

# KIER DISCUSSION PAPER SERIES

## KYOTO INSTITUTE OF ECONOMIC RESEARCH

Discussion Paper No.860

“Competition and the Bad News Principle  
in a Real Options Framework”

Katsumasa Nishide and Kyoko Yagi

April 2013



KYOTO UNIVERSITY

KYOTO, JAPAN

# Competition and the Bad News Principle in a Real Options Framework\*

Katsumasa Nishide<sup>†</sup> and Kyoko Yagi<sup>‡</sup>

(April 16, 2013)

**Abstract.** We study the investment timing problem where two firms that compete for investment preemption know in advance the time at which the economic condition changes. We show that the so-called Bad News Principle applies to the leader firm's investment decision near maturity in many cases. This result indicates that the option value to wait does have an impact even in a competitive situation, which is in contrast to the previous literature.

**Keywords:** Bad news principle, investment timing, competition, real options.

**JEL classification:** D92, G32, D43

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\*Numerical assistance by Dr. Hajime Fujiwara is greatly appreciated. The authors thank the participants at JAROS 2012 Conference, especially Makoto Goto and Motoh Tsujimura for their comments. The first author wishes to thank the financial support by the Japanese Ministry of Education, Culture, Sports, Science and Technology (MEXT) Grand in Aid for Scientific Research (B) #23310098 and (C) #23530362. This research is also supported in part by the 2012 Project Research at KIER as the Joint Usage and Research Center.

Any errors found are, of course, of the authors.

<sup>†</sup>Corresponding author. Department of Economics, Yokohama National University. 79-4 Tokiwadai, Hodogaya-ku, Yokohama 240-8501, Japan. E-mail: knishide@ynu.ac.jp.

<sup>‡</sup>Department of Management Science and Engineering, Akita Prefectural University. 84-4 Ebinokuchi, Tsuchiya, Yurihonjo-shi, Akita 015-0055, Japan. E-mail: yagi@akita-pu.ac.jp.

# 1 Introduction

The real options theory studies how uncertainty and irreversibility affect the timing of investment. For example, McDonald and Siegel (1986) show that the investment threshold is higher than the one of the NPV criteria when the option value of waiting is considered.

Recall that many papers in the real options literature consider the investment timing for a monopolistic firm and how multiple firms under competition interact for investment is examined just recently. Nielsen (2002) considers the case where two identical firms compete against each other to get preemptive cash flows as a leader. He shows that the threshold of a leader firm is strictly lower than that of a follower in equilibrium due to the strategic interaction. Grenadier (2002) studies an incremental investment problem and derives the equilibrium investment strategies of firms in a Cournot-Nash framework where the price of output depends on the total industry supply. He shows that the option value to wait converges to zero at a rapid rate as the number of firms goes to infinity. Among other papers studying competition in the real options framework are Joaquin and Khanna (2001), Weeds (2002), Pawlina and Kort (2006) and Bouis et al. (2009).

In this paper we consider the investment timing of two identical firms in a situation where the time of a regime change is known in advance. Our model can be applied to an actual case where the time of a regime change can be estimated. One typical example is that economic policies are often replaced when a new president is elected and the election date is known to everyone. A firm whose profit is affected by such a regime shift should jointly consider the present regime, the structure of future possible regimes with their probabilities, and the remaining time to the regime shift. To examine such cases, it is assumed that the regime change can take place only at a pre-specified deterministic time.

Nishide and Nomi (2009) examine the case with a monopolistic firm and this paper is regarded as an extension of their model by introducing competition. Recall that studying simultaneously both competition and regime switching is not new in the literature. For example, Goto et al. (2012) study a similar problem in which parameters describing the dynamics of the state variable are modulated by a Markov chain. In their setting the problem becomes time-homogeneous and so the value function is characterized by a simultaneous ordinary differential equation system. On the other hand, each firm needs to take the remaining time to the regime shift into account in our setting and the value function is characterized by a partial differential equation with a time derivative. This paper clarifies how not only competition but also the time to a regime shift affect the optimal investment timing for a leader and a follower.

One of the important results in Nishide and Nomi (2009) is that the investment threshold of a monopolistic firm converges to the highest one for the future possible regimes as the time approaches to the regime change. This means that the firm behaves as if the worst case scenario was about to occur just before the time of the regime change. This finding echoes the so-called *Bad News Principle* proposed by Bernanke (1983). He states in page 91 of the paper that

Given the current return, the willingness to invest in the current period depends only on the severity of bad news that may arrive. Just how good is the potential future good news for the investment does not matter at all.

Anyone might agree that the principle is one of the most important proposition in the modern investment theory. Our question in this paper is “Does the Bad News Principle hold for firms facing investment competition?”.

This paper gives an answer to the question as follows. The principle always holds for a follower firm, while it partly holds for a leader firm. As we shall see later, the problem of the follower’s investment timing is essentially the same as the one of monopolistic firm, and so the follower firm naturally conforms to the principle near the time of a regime change. For the leader’s case, the firm easily invests in the project only when a good scenario is significantly likely and the magnitude of investment preemption is large enough. Otherwise, the leader behaves in accordance with the principle.

As Nishide and Nomi (2009) mentioned, a follower firm should optimally wait for the investment to avoid the ex-post regret near the time of regime change even when the prospect of the future regime is optimistic. On the other hand, a leader firm should consider not only the ex-post regret but also the preemptive cash flows that would be reduced or lost by the entry of the other firm. More concretely, the leader firm faces a trade-off between the option value to wait and the preemption value for immediate investment as a leader. Our results indicate that the leader does not adopt the principle and is willing to invest if the preemption value is sufficiently large relative to the option value.

From another view point, we can say that a leader firm conforms to the principle unless the preemption value dominates the option value. Most papers in a real options framework show that competition lowers the investment threshold of a leader firm but it has been still unclear how a leader firm optimally takes competition and the option value into account for the investment decision. It will be effectively shown that the option value to wait *does* have an impact on the decision in a competitive situation. This paper sheds new light on the meaning of the Bad News Principle in the literature.

Another finding in the paper is that an exogenous parameter can influence the leader's strategy similarly to or differently from the follower's, depending on the structure of the future regime. This observation comes from the fact that a dynamic interaction between two firms can cause a different impact on the investment timing.

Our results coincide with other theoretical papers like Grenadier (2002) and Bouis et al. (2009) in that strategic interactions affect the strategies of firms facing competition for the investment. However, this study is the first to theoretically clarify how competition and the Bad News Principle are related from a standpoint of dynamic evolution. Our paper contributes to the literature in this sense.

Several papers empirically examine the relationship between economic conditions and investment behaviors in the industry, especially the effect of economic policy on the investment. The papers include Knack and Keefer (1995), Alesina and Perotti (1996), Servén (1997) and Lensink (2002). Recently Julio and Yook (2012) show that during election years, firms reduce investment expenditures by an average of 4.8% relative to non-election years, controlling the growth opportunities and economic conditions. Our theoretical results are in fact consistent with empirical findings reported by the papers.

The remaining part of the paper is organized as follows. Section 2 describes our model. The value functions and the investment thresholds of the two firms are derived in Section 3. We numerically calculate the value functions to analyze the optimal investment strategies in Section 4. Finally Section 5 gives some concluding remarks of this paper.

