KIER DISCUSSION PAPER SERIES

KYOTO INSTITUTE
OF
ECONOMIC RESEARCH

Discussion Paper No.890

"A Microeconomic Analysis toward Building Liability Law
in the Post-Earthquake Era"

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March 2014

KYOTO UNIVERSITY

KYOTO, JAPAN

A microeconomic analysis toward building liability law in the post-earthquake era*

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March, 2014

Abstract

First, we investigate which type of the liability law that protects nuclear power plants against catastrophe is socially optimal. We show that when a firm with a few funds is managed under the strict liability, maximization of social welfare is impossible since the firm may not cope with the damage by a catastrophic disaster. On the other hand, when the government sets suitable safety standards in the case where a firm has a few funds, maximization of social welfare is possible by using the negligence rule. Second, we investigate a normative analysis of liability problems that deal with how to share joint liability among agents. We axiomatize the Shapley value for this liability problem.

JEL Classification: C71; K32, Q54

Keywords: the Great East Japan Earthquake, law and economics, catastrophic risk, liability game, the Shapley value

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*This research is finacially supported by Institute of Economic Research, Kyoto University Joint Usage and Research Center.

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1 Introduction

Great East Japan Earthquake, the big earthquake of magnitude 9.0, occurred at 2:46 p.m. on March 11, 2011. The center of the quake is the offing of Sanriku, and massive tsunami struck Japan. As a result, serious damage arose mainly on the Pacific coast of Japan. Furthermore, in the Fukushima Daiichi nuclear power plant, the meltdown by a station blackout generated many radioactive materials. Even now, it is difficult to live in the partial area in Fukushima Prefecture because of the diffusion of radioactive materials. The Great East Japan Earthquake did Japan great damage that resulted from a nuclear plant disaster[1].

In order to prevent the damage caused by such a major accident, one of the most effective methods is earthquake forecasting. Recent earthquake forecasting has been progressing by research of asperity (e.g., Kikuchi, 2002). However, in the case of the Great East Japan Earthquake, it is hard to say that the forecasting could be employed efficiently. Asperity means the place from which it usually adheres strongly, shifts rapidly at a certain time, and takes out seismic waves, and it is supposed that the position and size can be presumed by analysis of seismic waves. Off a northeast, research of asperity distribution has progressed and forecasting of the earthquakes off the Coast of Miyagi Prefecture which occur every about 30 years has also been used. Thus, this approach may function to the earthquake in a cycle of tens of years, but it is quite difficult to forecast great earthquake that occurs once in all 1000 years. The reasons for this are as follows: (i) Although it is necessary to take sufficient data as a premise for looking for asperity, in an earthquake when a period of a cycle is 1000 years, the data of the past earthquake cannot be taken and there is a possibility that it may be considered that the area which is originally danger as the result is regarded as "safety." (ii) Since the possibility of linkage between different asperities in the wide area by huge asperity was ignored conventionally, the occurrence of the big earthquake by chaotic linkage of asperities could not be taken into consideration. According to asperity forecasting, near the focus of the Great East Japan Earthquake was actually made into the blank zone (Shizuoka University, 2011). In order to prevent the blank of such data, it is necessary to take various approaches, such as fault discovery by seabed measurement and the estimation of past tsunami damage caused by stratum investigation. For this purpose, large-scale research is required.

On the other hand, from the view of electric power firms, though they should carry out the preparation to an earthquake, it is actually difficult for the firms to decide how

[1] If decommissioning expense, a decontamination related cost, reparations expense, etc. are taken into consideration also by the trial calculation in the verification committees (2011), such as the National Policy Unit energy and environmental meeting cost, it will be supposed that it changes the amounts of damage by the nuclear power plant disaster of a Fukushima Daiichi nuclear power plant to 5 trillion yen or more.

much investment should be done for the big earthquake that has not been forecasted. Of course, each electric power firm has been strengthening various safety measures, such as making high tide embankment and preventive measures to a station blackout, in order to protect nuclear power plants. Furthermore, when large-scale damage occurs by such a big earthquake and massive tsunami that have not been forecasted, it is also necessary to consider the problem about who takes the liability. In the case of an unusually huge natural disaster, liability of the electric power firm is exempted[2]. In the case of the Great East Japan Earthquake the electric power firm takes the unlimited liability based on the law about compensation for nuclear damage[3]. The electric power firm is compensating the nuclear power plant disaster under this compensation scheme. However, the government has lent the electric power firm many funds. Furthermore, Nuclear Damage Liability Facilitation Fund Act was enacted on August 3, 2011, and governmental support will be offered about compensation of nuclear damage (METI, 2011b). They are the emergency plans about this nuclear power plant disaster. When future nuclear power plant re-operation are considered[4], we should reconsider whether this liability law is reasonable or not. This is the main motivation of this paper.

The purpose of this paper is to investigate which type of the liability law that protects nuclear power plants against catastrophe is socially optimal. Furthermore, we investigate a normative analysis of liability problems that deal with how to share joint liability among agents.

