KIER DISCUSSION PAPER SERIES

KYOTO INSTITUTE
OF
ECONOMIC RESEARCH

Discussion Paper No.850

"Interest in Private Assets and the Voracity Effect"

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February 2013

KYOTO UNIVERSITY

KYOTO, JAPAN

Interest in Private Assets and the Voracity Effect*

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December 24, 2012

Abstract

Using a differential game, we analyze a multiple agent economy in which there are common and private capital stocks. Each interest group can access the common capital and its own private capital stocks but not anyone else's private capital stocks. Considering the situation in which each interest group can observe and has interest in the opponents' private capital stocks, we show the following. The capital stocks have a negative effect on the consumption of each agent. The growth rate of the common capital does not depend on the technology level of the common sector; that is there is no voracity effect. Each agent's welfare is always lower than it is in the case that each agent has no interest in the opponents' private capital stocks.

Keywords differential game, Markov-perfect equilibrium, the voracity effect
JEL Classification Numbers C73, O10, O40

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*We are grateful to Akihisa Shibata, Akitoshi Muramoto, and participants in the Brown Bag Lunch (BBL) seminars at Kyoto for their helpful comments and suggestions. All errors are our own. Our research is financially supported by Grant-in-Aid for Specially Promoted Research (No. 23000001) and the G-COE program of Osaka University, "Human Behavior and Socioeconomic Dynamics," as well as the Keio-Kyoto joint G-COE program, "Raising Market Quality-Integrated Design of Market Infrastructure."

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1 Introduction

There has been increased interest in studying the economic growth of less developed countries with multiple groups and without secure property rights. It can be analyzed by using the model of a dynamic common pool problem, which is represented by growth models with multiple interest groups or agents who share access to an economy's capital stock. Since capital stock is interpreted as a common property asset, the well-known issues of the tragedy of commons are raised, i.e., the interest groups do not consider the fact that their excessive appropriation behavior has a negative effect on the common capital stock, and, thereby, the growth rate of the economy is lower than that obtained by a standard one-sector growth model.[1]

Tornell and Velasco (1992), Tornell and Lane (1999), and Long and Sorger (2006) consider a two-sector economy adding a second private and secure capital stock to the dynamic common pool problem described above; a common sector and a private sector. The interest groups can appropriate resources from open-access and insecure capital stock and invest in and accumulate private and secure capital stock. Tornell and Velasco (1992) explain why the capital in developing countries flows to developed countries and show that the occurrence of capital flight does not necessarily imply the reduction in the growth rate and welfare of the developing countries. Tornell and Lane (1999) apply the same model to the analysis of the economy in which interest groups interact via a fiscal process. Long and Sorger (2006) extend the Tornell and Velasco (1992) model by introducing a private appropriation cost and the wealth effect as a social status on utility.

In the analysis of these kinds of economies, the Markov perfect equilibrium is commonly used as a solution concept, which restricts strategies to be the choice of each group's current action conditioned on state variables. According to the definition of Markov strategies, it is natural that each group derives utility not only from its wealth but also the opponents 'asset holdings, i.e., the consumption strategy of each interest group is the function of the common capital stock, its own private capital stock, and the opponents' capital stocks. The existing literature (e.g., Tornell and Velasco (1992); Tornell and Lane (1999)), however, considers a consumption strategy that does not depend on the opponents' private capital stocks, i.e., they assume that each

[1] In the standard one-sector growth model, capital stocks which multiple interest agents have are secure property.

interest group cannot observe them or that, even if each group can observe, it has no interest in them. This may be due to the following. The private sector is interpreted to be consist of the domestic informal sector or of bank accounts in foreign countries in which property rights are not violated. It is considered difficult to correctly ascertain the amount of the private capital stock in the informal sector or overseas bank accounts.