## 2 Model Setup

This section sets up our model that extends Nishide and Nomi (2009) to the case of competition between two firms.

Suppose that two identical and risk-neutral firms are considering the investment in a project. To earn a cash flow from the project, each firm needs to pay an irreversible cost denoted by  $K$ . When one of the two firm has invested, it earns the instantaneous cash flow given by  $D_1P$ , where  $P$  denotes the industry-wide demand shock. When both firms have already invested, each of the two firms receives the cash flow  $D_2P$ . To describe a preemptive situation where a leader firm earns more profit due to less competition, we assume the inequality  $D_1 > D_2$ . The ratio  $D_1/D_2$  represents how preemptive the investment by a leader is. In summary, the instantaneous cash flow from the project is

written as

$$1_{\{t \geq \tau_F\}} D_2 P$$

if the firm is a follower, and

$$1_{\{\tau_L \leq t < \tau_F\}} D_1 P + 1_{\{t \geq \tau_F\}} D_2 P$$

if it is a leader, where  $\tau_F$  and  $\tau_L$  are the investment times of the follower and the leader, respectively.

The demand shock  $P$  and the risk-free interest rate  $r$  are subject to a regime shift. Contrary to a time-homogeneous setting as Hassett and Metcalf (1999), Guo et al. (2005) or Goto et al. (2012), the time of the regime shift is deterministic and known in advance to both firms. One example that our model is applicable to is a presidential election that can affect the economic policy, trade negotiation between two countries with an explicit deadline, and so on. In such situations, both firms should consider not only what the future economic conditions are like but also the remaining time to the time of the regime change. See Nishide and Nomi (2009) for detailed discussions.

Let  $\hat{T}$  be the time of the regime change. For  $t < \hat{T}$ , the demand shock of the project follows the geometric Brownian motion

$$dP(t) = \mu_0 P(t) dt + \sigma_0 P(t) dz(t),$$

where  $\mu_0$  is the expected growth rate of the demand,  $\sigma_0$  is its volatility, and  $z(t)$  is a standard Brownian motion describing randomness. The risk-free rate (discount factor) before  $\hat{T}$  is assumed to be a constant  $r_0$ .

At time  $\hat{T}$ , a regime change happens. Suppose that there are  $S$  possible states after  $\hat{T}$ , and let  $q_s$  be the probability of state  $s$ ,  $s = 1, \dots, S$ . When state  $s$  is realized, the demand shock  $P(t)$  satisfies the stochastic differential equation as

$$dP(t) = \mu_s P(t) dt + \sigma_s P(t) dz(t),$$

and the risk-free rate  $r(t)$  is equal to a constant  $r_s$  for  $t \geq \hat{T}$ . We also assume that  $r_s > \mu_s$  to guarantee the problem to be well-posed.

The firms do not know what state will be formed in advance, but know the structure of the future regimes and the probabilities  $\{(r_s, \mu_s, \sigma_s, q_s)\}_{s=1}^S$ . Assume for simplicity that once state  $s$  is realized, the parameters  $r_s$ ,  $\mu_s$ , and  $\sigma_s$  remain there forever and both firm have no power to influence the regime.

We illustrate the structure of the regime in Figure 1.

[Figure 1 is inserted here.]

The optimization problem of each firm is described as follows. If a firm is a follower and will invest in the project after the other firm, then the function to be maximized is given by

$$\mathbb{E}_t \left[ \int_{\tau_F}^{\infty} e^{-\int_t^u r(v)dv} D_2 P(u) du - e^{-\int_t^{\tau_F} r(u)du} K \right] \quad (2.1)$$

where the maximization is taken with respect to the stopping time  $\tau_F$ . On the other hand, the objective function of a leader firm is

$$\mathbb{E}_t \left[ \int_{\tau_L}^{\tau_F} e^{-\int_t^u r(u)du} D_1 P(u) du + \int_{\tau_F}^{\infty} e^{-\int_t^u r(u)du} D_2 P(u) du - e^{-\int_t^{\tau_L} r(u)du} K \right] \quad (2.2)$$

given the strategy of the follower  $\tau_F$ .

### 3 Model Solutions

Following Nielsen (2002) and other papers, we firstly consider the investment timing of the follower and then examine the strategy of the leader.

After the time of regime change, the objective function of the follower (2.1) becomes time-homogeneous and so the problem reduces to a standard one studied by McDonald and Siegel (1986). That is, the follower's value function for  $t \geq \hat{T}$  and state  $s$  is given by

$$\phi_s(P) = 1_{\{P \geq P_{Fs}^*\}} \left( \frac{D_2 P}{r_s - \mu_s} - K \right) + 1_{\{P < P_{Fs}^*\}} \left( \frac{P}{P_{Fs}^*} \right)^{\beta_s} \left( \frac{D_2 P_{Fs}^*}{r_s - \mu_s} - K \right), \quad (3.1)$$

where  $\beta_s$  and  $P_{Fs}^*$  are defined by

$$\begin{aligned} \beta_s &= \frac{1}{2} - \frac{\mu_s}{\sigma_s^2} + \sqrt{\left( \frac{1}{2} - \frac{\mu_s}{\sigma_s^2} \right)^2 + \frac{2r_s}{\sigma_s^2}}, \\ P_{Fs}^* &= \frac{\beta_s}{\beta_s - 1} (r_s - \mu_s) \frac{K}{D_2}. \end{aligned} \quad (3.2)$$

The first term of (3.1) represents the present value of immediate investment and the second term is the option value to wait until the random demand  $P$  goes up to the optimal threshold (3.2).

Before deriving the solutions of (2.1) for  $t < \hat{T}$ , we define the following function.

$$G_F(t, P) = D_2 P \left( \frac{1 - e^{-(r_0 - \mu_0)(\hat{T} - t)}}{r_0 - \mu_0} + \sum_{s=1}^S q_s \frac{e^{-(r_0 - \mu_0)(\hat{T} - t)}}{r_s - \mu_s} \right).$$

The function  $G_F$  represents the present value of immediate investment before the time of the regime shift  $\hat{T}$ .

The problem for the follower at  $t < \hat{T}$  is solved by Nishide and Nomi (2009).

**Proposition 1** (Nishide and Nomi, 2009). *Let  $V_F = V_F(t, P)$  be the value function of (2.1) for  $t < \hat{T}$ . Then  $V_F$  satisfies the PDE*

$$\frac{\partial V_F}{\partial t} + \mu_0 P \frac{\partial V_F}{\partial P} + \frac{\sigma_0^2}{2} P^2 \frac{\partial^2 V_F}{\partial P^2} - r_0 V_F = 0 \quad (3.3)$$

with boundary conditions

$$\begin{cases} V_F(t, 0) = 0 & \text{(condition at } P = 0), \\ V_F(t, B_F(t)) = G_F(t, B_F(t)) - K & \text{(value-matching),} \\ \frac{\partial}{\partial P} V_F(t, B_F(t)) = \frac{\partial}{\partial P} G_F(t, B_F(t)) & \text{(smooth-pasting),} \\ V_F(\hat{T}, P) = \sum_s q_s \phi_s(P) & \text{(value at maturity).} \end{cases}$$

The proof is shown in their paper.