Our first strategy is as follows: Let us consider the situation where an electric power firm with a nuclear power plant maximizes its expected profit under the risk of a big earthquake. In this situation, we analyze the liability law that yields the socially optimal outcome. Especially, we focus on a situation in which a firm with a few funds loses the incentive to prevent an accident. This is one of judgment-proof problems (Shavell, 1986) where a compensatory burden of the firm may exceed its funds. The fundamental theoretic model about the accountability system has been developed by generalizations of the Shavell's model (e.g., Pitchford, 1995; Sakaue, 2011). An electric power firm with a nuclear power plant is an economic unit, and the government is interested in the damage to peripheral people. Furthermore, unlike the existing model, we consider two types disasters. The one type disaster is the middle-scale disaster where the probability of occurrence of the disaster is estimated to be once in tens of years. The other type disaster is the catastrophic disaster which may bring about such a serious damage that the probability of occurrence cannot be estimated easily and a firm cannot pay since it happens only once in hundreds of years. Next, we focus on the actions of an electric

[2] It is referring to JAEC (1998) about this definition.

[3] See e.g., Takemori (2011).

[4] Units 3 and 4 of Oi nuclear power plant were restarted in July 2012.

power firm, including the measure against damage by an accident, in the situation where the probability of occurrence of a catastrophic disaster can be estimated to some extent by investment in earthquake researches such as investigation. We show that when a firm with a few funds is managed under the strict liability, maximization of social welfare is impossible since the firm may not cope with the damage by a catastrophic disaster. On the other hand, when the government sets suitable safety standards in the case where a firm has a few funds, maximization of social welfare is possible by using the negligence rule.

Next, our second strategy is as follows: we consider a normative analysis of liability problems, proposed by Dehez and Ferey (2013), from the viewpoint of cooperative game theory. These problems deal with how to share joint liability among agents. For example, in the case of the nuclear plant disaster, the government and the power plants face with the problem on how to share joint liability among them. We are interested in the "Shapley value" for liability problems. The Shapley value, proposed by Shapley (1953), is the most well-known solution of cooperative games, and it has many applications to economic and political problems. We axiomatize the Shapley value for liability problems by two liability bounds properties and a consistency. Each property involved in this axiomatization is derived from the "dual" of each axiom involved in the axiomatization of the Shapley value for airport problems in Fragnelli and Mariana (2010). This "duality approach" is developed by Oishi et al. (2013), and it is useful for generating new axiomatizations automatically in both cooperative game theory and the theory of fair problems.

This paper is organized as follows. Section 2 describes a model of a firm under strict liability. Definition of a negligence rule and comparison of strict liability and a negligence rule in the model are given in section 3. Section 4 presents comparison of strict liability and a negligence rule when the government cannot estimate probability of a catastrophic disaster. Section 5 establishes an axiomatization of the Shapley value for liability problems.

2 Theoretical analysis of strict liability

We consider an economy consisting of an electric power firm that manages one nuclear power plant, and the government that regulates to the firm for maximizing social welfare. The social welfare includes benefits and damages of residents. We assume that a firm and the government are risk-neutral. We define in order the profits of the firm which manages nuclear power plant and a social welfare function.

First, we define benefit and cost functions. B expresses the net benefit which a firm obtains by an economic activity, when there is no disaster. We assume $B > 0$. Let c be the amounts of investments for reducing damage by a middle-scale disaster. Similarly,

r expresses the amounts of investments for reducing damage by a catastrophic disaster. $D(c)$ is defined as the amounts of social damage when a middle-scale disaster occurs. That is, not only the damage to a nuclear power plant but also damage to the neighborhood, such as personal suffering, by an accident are included in $D(c)$. We assume $D(c) > 0$. The amounts of damage by a middle-scale disaster decrease as investments for reducing damage by a middle-scale disaster increase ($D'(c) < 0$), the margin of decline of marginal damage decreases gradually ($D''(c) > 0$), and the effect of investment is infinitely large when a firm does not make an investment at all ($\lim_{x \to 0} D'(x) \to -\infty$). Let i be the amounts of investments to the investigation for estimating the probability that a catastrophic disaster will occur.

$H(c, r)$ expresses the amounts of social damage when a catastrophic disaster occurs. We assume $H(c, r) > 0$. The amounts of damage by a catastrophic disaster also decrease as investments for reducing damage by a middle-scale disaster increase ($H_c(c, r) < 0$), and the margin of decline of the marginal damage decreases gradually ($H_{cc}(c, r) > 0$). In addition, by investment for reducing damage by a catastrophic disaster, the amounts of the damage decreases ($H_r(c, r) < 0$), and it decreases gradually the margin of decline of the marginal amounts of damage ($H_{rr}(c, r) > 0$). Finally, let K be funds of a firm, i.e., the maximum solvency to the damage when an accident occurs.