The main purpose of this paper is, therefore, to relax the assumption imposed in the existing literature and consider a more general consumption strategy; i.e., we examine an economy in which each interest group can observe and has interest in the opponents' private capital stock, and to compare the economy with the Tornell and Velasco (1992) economy with respect to the growth rates and welfare. We show that the consumption strategy of each interest group becomes Markov control-state substitutability; that is the opponents' capital stock holdings have a negative effect on each group's consumption, and the voracity effect[2] is not obtained, which means that the growth rate of the common capital stock does not depend on the technology level of the common sector. We then show that each agent's welfare is always lower than it is in the case that each agent cannot observe or, even if it observes, has no interest in the opponents' private capital stocks. This result implies that, even if the interest groups can actually observe their opponents' private capital stocks, they will not have interest in them.

Other related papers consider one-sector economies without secure property rights. Lane and Tornell (1996) show that, if the elasticity of intertemporal substitution is high enough, the voracity effect operates and then provide some empirical evidence in support of the mechanism. Mino (2006) introduces a variable labor supply and increasing returns to scale and shows that, if the scale effect due to the increasing returns dominates the overconsumption under insecure property rights, no voracity effect is obtained. The effect of the use of Stone-Geary utility function on the voracity effect is analyzed by Strulik (2012). In the setting, he shows that, since the rate of intertemporal substitution in consumption depends on the level of consumption, the voracity effect occurs when an economy is in decline and sufficiently close to stagnation. Tornell (1999) analyzes the discrete time model similar to that created by Tornell and Lane (1999) and obtain the same result.

[2]In the standard growth model or in the situation that all interest groups can coordinate, an increase in the technology level increases the return on investment and the growth rate. However, the voracity effect counteracts this standard effect and a higher technology leads to slower economic growth.

The paper is organized as follows. Section 2 is the basic model. In Section 3, the equilibrium conditions of the model are derived and characterized. This section is an explanation of the two equilibrium candidates in our model, which are the same equilibrium obtained by Tornell and Velasco (1992) and Tornell and Lane (1999) and another equilibrium. The latter equilibrium is analyzed in Section 4. Section 5 contains a discussion of the balanced growth rates and a welfare comparison of the two equilibria and the comparative statics. Section 6 is the conclusion.

2 The Model

There are n (≥ 2) symmetric representative groups in an economy. Each group i ($i = 1, 2, \cdots, n$) has the same CRRA utility function. The discounted sum of the utility is represented as follows.

$$U_i = \int_0^\infty \frac{c_i(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt, 0 < \theta, \theta \neq 1, \; i = 1, 2, \cdots, n \tag{1}$$

where $c_i(t) \in \mathbb{R}_+$ is group i's consumption at time t, θ is the inverse of the intertemporal elasticity of substitution in consumption, and ρ is the subjective rate of time preference.

There is no property right for the common capital stock so that each group i can use this freely given the other groups' behavior. The state equation of the common capital is described by

$$\dot{K}(t) = AK(t) - d_i(t) - \sum_{j \neq i} d_j(t), \tag{2}$$

where $A \in \mathbb{R}_{++}$ is the technology level of the common sector, $K(t) \in \mathbb{R}_+$ is the common capital at time t, and $d_i(t) \in \mathbb{R}_+$ is group i's amount of appropriation from the common capital pool at time t.

Each group i can use the capital appropriated from the common sector to consume and accumulate as its private asset. The private capital of group i is accumulated according to

$$\dot{h}_i(t) = Bh_i(t) + d_i(t) - c_i(t), i = 1, 2, \cdots, n, \tag{3}$$

where $B \in \mathbb{R}_{++}$ is the technology level of the private sector, $h_i(t) \in \mathbb{R}_+$ is the private capital of group i at time t, $d_i(t)$ is the amount appropriated from the common pool by group i at time t. The feasibility conditions can be summarized by the following requirement. For all t and all i,

$$K(t) \geq 0, \quad h_i(t) \geq 0, \quad d_i(t) \geq 0, \quad c_i(t) \geq 0. \tag{4}$$

As for the initial stock, we assume that the initial stock of common capital is positive, $K(0) = K_0 > 0$, and assume that the initial stock of the private capital of group i is zero, $\vec{h}(0) = \vec{h}_0 = \vec{0}$, to ease comparisons with the case of Tornell and Velasco (1992). In what follow, we define the solution concept.