In Proposition 1, the investment threshold  $B_F(t)$  is derived by solving the free boundary problem. Although it can be solved only numerically,  $B_F$  has an important analytical property as follows.

**Proposition 2** (Nishide and Nomi, 2009). *The threshold at  $\hat{T}$  satisfies*

$$\lim_{t \nearrow \hat{T}} B_F(t) = \max_{\{s: q_s > 0\}} P_{F_s}^*.$$

Proposition 2 states that just before the time of a regime change, the optimal investment strategy of the follower is such that a firm acts as if the worst case scenario was about to happen, even if the firm is risk-neutral and the probability of the worst state is small. This result is quite consistent with the so-called *Bad News Principle* proposed by Bernanke (1983). The detailed explanations are presented in the introductory section of this paper and Section 3 of Nishide and Nomi (2009).

Now we examine the investment strategy of a leader firm. Denote by  $G_L$  the present value of the cash flow after investment at  $t$ :

$$G_L(t, P) = \mathbb{E}_t \left[ \int_t^{\tau_F} e^{-\int_t^u r(u) du} D_1 P(u) du + \int_{\tau_F}^{\infty} e^{-\int_t^u r(u) du} D_2 P(u) du \right]. \quad (3.4)$$

After the regime change, the problem reduces to the one studied by Nielsen (2002). That is, when state  $s$  is realized at  $\hat{T}$ , (3.4) is equal to

$$\psi_s(P) := \frac{D_1 P}{r_s - \mu_s} - \left( \frac{P}{P_{F_s}^*} \right)^{\beta_s} \frac{(D_1 - D_2) P_{F_s}^*}{r_s - \mu_s} \quad (3.5)$$

for  $t \geq \hat{T}$ . The first term of (3.5) represents the present value of the cash flow in the case the other firm would never invest in the project. The second term describes the negative

option value reflecting the possibility that the cash flow will be reduced due to the follower firm's entry.

We here assume that each firm has an incentive to become a leader for state  $s$  and  $t \geq \hat{T}$  if  $\psi_s(P) - K \geq \phi_s(P)$ . Consequently the investment threshold of a leader firm for state  $s$  and  $t \geq \hat{T}$  satisfies

$$\psi_s(P_{L_s}^*) - K = \phi_s(P_{L_s}^*).$$

Figure 2 illustrates how the leader's threshold is determined.

[Figure 2 is inserted here.]

See also Nielsen (2002) and Fudenberg and Tirole (1985) for the notion of equilibrium.<sup>1</sup> The thresholds  $P_{L_s}^*$  are easily obtained by numerical calculations.

A similar discussion to the previous paragraph can be applied to the case  $t < \hat{T}$ . That is, each firm has an incentive to invest in the project as a leader if  $G_L(t, P) - K \geq V_F(t, P)$  and the investment threshold at  $t < \hat{T}$ , denoted by  $B_L(t)$ , satisfies  $G_L(t, B_L(t)) - K = V_F(t, B_L(t))$ . Since we already have the function  $V_F$  in hand from Proposition 1, we can calculate  $B_L = \{B_L(t)\}_{t < \hat{T}}$  if we obtain the function  $G_L$  for  $t < \hat{T}$ . The following proposition leads to the solution of this problem.

**Proposition 3.** *For  $t < \hat{T}$ ,  $G_L$  satisfies the PDE*

$$\frac{\partial G_L}{\partial t} + \mu_0 P \frac{\partial G_L}{\partial P} + \frac{\sigma_0^2}{2} P^2 \frac{\partial^2 G_L}{\partial P^2} - r_0 G_L + D_1 P = 0 \quad (3.6)$$

*with boundary conditions*

$$\begin{cases} G_L(t, 0) = 0 & \text{(condition at } P = 0), \\ G_L(t, B_F(t)) = G_F(t, B_F(t)) & \text{(value-matching),} \\ G_L(\hat{T}, P) = \sum_{s=1}^S q_s \psi_s(P) & \text{(value at maturity).} \end{cases} \quad (3.7)$$

Contrary to Proposition 1, we have a Dirichlet boundary condition with a fixed upper boundary  $B_F$  as in the second line of (3.7). Note also that the PDE (3.6) contains the final term  $D_1 P$  unlike (3.3). This term represents a preemptive cash flow the leader firm receives as a first investor.

The leader's investment threshold  $B_L$  seems hard to obtain analytically. However we can verify one important property as follows. We have observed that leader's investment

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<sup>1</sup>Nielsen (2002) does not investigate which firm actually becomes a leader, neither does this study.

threshold is determined by the condition that the NPV of the immediate investment as a leader is equal to the value function as a follower. The leader's NPV and the follower's value function at  $t \nearrow \hat{T}$  is easily calculated and so the condition for  $B_L(\hat{T})$  to satisfy is given by

$$\sum_{s=1}^S q_s(\psi_s(B_L(\hat{T})) - K) = \sum_{s=1}^S q_s\phi_s(B_L(\hat{T})).$$

Therefore we have the following result.

**Proposition 4.** *The leader's threshold at maturity satisfies*

$$\min_{\{s:q_s>0S\}} P_{Ls}^* < \lim_{t \nearrow \hat{T}} B_L(t) < \max_{\{s:q_s>0\}} P_{Ls}^*.$$

We expect from Proposition 4 that contrary to the follower's case, the leader firm does not necessarily act in accordance with the *Bad News Principle* for the investment decision near the time of a regime change. However it is still unclear how the leader actually invests in the project given the regime structure and the follower's strategy. To see this, we shall implement a numerical analysis in the next section.

## 4 Numerical Results

In this section we conduct comparative statics with some numerical calculations to examine how the regime structure affects the investment thresholds of both firm, especially of the leader.

Throughout the analysis we suppose that  $S = 2$  to simplify the problem and set  $K = 10$ ,  $D_1 = 1.5$  and  $D_2 = 1.0$ . Other parameters characterizing the regime structure are given in Table 1. The thresholds of the leader and the follower after  $\hat{T}$  are also

Table 1: Parameter values in the base case. Most parameter values are chosen from Case II of Nishide and Nomi (2009).

	$r_s$	$\mu_s$	$\sigma_s$	$P_{Fs}^*$	$P_{Ls}^*$
Current state	0.1	0.00	0.7	4.21	2.01
State 1	0.1	0.05	0.5	2.55	1.21
State 2	0.1	-0.05	0.8	5.42	2.61

presented in the table. From the regime structure we can say that state 1 is good and state 2 is bad for investment. We should also note that the current state is intermediate between the two possible states for  $t \geq \hat{T}$ .

To clarify the novelties and contributions of the paper in the literature, we only give comparative static results that lead to important implications. Other numerical calculations are available upon request to the authors.

**Effect of regime probability** First we study the effect of the regime probability on the investment decisions by both firms to see how the investment strategies before the regime change depend on the future prospects. Figure 3 is presented to describe the effect of  $q \equiv q_1$ , the probability that a good scenario will happen at  $\hat{T}$ .

[Figure 3 is inserted here.]