Next, probabilities of occurrence of disasters are defined. Let $p > 0$ be the probability that the middle-scale disaster will occur. We assume that it can be estimated. Similarly, $q(i)$ is the probability of occurrence of a catastrophic disaster which the firm estimates. We assume that a middle-scale disaster and a catastrophic disaster do not occur simultaneously ($1 - p \geq q(i) \geq 0$). Furthermore, the probability of occurrence of a catastrophic disaster is going up ($q'(i) > 0$) when investigation is conducted. We suppose that the effect decreases gradually ($q''(i) < 0$) and converges on a certain given probability $\bar{q}$ ($\lim_{i \to \infty} q'(i) = \bar{q}$)[5]. $\bar{q}$ is defined as the true probability that a catastrophic disaster will occur. Finally $\tilde{q}$ is made into the provisional estimated probability of occurrence of a catastrophic disaster which the government considers. It is natural to assume that $\tilde{q} \geq q(i)$. That is, the government sets the same probability of occurrence of a catastrophic disaster which the firm estimates, or sets the higher probability $\tilde{q}$.

Next, we define profits of a firm and social welfare based on this setup for our analysis.

[5] We consider investigation here rather as the new investigation in the area which have not been investigated or a catastrophic disaster risk is very low conventionally than as that in already investigated area where the investigation about a catastrophic disaster is sufficient and a catastrophic disaster risk is regarded as high. Since uninvestigated areas, which were presupposed that there was no catastrophic disaster risk conventionally, decrease in number as the investment in investigation increases at this time, it can become clear that that is an area where a catastrophic disaster risk is high by investigation. The probability of occurrence of a catastrophic disaster goes up as this result.

The government maximizes a social welfare function based on given estimated probability,

$$\tilde{W}(\tilde{q}) \equiv B - c - i - r - pD(c) - \tilde{q}H(c, r). \quad (1)$$

Since it is clear that $i = 0$, first order condition is that

$$\frac{\partial \tilde{W}}{\partial c} = -1 - pD'(c) - \tilde{q}H_c(c, r) \le 0, \frac{\partial \pi^S}{\partial c} c = 0 \quad (2)$$

$$\frac{\partial \tilde{W}}{\partial r} = -1 - \tilde{q}H_r(c, r) \le 0, \frac{\partial \pi^S}{\partial r} r = 0. \quad (3)$$

The solution for the maximization is denoted by $(\tilde{c}, \tilde{r})$. If $\tilde{q} = \bar{q}$, it is socially optimal. If not, we have the loss of social welfare, given by $\max_{c,i,r} \tilde{W}(\bar{q}) - \tilde{W}(\bar{q})|_{(c,i,r)=(\tilde{c},0,\tilde{r})}$.

Next, we define an expected profit function of a firm. First, we define strict liability. Under strict liability, when a disaster occurs a firm owes a duty to pay compensation at the fixed rate of damage by disaster in spite of the level of its investments for the disaster. In this case, a firm obtains the following expected profits:

$$\pi^S \equiv B - c - i - r - p\alpha_1 D(c) - q(i) \min\{\alpha_2 H(c, r), K\} - [\tilde{q} - q(i)] \min\{\alpha_3 H(c, r), K\} \quad (4)$$

where α_1, α_2, and α_3 are given parameters in nonnegative, which are the ratios of compensation of a firm to damage by a disaster in each case. $\alpha_1 D(c)$ expresses the amounts of compensation of a firm when the middle-scale disaster occurs. $\alpha_2 H(c, r)$ expresses the amounts of compensation of a firm when a catastrophic disaster occurs within the limits of estimated probability. Finally, $\alpha_3 H(c, r)$ expresses the amounts of compensation of a firm when an unexpected catastrophic disaster occurs[6]. We adopt this formularization since it is very likely that the amounts of compensation a firm exceeding its funds K in the case of a catastrophic disaster. We will consider profit maximization problems of a firm in several cases.

2.1 When there are a few funds under strict liability

(S-1) Consider a case where $\alpha_2 H(c, r) \ge K$ and $\alpha_3 H(c, r) \ge K$ about arbitrary investment c and r. In this situation, the amounts of compensation for damage by a catastrophic disaster always exceed its funds since it is impossible to compensate damage even if the firm performs any measures to abate damage. In this case, the first order condition for maximization is that

$$\frac{\partial \pi^S}{\partial c} = -1 - p\alpha_1 D'(c) \le 0, \frac{\partial \pi^S}{\partial c} c = 0 \quad (5)$$

$$\frac{\partial \pi^S}{\partial i} = -1 \le 0, \frac{\partial \pi^S}{\partial i} i = 0 \quad (6)$$

$$\frac{\partial \pi^S}{\partial r} = -1 \le 0, \frac{\partial \pi^S}{\partial r} r = 0. \quad (7)$$

[6] "The unexpected catastrophic disaster" is a catastrophic disaster that occurs in the area where it is considered by investigation that there has been no risk of occurrence of a catastrophic disaster.

By arrangement, we obtain a solution $c_1^S = D'^{-1}(1/(p\alpha_1))$ and $i_1^S = r_1^S = 0$. That is, a firm does not invest at all in the case of a catastrophic disaster, but it make investments for abating damage by a middle-scale disaster. This is because the firm loses an incentive for reducing the damage by a catastrophic disaster when the firm is obligated to pay enormous amounts of compensation exceeding its funds. This is called a judgment-proof problem.

