the payout and replacement boundary conditions are identical, the main results of our model are unchanged, even when contracts are constrained to be renegotiation-proof. Furthermore, the renegotiation boundary* $\widetilde{w}^n_{RP}$ *is exactly equal to the replacement boundary* $\underline{w}^n_{RP}$ *for* $n = 1, \ldots, N-1$.

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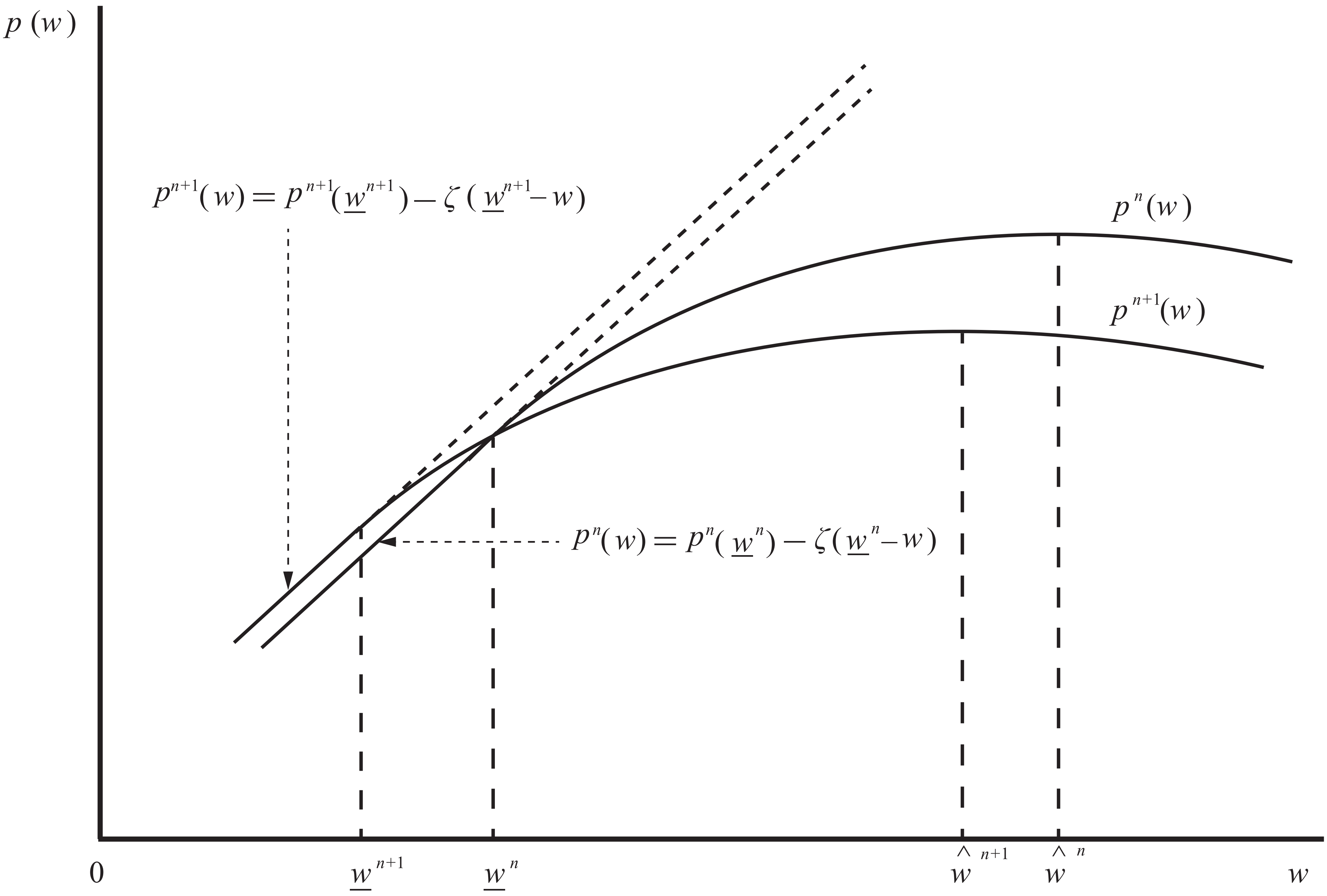

Figure 1. The investors' value function $p(w)$