As we observe from the figure, the follower firm's threshold for  $q < 1$  converges to  $P_{F2}^*$ , the higher threshold of the two. This result is in accordance with the *Bad News Principle* in that the investment decision is dependent only on the worst case scenario near the time of regime shift.

Contrarily, the value of the threshold for the leader differs near maturity, depending on the regime probability. For example, the threshold is decreasing as the time approaches to the regime change if  $q = 0.75$ . This finding indicates that the bad news principle does not always hold for the leader's investment decision and the leader firm is willing to invest when a good scenario would happen with a significantly high probability.

Notice also that the threshold in the case  $q = 0.5$  is increasing and takes a relatively high value near maturity. Intuitively we could imagine that  $B_L$  converges to around the midpoint of  $P_{L1}^*$  and  $P_{L2}^*$  as  $t \rightarrow \hat{T}$ . Figure 3, however, indicates that this conjecture is not true. When  $q = 0.25$ , the threshold at maturity is quite close to  $P_{F2}^*$ , even though the probability of a good state is away from zero. It is concluded from the above findings that the bad news principle may hold under competition unless a good scenario for investment is non-negligible.

**Effect of preemption** Second, we investigate the effect of  $D_2$ , describing the magnitude of preemption for the leader's investment, while  $D_1$  is fixed. An increase in  $D_2$  indicates that the leader's cash flow is less preemptive and the effect of the entry by the follower becomes smaller. The probability of state 1 is set to be 0.75.

[Figure 4 is inserted here.]

Figure 4 shows that the Bad News Principle appears remarkably in the leader's investment strategy as the value of  $D_2$  increases. On the other hand, the leader firm does not conform to the principle when  $D_2 = 1.0$ , or equivalently the cash flow of the leader is more preemptive.

In summary, we can conclude that the Bad News Principle partly holds for the leader's investment strategy. More specifically, a leader firm acts as if a bad case scenario was about to happen if the probability of a bad scenario is non-negligible or the effect of the other firm's entry on the leader's cash flow is not large.

The intuitive explanation is the following. The value of the leader firm's investment is mainly affected by the two factors: the demand shock and the entry by the other firm. By waiting for the investment until  $\hat{T}$ , a leader firm is able to see how the random demand and its regime actually change over time. On the other hand, a firm can receive the preemptive cash flow by immediate investment if it invests in the project before the other firm. If a firm did not invest now, the preemptive cash flow could be lost by the other firm's entry. Each firm faces a trade-off between the values of option and preemption.

Suppose that  $D_1 \gg D_2$  and  $q$  is large. This assumption means that the preemption value dominates the option value and so a leader firm has an incentive to invest in the project even just before the time of a regime shift. However, if the assumption is not the case, the leader firm should optimally wait and see which state is realized at the time of the regime shift. The Bad News Principle holds in such cases.

Many extant papers investigate the effect of competition on the strategy of each firm but it is not apparent how the investment principle differs, depending on whether the firm is a leader or a follower. This paper sheds new light on the Bad News Principle in the literature by clarifying how and when competition affects the leader's strategy.

**Effect of other parameters** As stated above, we provide only some figures with interesting observations to focus on what we get as a new finding in the paper.

Figure 5 depicts the investment thresholds with different values of the volatility in the current regime. We set  $q = 0.5$  in this calculation.

[Figure 5 is inserted here.]

We observe from the figure that thresholds of both leader and follower firms are monotonically increasing in the current volatility  $\sigma_0$ . This property is already found in the real options literature (See, for example, Dixit and Pindyck, 1994). It should be also noted that the shapes of the investment threshold are similar between the leader and the

follower. This is because an increase in the volatility always raises the option values to wait for both firms.

We next show Figure 6, describing the effect of the current discount rate  $r_0$  on the thresholds. We set  $q = 0.25$ .

[Figure 6 is inserted here.]

We observe from the figure that a higher value of the discount rate leads to a higher value of the thresholds of both leader and follower firms. As in the analysis on the volatility, the monotonicity with respect to the discount rate is not new in the literature and explained by the fact that an increase in the discount rate raises the cost of capital and reduces investment (see Chapter 5 of Dixit and Pindyck, 1994). Note also that the shapes of the thresholds are very similar between the two firms as in Figure 5.

Finally we present Figure 7, describing the effect of the current discount rate  $r_0$  on the thresholds in the case  $q = 0.75$ .

[Figure 7 is inserted here.]

An interesting observation from the figure is the following. When  $r = 0.05$  and  $q = 0.75$ , the threshold of the leader takes a hump-shape while the follower's one is of U-shape. This result indicates that the magnitude of the effect of each parameter can vary, depending on the time to maturity. This case is in stark contrast to the ones in Figures 5 and 6.

An explanation of this observation is as follows. Consider the case where  $r = 0.05$  in Figure 7. Note first that the time when the threshold of the leader is at its peak matches the time when the threshold of the follower starts increasing. The leader knows that the other firm hesitates to invest from this period of time to maturity  $\hat{T}$ . Therefore the firm does not need to hasten to become a leader. However, as the time approaches to maturity, the expected preemptive cash flow after the time of regime shift has a larger effect on the objective function (2.2). The firm also knows that a good scenario is highly likely and so is becoming more willing to invest in the project as a first investor, leading to a leader's hump-shaped threshold. The above observation effectively illustrates how the investment strategies of two firms dynamically interact over the time to regime shift.

**Comments on the literature** As we mentioned in the introductory section, the analysis of competition is not new in the real options literature. Such papers include Joaquin and Khanna (2001), Nielsen (2002), Grenadier (2002), Weeds (2002), Pawlina and Kort (2006), Bouis et al. (2009) and Goto et al. (2012). These papers show that a firm should hasten the investment under competition due to the preemption value. However, it has

been still unclear how a firm optimally decides the investment under competition and how the decision relates to the option value to wait. This paper effectively shows that the option value in fact has a significant impact on the leader firm's strategy in a competitive situation. In other words, the Bad News Principle applies to the leader's investment in many actual situations.

It is worth mentioning that all of these papers construct a time homogeneous model to investigate the effect of competition. Our model is, however, time-inhomogeneous by assuming that the time of regime shift is deterministic. With our model, we are able to show that how investment strategies are affected via dynamic interactions. More concretely, a leader firm should simultaneously take into account the values of option and preemption, which evolve over time and are affected by the regime structure. Therefore, some exogenous parameters have different effects on the thresholds of both firms, depending on the time of a regime shift and the regime structure. This implication can be obtained only by a time-inhomogeneous setting. Our paper theoretically contributes to the literature in this sense.

In addition, our theoretical results are quite consistent with empirical papers. For example, Alesina and Perotti (1996) find an inverse relationship between income inequality and investments, and conclude that the negative correlation is linked through socio-political instability. Recently Julio and Yook (2012) show that during election years, firms reduce investment expenditures by an average of 4.8% relative to non-election years, controlling the growth opportunities and economic conditions. Our model theoretically explains these empirical results.

## 5 Conclusion

In this paper, we have studied the investment decisions of two firms in a situation where they compete against each other to get preemptive cash flows as a leader. Our results shed new light on how competition among firms affects the investment decisions with strategic interactions in a real options framework.