Proposition 1. *Since a firm does not invest at all to a catastrophic disaster when there are always more amounts of compensation for damage by a catastrophic disaster than funds under strict liability, maximization of the social welfare based on given estimated probability cannot be attained.*

2.2 When there are many funds under strict liability

(S-2) Next, consider a case where $\alpha_2 H(c,r) < K$ and $\alpha_3 H(c,r) < K$ about arbitrary investment c and r. This expresses the case where a firm can pay compensation for all damages by a catastrophic disaster. The first order condition in this situation is that

$$\frac{\partial \pi^S}{\partial c} = -1 - p\alpha_1 D'(c) - q(i)\alpha_2 H_c(c,r) - [\tilde{q} - q(i)]\alpha_3 H_c(c,r) \leq 0, \frac{\partial \pi^S}{\partial c} c = 0 \quad (8)$$

$$\frac{\partial \pi^S}{\partial i} = -1 - q'(i)\alpha_2 H(c,r) + q'(i)\alpha_3 H(c,r) \leq 0, \frac{\partial \pi^S}{\partial i} i = 0 \quad (9)$$

$$\frac{\partial \pi^S}{\partial r} = -1 - q(i)\alpha_2 H_r(c,r) - [\tilde{q} - q(i)]\alpha_3 H_r(c,r) \leq 0, \frac{\partial \pi^S}{\partial r} r = 0. \quad (10)$$

The interior solution is denoted by c_2^S, i_2^S, and r_2^S. When $\alpha_2 = \alpha_3$ (i.e., when a catastrophic disaster occurs, whether a catastrophic disaster is unexpected does not affect the amounts of compensation of a firm at all), all the portions relevant to $q(i)$ are offset. Especially, if $\alpha_1 = \alpha_2 = \alpha_3 = 1$, a solution will become the same as solution of the maximum problem of social welfare $\tilde{c}$ and $\tilde{r}$, where $i = 0$. That is, if the amounts of compensation for damage by a catastrophic disaster are less than those of funds of the firm, a firm has an incentive to make an investment under strict liability.

In addition, $i_2^S > 0$ is only a case where $\alpha_3 > \alpha_2$, that is, when firm pays more amounts of compensation for unexpected damage by a catastrophic disaster.

Proposition 2. *When the amounts of compensation for damage by a catastrophic disaster are less than funds under strict liability, maximization of the social welfare based on given estimated probability is attained.*

2.3 General cases under strict liability

Next, we consider a case of the medium level of funds. If c and r are small, it may specifically become $\alpha_2 H(0,0) > K$ and $\alpha_3 H(0,0) > K$, but if c and r are large, it is a case where it can become $\alpha_2 H(c,r) < K$ and $\alpha_3 H(c,r) < K$. This is classified into four cases.

(S-3) (i) When $\alpha_2 H(c_3^S, r_3^S) \geq K > \alpha_3 H(c_3^S, r_3^S)$ in a solution. Since $\alpha_2 > \alpha_3$, we solve the following Lagrangian function

$$L \equiv B - c - i - r - p\alpha_1 D(c) - q(i)K - [\tilde{q} - q(i)]\alpha_3 H(c,r) - \lambda[K - \alpha_2 H(c,r)] \tag{11}$$

and we obtain the following first-order conditions:

$$\frac{\partial L}{\partial c} = -1 - p\alpha_1 D'(c) - [\tilde{q} - q(i)]\alpha_3 H_c(c,r) + \lambda\alpha_2 H_c(c,r) \leq 0, \frac{\partial L}{\partial c}c = 0 \tag{12}$$

$$\frac{\partial L}{\partial i} = -1 - q'(i)K + q'(i)\alpha_3 H(c,r) \leq 0, \frac{\partial L}{\partial i}i = 0 \tag{13}$$

$$\frac{\partial L}{\partial r} = -1 - [\tilde{q} - q(i)]\alpha_3 H_r(c,r) + \lambda\alpha_2 H_r(c,r) \leq 0, \frac{\partial L}{\partial r}r = 0. \tag{14}$$

Since $\alpha_2 H(c_3^S, r_3^S) = K > \alpha_3 H(c_3^S, r_3^S)$ and there is no incentive to investigate, we obtain $i_3^S = 0$.

(ii) When $\alpha_3 H(c_3^S, r_3^S) \geq K > \alpha_2 H(c_3^S, r_3^S)$ in a solution. Since $\alpha_3 > \alpha_2$, we solve the following Lagrangian function

$$L \equiv B - c - i - r - p\alpha_1 D(c) - q(i)\alpha_2 H(c,r) - [\tilde{q} - q(i)]K - \lambda[K - \alpha_3 H(c,r)] \tag{15}$$

and we obtain the following first-order conditions:

$$\frac{\partial L}{\partial c} = -1 - p\alpha_1 D'(c) - q(i)\alpha_2 H_c(c,r) + \lambda\alpha_3 H_c(c,r) \leq 0, \frac{\partial L}{\partial c}c = 0 \tag{16}$$

$$\frac{\partial L}{\partial i} = -1 - q'(i)\alpha_2 H(c,r) + q'(i)K \leq 0, \frac{\partial L}{\partial i}i = 0 \tag{17}$$

$$\frac{\partial L}{\partial r} = -1 - q(i)\alpha_2 H_c(c,r) + \lambda\alpha_3 H_r(c,r) \leq 0, \frac{\partial L}{\partial r}r = 0. \tag{18}$$

By these conditions, $\alpha_3 H(c_3^S, r_3^S) = K > \alpha_2 H(c_3^S, r_3^S)$. We obtain $i_3^S > 0$ since the firm has an incentive to investigate.