One of the important findings is that the so-called *Bad News Principle* partly holds for the leader firm's investment strategy. That is, a leader becomes reluctant to invest in the project just before the time of a regime shift unless a good scenario is highly likely and the leader's cash flow is preemptive enough. This is because even a leader firm has an incentive to wait and see the realization of the future scenario in a situation where the option value is non-negligible relative to the preemption value.

Another observation is that the leader's investment threshold can be influenced by a change of some parameters differently from the follower's threshold. This result comes from the fact that a change in the current economic conditions has multiple effects on the thresholds and the effect directions vary, dependent on the regime structure and the time of a regime change.

It should be mentioned that our model can be extended to a more general setting where the regime change occurs at several times and the times of regime change are deterministic. However, it can be shown with numerical calculations that the obtained results still hold in such settings. Our model covers a wide variety of situations and our results are robust even in more general settings.

One limitation of our model is that two firms are symmetric in profits and costs. In a time-homogeneous setting, Pawlina and Kort (2006) find that there are three equilibrium types, depending on how one firm is advantaged in profits and costs to the other. A generalization of our model to the case of asymmetric firms does not seem easy because the equilibrium type can vary in time. Another possible extension is to introduce some interaction between the strategies of firms and the regime structures. For example, firms can influence a government's decision on economic policies in actual situations. Those are some of our future researches.

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Figure 1: The structure of a regime change. Both firms know the time of the regime change  $\hat{T}$ , the possible regimes after  $\hat{T}$ , and their respective probabilities in advance.

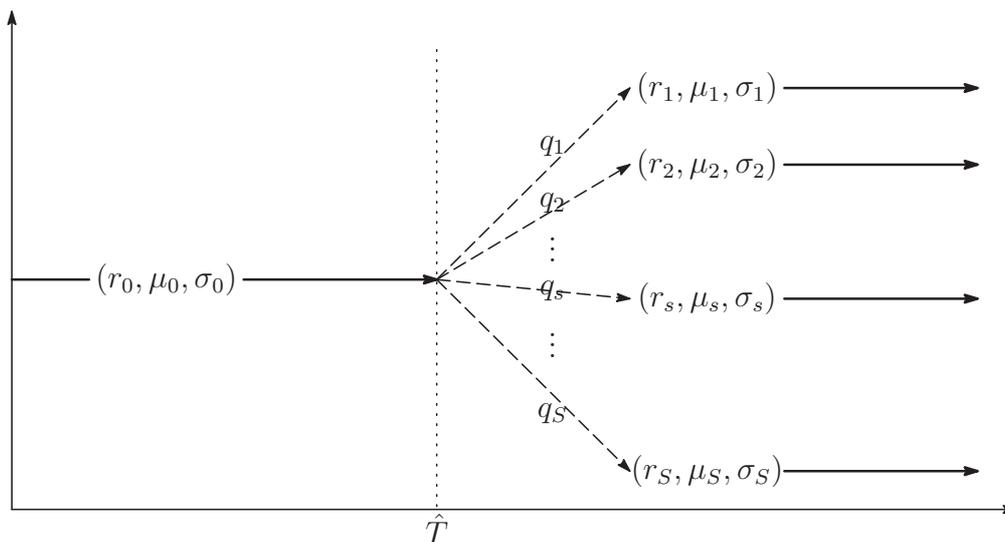


Figure 2: The leader's NPV of immediate investment and investment threshold. Each firm has an incentive to become a leader if  $\psi_s(P) - K \geq \phi_s(P)$ .

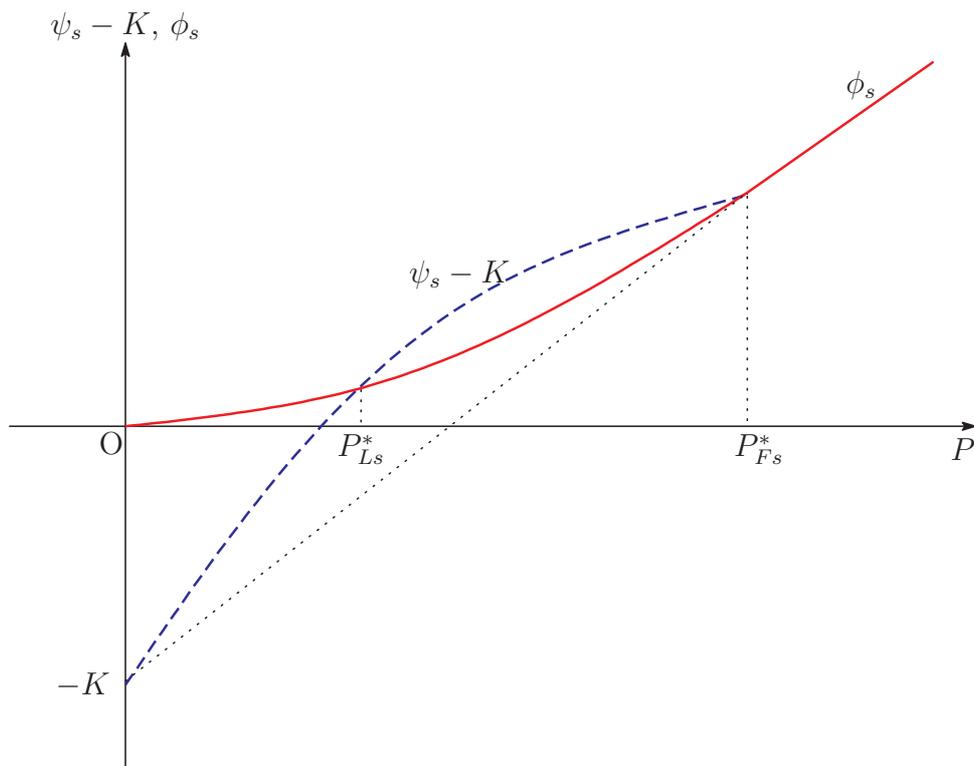
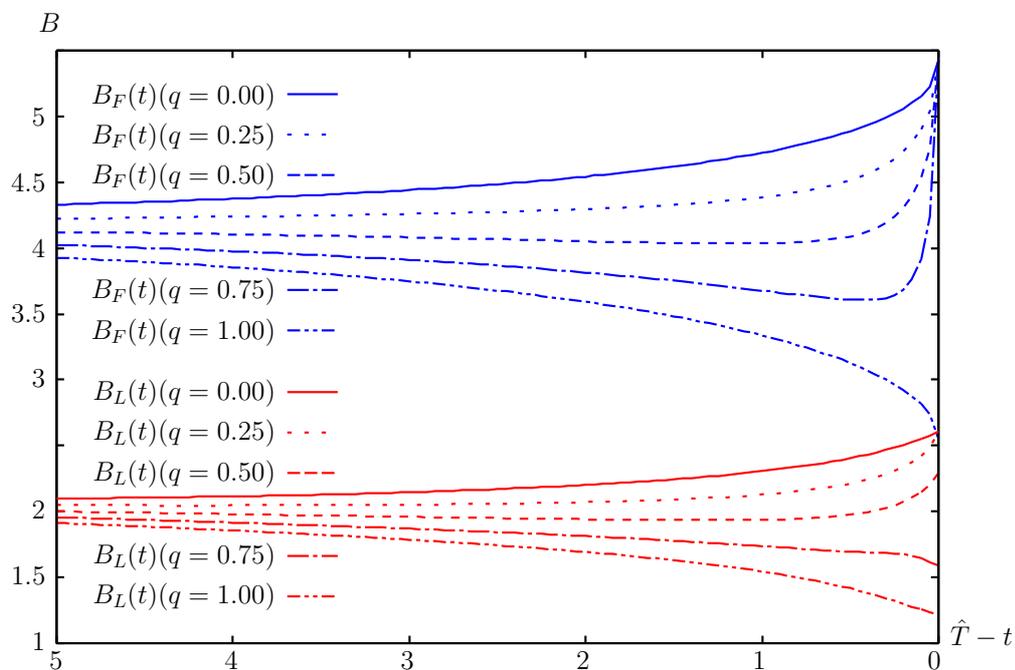
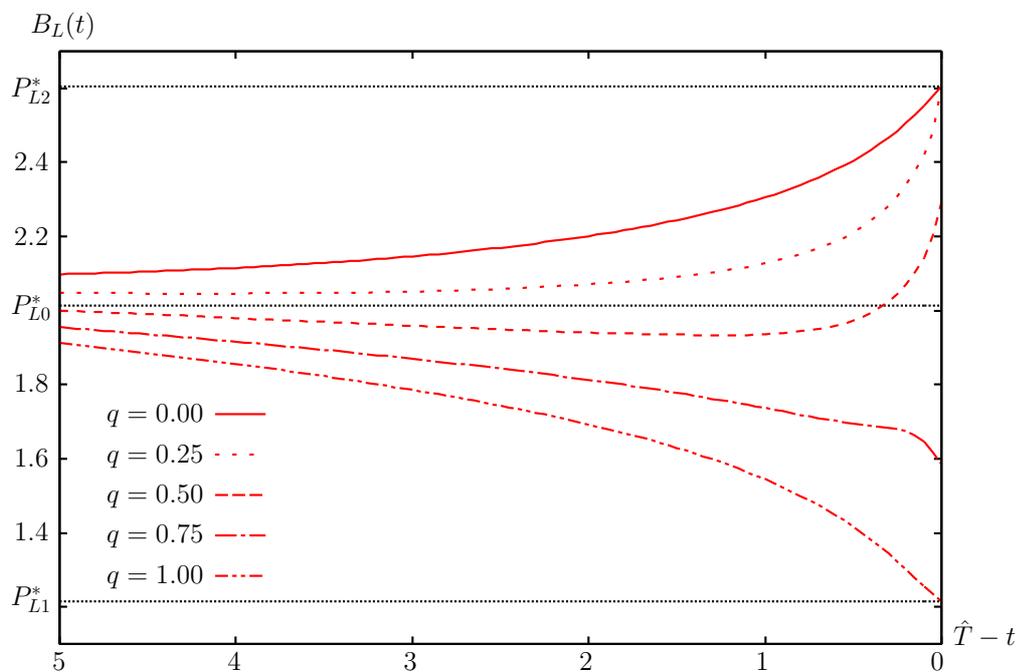


Figure 3: Investment thresholds with various values of the regime probabilities. Here  $q$  denotes the probability of a good state.



Investment thresholds for both firms.



Investment thresholds for the leader firm.

Figure 4: Investment thresholds with various values of  $D_2$ , describing the magnitude of preemption for the leader. We set  $q = 0.75$ .

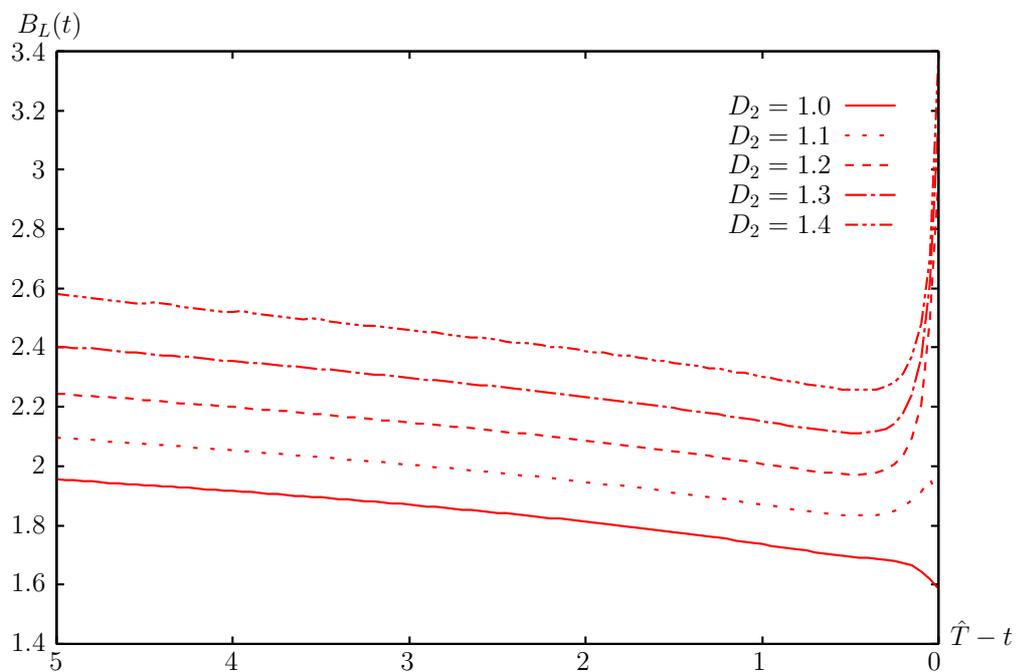
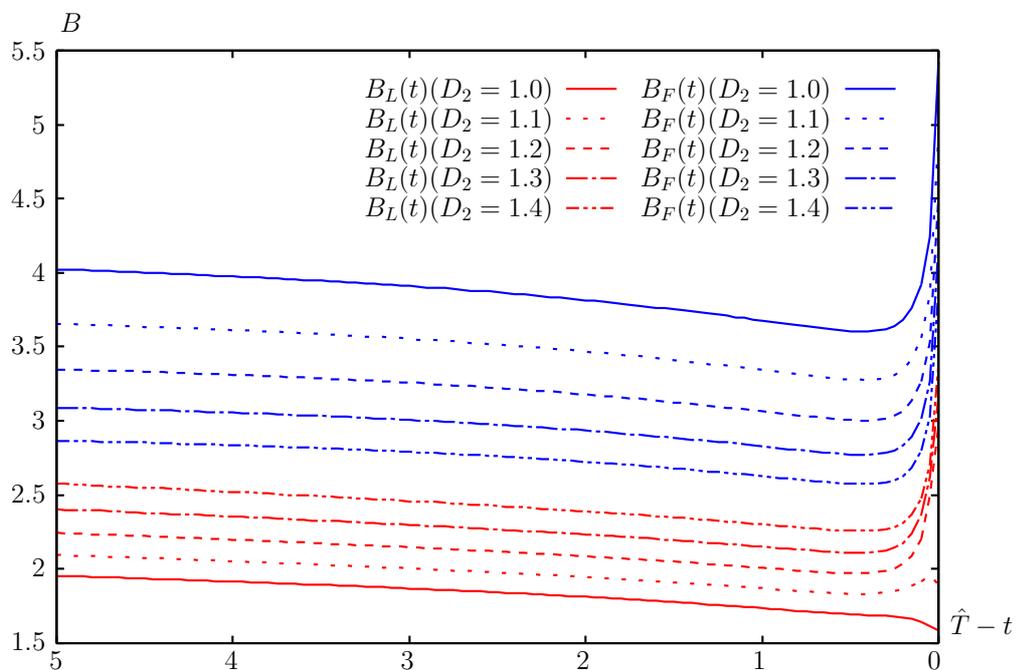
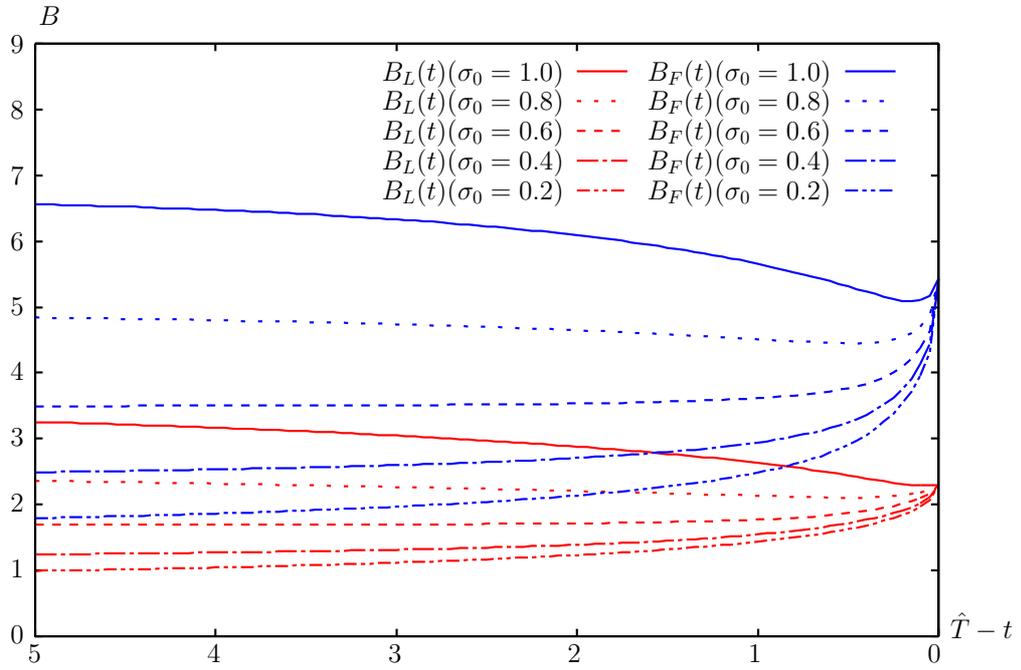
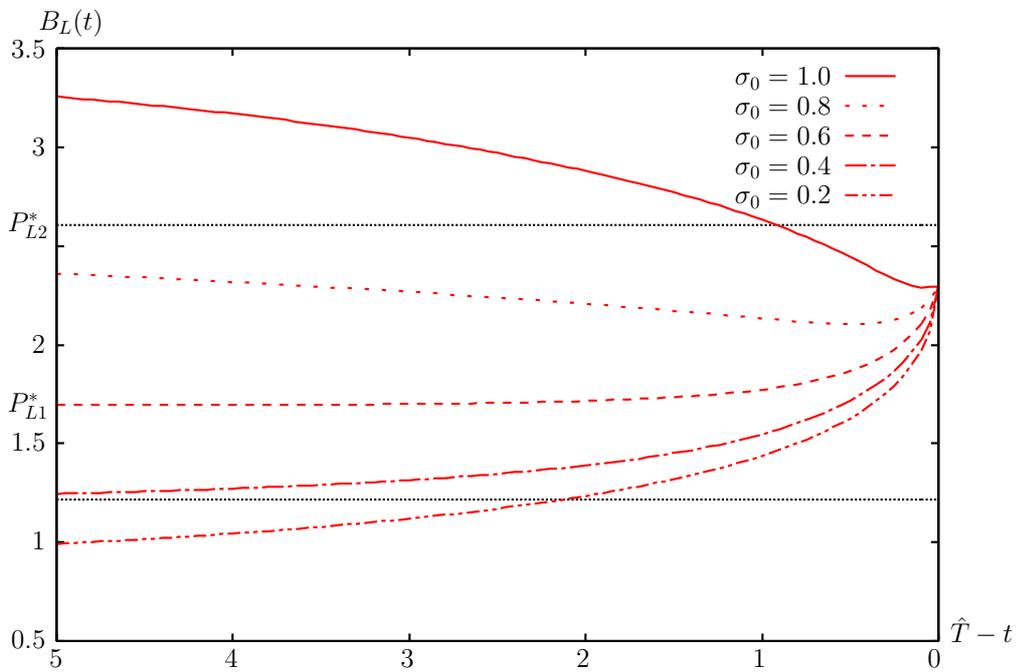


Figure 5: Investment thresholds with various values of the volatility.

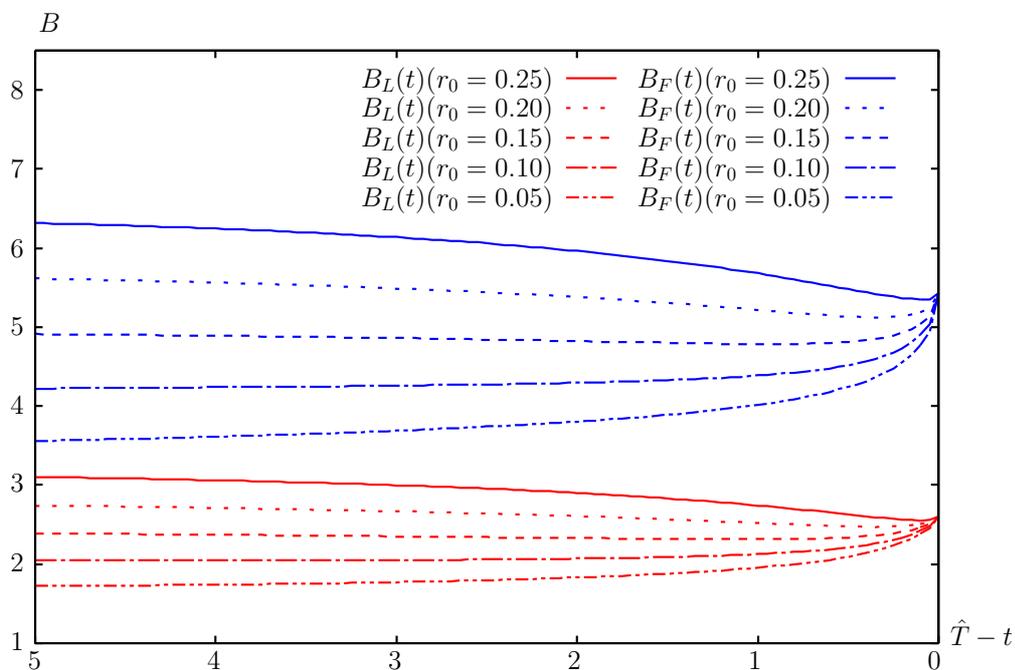


Investment thresholds for both firms.