(iii) When $\min\{\alpha_2, \alpha_3\}H(c_3^S, r_3^S) \geq K$ in a solution. The solution is the same as (S-1).

(iv) When $\max\{\alpha_2, \alpha_3\}H(c_3^S, r_3^S) < K$ in a solution. The solution is the same as (S-2).

In addition, combination with the highest profits serves as a solution of a profit maximization problem among these four cases. The following proposition is realized from the above results.

Proposition 3. *When the amounts of compensation for unexpected damage with underestimated probability are smaller than those for expected damage, a firm does not investigate at all under strict liability.*

3 Theoretical analysis of a negligence rule

By (S-1), when there are a few funds of a firm, maximization of social welfare cannot be attained under strict liability. In stead, we try to maximize social welfare by using the negligence rule.

3.1 In the case that a firm is exempted from liability when the investment to both of disasters meets the fixed standard

First, when investments of a firm to both type of disasters exceed safety-standards $\bar{c}$ and $\bar{r}$ defined by the government, even if a disaster actually occurs, a firm is exempted from liability. In this case, profits are as follows:

$$\pi^N \equiv \begin{cases} B - c - i - r & \text{if } c \geq \bar{c} \ \& \ r \geq \bar{r} \\ B - c - i - r - p\min\{\alpha_1 D(c), K\} - q(i)\min\{\alpha_2 H(c,r), K\} & \text{otherwise} \\ \qquad -[\tilde{q} - q(i)]\min\{\alpha_3 H(c,r), K\} & \end{cases}$$

3.2 When there are a few funds under a negligence rule

(N-1) First, we consider a maximum problem paying attention to a few funds cases where $\alpha_2 H(c,r) \geq K$ and $\alpha_3 H(c,r) \geq K$. Either of the following two investments is chosen by profit maximization.

(i) When $c_1^N = \bar{c}$, $r_1^N = \bar{r}$, and $i_1^N = 0$. That is, sufficient investment is performed to both disasters.

(ii) When $c_1^N = c_1^S$, $r_1^N = 0$, and $i_1^N = 0$. That is, a firm invests only to a middle-scale disaster similarly to (S-1). Since it will certainly go bankrupt if a catastrophic disaster actually occurs even if it takes the measures against a catastrophic disaster, the firm does not invest at all.

In this case, if the government sets with $\bar{c} = \tilde{c}$, $\bar{r} = \tilde{r}$ and $\alpha_1 = \alpha_2 = \alpha_3 = 1$ and if the firm chooses (i), social welfare $W(\tilde{q})$ will become the maximum, but (i) is not necessarily chosen. Choice of the firm depends on a relationship of the size of the following profits:

$$\pi^N \equiv \begin{cases} B - \tilde{c} - \tilde{r} & \text{if } c \geq \bar{c} \ \& \ r \geq \bar{r} \\ B - c_1^S - p\alpha_1 D(c_1^S) - \tilde{q}K & \text{otherwise} \end{cases} \tag{19}$$

Here, α_1 is large enough, when filling $\alpha_1 > (\tilde{c} - c_1^S + \tilde{r} - \tilde{q}K)/(pD(c_1^S))$, (i) is certainly chosen and maximization of social welfare is attained under a negligence rule.

Proposition 4. *When the amounts of compensation for damage by a catastrophic disaster are larger than funds under a negligence rule, if safety standards are set up with the suitable level by enlarging the ratio of compensation to damage by a middle-scale disaster enough, maximization of the social welfare based on given estimated probability will be attained.*

This condition looks natural since maximization of social welfare can be attained more easily when there are less amounts of an additional investment for meeting safety standards and when the expected value of compensation of a firm for damage by a catastrophic disaster is larger.

3.3 When there are many funds under a negligence rule

(N-2) Next, consider a maximum problem paying attention to the case $\alpha_2 H(c,r) < K$ and $\alpha_3 H(c,r) < K$. Either of the following patterns is chosen by profit maximization.

(i) When $c_2^N = \bar{c}$, $r_2^N = \bar{r}$, and $i_2^N = 0$. That is, sufficient investment is performed to both disasters.

(ii) When $c_2^N = c_2^S$, $r_2^N = r_2^S$, and $i_2^N = 0$. That is, it is the same as (S-2), and performs a certain amounts of investments to both disasters.

In this case, from (S-2), if the government sets with $\bar{c} = \tilde{c}$, $\bar{r} = \tilde{r}$ and $\alpha_1 = \alpha_2 = \alpha_3 = 1$, a firm will make an investment at the optimal level so that $W(\tilde{q})$ is maximized under a negligence rule.

Proposition 5. *When the amounts of compensation for damage by a catastrophic disaster are less than funds under a negligence rule, if safety standards are set up with the suitable level, maximization of the social welfare based on given estimated probability will be attained.*

3.4 When the investment to a middle-scale disaster meets the fixed standard and it is exempted from liability

We have considered both types of investments c and r as safety standards under a negligence rule until now. In this subsection, we consider only c as another safety standard.

(N-3) In this case, profits can be formalized as follows:

$$\pi^N \equiv \begin{cases} B - c - i - r & \text{if } c \geq \bar{c} \\ B - c - i - r - p\min\{\alpha_1 D(c), K\} - q(i)\min\{\alpha_2 H(c,r), K\} & \text{otherwise} \\ \qquad -[\tilde{q} - q(i)]\min\{\alpha_3 H(c,r), K\} & \end{cases} \tag{20}$$

When there are a few funds of a firm, we obtain $r_3^N = 0$ from (S-1). Therefore, maximization of social welfare cannot be attained in this case.

4 When the government cannot estimate probability of a catastrophic disaster

We have assumed that the government can estimate probability that a catastrophic disaster occurs in the given cases. That is, it was premised on the ability of the government to expect prior probability in a certain form. However, it is difficult to know the probability $\tilde{q}$ in reality. In this situation, the government sometimes adopts a probability estimated by a firm. The case where the estimated probability by the government becomes equal to probability $q(i)$ based on investigation of a firm instead of $\tilde{q}$. Note that a case of $\tilde{q} = q(i)$ is similar to a case of $\alpha_3 = 0$ considering the profits of a firm. We consider liability under these restrictions.

First, in $\tilde{q} = q(i)$, social welfare is given by

$$\tilde{W}(q(i)) \equiv B - c - i - r - pD(c) - q(i)H(c,r) \tag{21}$$

By maximizing this, we have the following first-order conditions

$$\frac{\partial \tilde{W}}{\partial c} = -1 - pD'(c) - q(i)H_c(c,r) \leq 0, \frac{\partial \pi^S}{\partial c}c = 0 \tag{22}$$

$$\frac{\partial \tilde{W}}{\partial i} = -1 - q'(i)H(c,r) \leq 0, \frac{\partial \pi^S}{\partial i}i = 0 \tag{23}$$

$$\frac{\partial \tilde{W}}{\partial r} = -1 - q(i)H_r(c,r) \leq 0, \frac{\partial \pi^S}{\partial r}r = 0. \tag{24}$$

Note that $i = 0$. This is because the expected amounts of damage increase as probability goes up. That is, the government has no incentive to investigate.

4.1 In the case of strict liability

4.1.1 When there are a few funds

(OS-1) When there are a few funds of a firm, maximization of social welfare cannot be attained from (S-1).

Proposition 6. *When there are more amounts of compensation for damage by a catastrophic disaster under strict liability than funds, maximization of the social welfare based on the estimated probability of a firm is not attained.*

4.1.2 When there are many funds

(OS-2) When there are many funds of a firm, it is a case $\alpha_3 = 0$ by (S-2). If $\alpha_1 = \alpha_2 = 1$, maximization of social welfare can be attained.

Proposition 7. *When the amounts of compensation for damage by a catastrophic disaster are less than funds under strict liability, maximization of the social welfare based on the estimated probability of a firm is attained.*

4.2 In the case of a negligence rule

4.2.1 When there are a few funds

(ON-1) When there are a few funds of a firm, maximization of social welfare can be attained from (N-1).

Proposition 8. *When there are more amounts of compensation for damage by a catastrophic disaster than funds under a negligence rule, if safety standards are set up with the suitable level by enlarging the ratio of compensation to damage by a middle-scale disaster enough, maximization of the social welfare based on the estimated probability of a firm will be attained.*

4.2.2 When there are many funds

(ON-2) When there are many funds of a firm, if $\alpha_1 = \alpha_2 = 1$ from (S-2) and (N-2), maximization of social welfare can be attained.

Proposition 9. *When the amounts of compensation for damage by a catastrophic disaster are less than funds under strict liability, maximization of the social welfare based on the estimated probability of a firm is attained.*

Note that social welfare here was not necessarily based on true probability in this case since the probability is provisional. Furthermore, the loss of social welfare is always positive since the investigation to a catastrophic disaster is also insufficient.

5 Axiomatization of the Shapley value for liability problems

In this section, we consider a normative analysis of liability problems that deal with how to share joint liability among agents. This analysis is potentially applicable to a liability problem on how to share damages among the government, and the power companies if the nuclear plant disaster has caused jointly by them.

There is a universe of "potential" agents, denoted by $\mathcal{I} \subseteq \mathbb{N}$, where $\mathbb{N}$ is the set of natural numbers. Let $\mathcal{N}$ be the class of non-empty and finite subsets of $\mathcal{I}$. A "liability problem" is a pair (N, d), where $N \subseteq \mathcal{N}$ is a finite non-empty set of agents and $d = (d_i)_{i \in N}$ is the profile of additional damage parameters of the agents satisfying $d_i > 0$ for each $i \in N$. Let $\mathcal{D}$ be the class of all liability problems on $\mathcal{N}$.