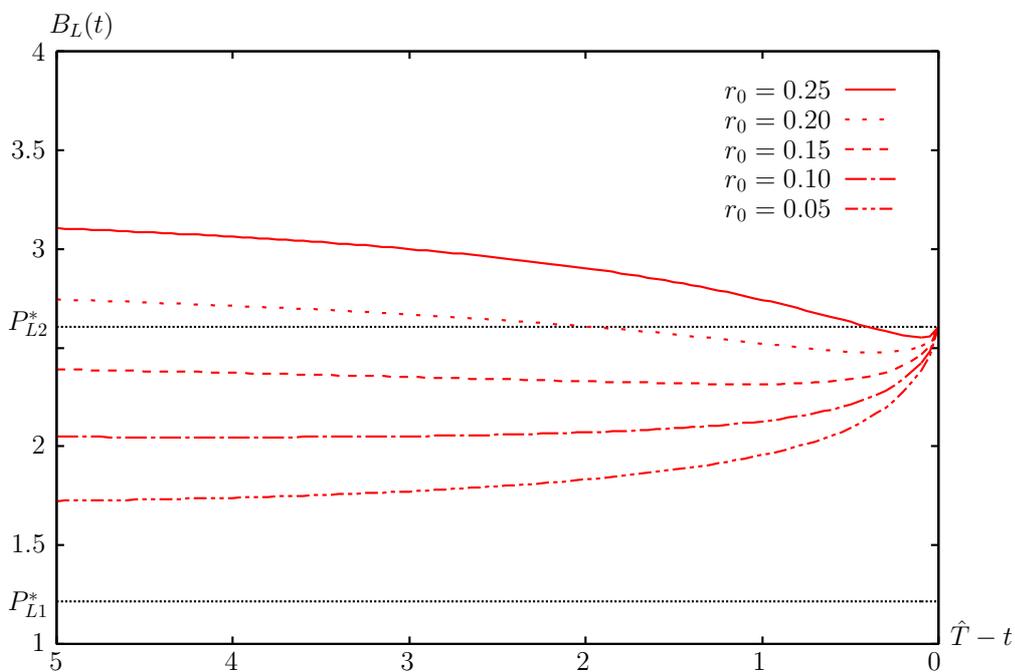


Investment thresholds for the leader firm.

Figure 6: Investment thresholds with various values of the discount rate. The probability  $q$  is set to be 0.25.

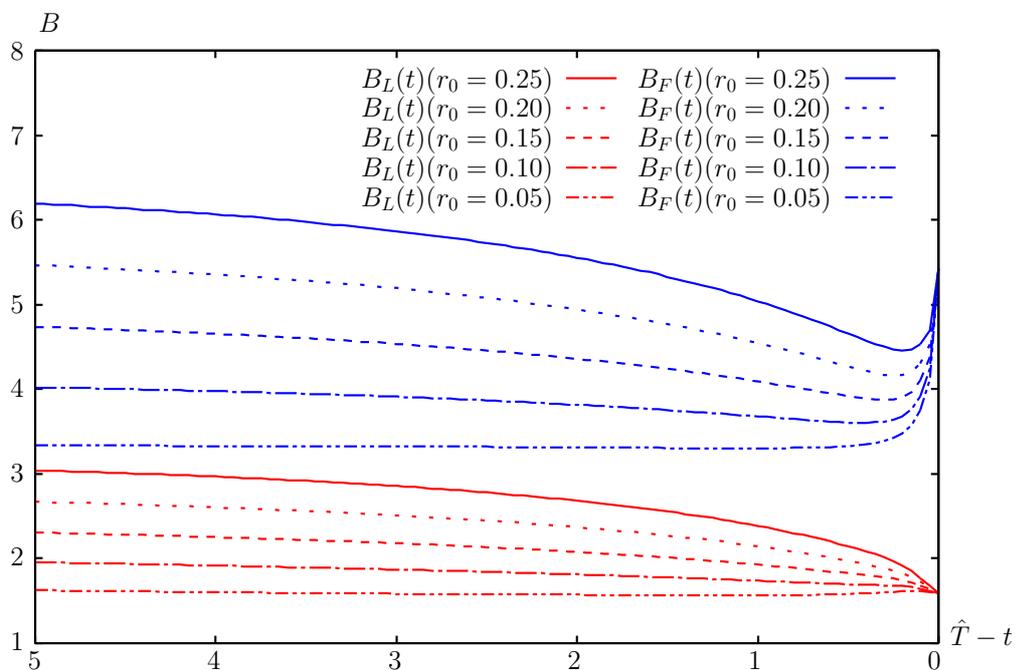


Investment thresholds for both firms.

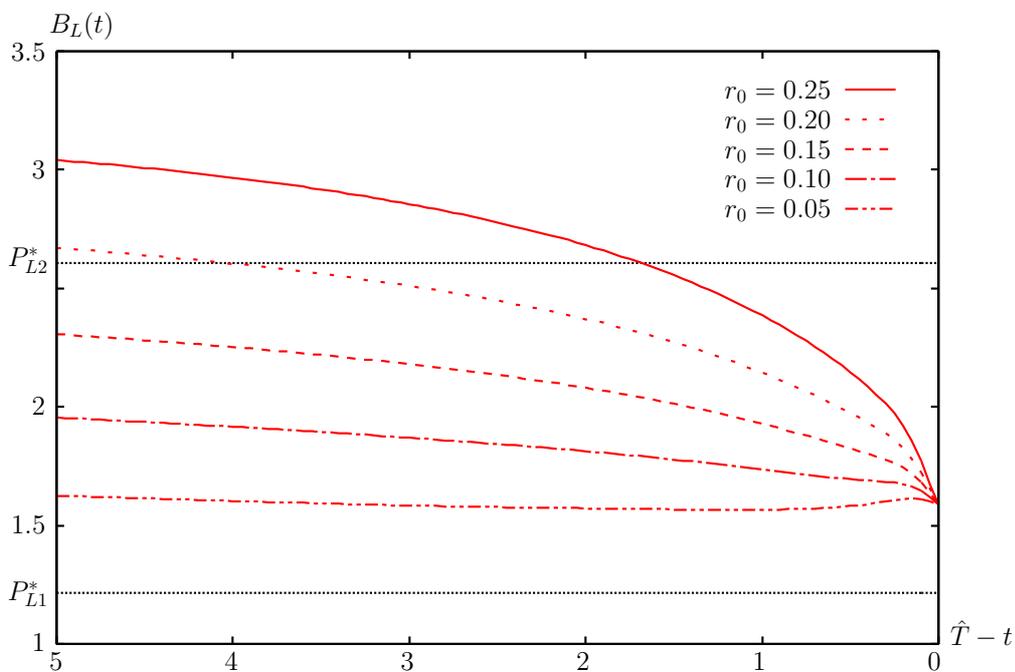


Investment thresholds for the leader firm.

Figure 7: Investment thresholds with various values of the discount rate. In this case we set  $q = 0.75$ .



Investment thresholds for both firms.



Investment thresholds for the leader firm.