An allocation rule for liability problems is denoted by $\varphi^L : \mathcal{D} \to \mathbb{R}^n_+$ such that $\sum_N \varphi^L_i(N, d) = \sum_N d_i$. It associates with each problem an n-dimensional payoff vector. Fix an arbitrary $(N, d) \in \mathcal{D}$. We derive a liability game, proposed by Dehez and Ferey (2013), from a liability problem, that is,

$$v^L(S) = \begin{cases} \sum_{k=1}^{n} d_k & \text{if} \quad S = N \\ \sum_{k=1}^{(\min N\setminus S)-1} d_k & \text{if} \quad 1 \in S \text{ and } S \neq N \\ 0 & \text{otherwise.} \end{cases}$$

For possible interpretations of this game, see Dehez and Ferey (2013). The amount of $v^L(S)$ is the cumulative damages that the sequential agents starting from agent 1 have caused. In other words, agent 1 is the most-upstream tortfeasor, and agent n is the most-downstream tortfeasor; and sharing cumulative damages are caused jointly by the sequential tortfeasors.

We consider the most well-known single-valued solution for coalitional games, namely the Shapley value (Shapley, 1953). The Shapley value has many applications to economic and political problems. An n-dimensional vector $x \in \mathbb{R}^n$ of a liability game is a "payoff vector" if it satisfies that $\sum_N x_i = v^L(N)$. Let $Sh(v^L)$ be the "Shapley value" of the liability game. The Shapley value is the payoff vector given by the following formula:

$$Sh_i(v^L) = \sum_{S \subseteq N,\ i \notin S} \frac{|S|!(n-|S|-1)!}{n!} \left(v^L(S \cup \{i\}) - v^L(S)\right) \quad \text{for each } i \in N.$$

We refer to $\varphi^{*L}(\cdot)$ as the Shapley value for the liability problem if $\varphi^{*L}(N,d) = Sh(v^L)$ for each $(N,d) \in \mathcal{D}$.

Our goal in this section is to axiomatize the Shapley value for the liability problem. For this purpose, we can utilize the "duality approach," proposed by Oishi et al. (2009), by considering an axiomatization of the Shapley value for airport problems (Fragnelli and Marina, 2010).

Airport problems are cost sharing problem of an airstrip among the airlines, which are well known in the game theoretic literature, e.g., see Thomson (2007). Let c_i be a cost parameter of agent (airline) i. We denote by $c = (c_i)_{i \in N}$ the profile of cost parameters of agents satisfying $c_i < c_j$ if $i > j$ and $i, j \in N$, and $c_h > 0$ for each $h \in N$. An "airport problem" is a pair (N, c). Let $\mathcal{C}$ be the class of all airport problems on $\mathcal{N}$. An allocation rule for airport problems is denoted by $\varphi^A : \mathcal{C} \to \mathbb{R}^n_+$ such that $\sum_N \varphi^A_i(N,c) = c_1$. It associates with each problem an n-dimensional payoff vector.

We derive an airport game, proposed by Littlechild and Owen (1973), from an airport problem, that is,

$$v^A(S) = \max_{i \in S} c_i \quad \text{for each } S \subseteq N.$$

For possible interpretations of this game, see Littlechild and Owen (1973). Let $Sh(v^A)$ be the Shapley value of the airport game. The formalization of $Sh(v^A)$ is in the same manner as the formalization of $Sh(v^L)$. We refer to $\varphi^{*A}(\cdot)$ as the Shapley value for the airport problem if $\varphi^{*A}(c) = Sh(v^A)$ for each $(N,c) \in \mathcal{C}$.

Next, we briefly explain the notion of duality. Using the notion of anti-duality, we uncover the hidden relationship between the Shapley value of the liability game and the Shapley value of the airport game. Let V be a generic notation of a coalitional game for N, that is, $V : 2^N \to \mathbb{R}$ with $V(\emptyset) = 0$. Given a coalitional game V for N, let us denote the "dual" of V by V^*. For each $S \subseteq N$, the dual V^* is defined by $V^*(S) := V(N) - V(N \backslash S)$. Dehez and Ferey (2013) showed that the liability game is the dual of the airport game. It is well known that the Shapley value is the "self-dual" solution in the sense that $Sh(V) = Sh(V^*)$. The following proposition summarizes these results.

Proposition 10. *For each $(N, c) \in \mathcal{C}$ and each $(N, d) \in \mathcal{D}$, the following assertion holds:*

1: The liability game v^L is the dual of the airport game v^A, and vice versa.
2: $Sh(v^L) = Sh(v^A)$.

Next, we apply the notion of duality to axiomatization of the Shapley value for allocation problems mentioned above. For each $(N, c) \in \mathcal{C}$, there exists a unique liability problem $(N, d) \in \mathcal{D}$ such that for each $i \in N$ $d_i = c_i - c_{i+1}$, where $c_{n+1} \equiv 0$. Conversely, for each $(N, d) \in \mathcal{D}$, there exists a unique airport problem $(N, c) \in \mathcal{C}$ such that for each $i \in N$ $c_i = \sum_{k=i}^{n} d_k$. Thus, the set $\mathcal{C}$ is "equivalent" to the set $\mathcal{D}$.

Let us call that the solution φ^{*L} is the "dual" of the solution φ^{*A} if whenever the set $\mathcal{C}$ is equivalent to the set $\mathcal{D}$,

$$\varphi^{*L}(N, d) = Sh(v^L) = Sh(v^A) = \varphi^{*A}(N, c).$$

"Two axioms are dual to each other" if whenever the set $\mathcal{C}$ is equivalent to the set $\mathcal{D}$ and the solution φ^{*A} satisfies one of them, the dual of φ^{*A} (i.e. the solution φ^{*L}) satisfies the other.

The duality operator, applied to the allocation rules, is useful for generating new axiomatizations automatically. Just by identifying the dual of each axiom involved in the axiomatization of solution φ^{*A}, we obtain an axiomatization of solution φ^{*L}. The resulting axiomatization of solution φ^{*L} might be found in the literature or not. In the former case, the duality approach provides us with a viewpoint for linking existing axiomatizations of solutions. In the latter case, the duality approach gives us a new axiomatization. In this section, we will find the latter case.

For an axiomatization of φ^{*A}, Fragnelli and Marina (2010) identified the following properties:

- **Individual equal sharing**: For each $(N, c) \in \mathcal{C}$,

  $$\varphi^A_i(N, c) \geq \frac{c_i}{n} \quad \text{for each } i \in N.$$

- **Collective usage right**: For each $(N, c) \in \mathcal{C}$,

  $$\varphi_i^A(N, c) \leq \frac{c_i}{i} \text{ for each } i \in N.$$

- **Last-airline consistency**: For each $(N, c) \in \mathcal{C}$,

  $$\varphi_i^A(N, c) = \varphi_i^A(N', c') \text{ for each } i \in N',$$

  where $N' = N \backslash \{n\}$, $c_i' = c_i - \varphi_n^A(N, c)$ for all $i \in N'$, and $(N', c') \in \mathcal{C}$.

For possible interpretations for these properties, see Fragnelli and Marina (2010). By considering that $\varphi^{*L}(N, d) = \varphi^{*A}(N, c)$ and for each $i \in N$ $c_i = \sum_{k=i}^{n} d_k$, we can identify the dual of each axiom involved in the axiomatization of solution φ^{*A}.

- **Individual equal liability lower bounds**: For each $(N, d) \in \mathcal{D}$,

  $$\varphi_i^L(N, d) \geq \frac{d_i + d_{i+1} + \cdots + d_n}{n} \text{ for each } i \in N.$$

  We refer to $d_i + d_{i+1} + \cdots + d_n$ as "individual marginal damages". This is because agent i has a responsibility for potential damages $d_i + d_{i+1} + \cdots + d_n$ in the sequential process. This property requires that each agent i should share at least as much as an equal division of individual marginal damages between the agents in N. This is the dual of the "individual equal sharing" axiom.

- **Collective equal liability upper bounds**: For each $(N, d) \in \mathcal{D}$,

  $$\varphi_i^L(N, d) \leq \frac{d_i + d_{i+1} + \cdots + d_n}{i} \text{ for each } i \in N.$$

  A group of agents $1, 2, \cdots, i$, denoted by U_i, has a responsibility for potential damages $d_{i+1} + d_{i+2} + \cdots + d_n$ in the sequential process. We refer to $d_{i+1} + d_{i+2} + \cdots + d_n$ as "collective marginal damages". This property requires that each agent i should share at most as much as the sum of an equal division of additional damages d_i between the members of U_i and an equal division of collective marginal damages between the members of U_i, namely $\frac{d_i}{i} + \frac{d_{i+1} + d_{i+2} + \cdots + d_n}{i} (= \frac{d_i + d_{i+1} + \cdots + d_n}{i})$. This property is the dual of the "collective usage right" axiom.

- **Last-tortfeasor consistency**: For each $(N, d) \in \mathcal{D}$,

  $$\varphi_i^L(N, d) = \varphi_i^L(N', d') \text{ for each } i \in N',$$

  where $N' = N \backslash \{n\}$ and $d' = (d_1, d_2, \cdots, d_{n-2}, d_{n-1} + d_n - \varphi_n^L(N, d)) \in \mathcal{D}$.

Imagine the last agent n shares the damage $\varphi_n^L(N,d)$ and it leaves N. Agent n's responsibility for sharing the damages $d_n - \varphi_n^L(N,d)$ is transferred to agent $n-1$. As a result, the modified additional damage of agent $n-1$ is given by $d_{n-1} + d_n - \varphi_n^L(N,d)$. This property requires that the outcome that the allocation rule chooses for each liability problem is invariant under the departure of the last agent. This is the dual of the "last-airline consistency" axiom.

Thus, we obtain the following axiomatization of solution φ^{*L} automatically:

Proposition 11. *The Shapley rule for liability problems is the only rule satisfying individual equal liability lower bounds, collective equal liability upper bounds, and last-tortfeasor consistency.*

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