2.1 The Solution Concept

Following the existing literature, we focus on symmetric Markov perfect equilibrium of the differential game specified above. A Markovian strategy for group i is a pair of functions $\psi^i : \mathbb{R}_+^{n+1} \to \mathbb{R}_+$ and $\phi^i : \mathbb{R}_+^{n+1} \to \mathbb{R}_+$, which implies that there are n-tuple of strategies, $\{(\phi_i, \psi_i)\}_{i=1}^n$. We call ϕ_i group i's appropriation strategy and ψ_i its consumption strategy. Each group i chooses its appropriation and consumption rates according to the feedback rules $d_i(t) = \phi^i(K(t), \vec{h}(t))$ and $c_i(t) = \psi^i(K(t), \vec{h}(t))$, where $\vec{h} = (h_1, h_2, \cdots, h_n) \in \mathbb{R}_+^n$ represents the n demential vector of private capital stocks. Strategies ϕ_i and ψ_i are called symmetric if, for all i and $j(\neq i)$, the relations $\phi_i = \phi_j$ and $\psi_i = \psi_j$ hold.

In this setting, each group i chooses the optimal levels of c_i and d_i at any instantaneous time to maximizes (1) subject to (2)$-$(4), the optimal behavior of the other groups, and any given initial stocks, K_0 and $\vec{h}_0$. Therefore, the model is a differential game among n groups in which the control variables of each group i are c_i and d_i and the state variables of the game are the common capital, K and the private capital, h_i, for $i = 1, 2, \cdots, n$. Furthermore, we define Markov perfect equilibrium as follows.

Definition 1. *The n-tuple of functions, $\{(\phi_i^*, \psi_i^*)\}_{i=1}^n$, forms a Markov perfect equilibrium if, for each group i, all K_0, and $\vec{h}_0$, it is a subgame perfect equilibrium for every realization of the state $(K(t), \vec{h}(t))$.*

2.2 Hamilton-Jacobi-Bellman Equation

Deriving Markov perfect equilibrium, we use the dynamic programming technique; i.e., we use the Hamilton-Jacobi-Bellman (HJB) equation. The Markov perfect equilibrium must satisfy the HJB equation. The HJB equation of group i is as follows,

$$\rho V_i(K,\vec{h}) = \max_{\psi_i,\phi_i}\left\{\frac{\psi_i^{1-\theta}}{1-\theta} + \frac{\partial V_i}{\partial K}\left(AK - \phi_i - \sum_{j\neq i}^{n}\phi_j^*\right)\right.$$
$$+ \frac{\partial V_i}{\partial h_i}\left(Bh_i + \psi_i - \phi_i\right)$$
$$\left. + \sum_{j\neq i}^{n}\frac{\partial V_i}{\partial h_j}\left(Bh_j + \phi_j^* - \psi_j^*\right)\right\}, \tag{5}$$

where $V_i(K,\vec{h})$ is the value function of group i, variables with asterisks represent the opponents' optimal strategies. In addition, the value function $V_i(K,\vec{h})$ must satisfy the following boundary condition:

$$\lim_{t\to\infty} V_i(K,\vec{h})\exp(-\rho t) = 0. \tag{6}$$

Differentiating the HJB equation with respect to c_i and d_i, the optimal conditions are given as

$$\psi_i^{-\theta} = \frac{\partial V_i(\cdot)}{\partial h_i}, \tag{7}$$
$$\frac{\partial V_i(\cdot)}{\partial h_i} = \frac{\partial V_i(\cdot)}{\partial K} \tag{8}$$

for all i. Equations (6) and (7) yield a set of Markov perfect equilibrium solutions.

Substituting these into the HJB equation and using the envelope theorem, we find that differentiating both sides of the HJB equation with respect to K, h_j, and h_j, respectively, yields

$$\rho\frac{\partial V_i(\cdot)}{\partial K} = \frac{\partial^2 V_i(\cdot)}{\partial K^2}\left(AK - \phi_i^* - \sum_{j\neq i}\phi_j^*\right) + \frac{\partial V_i(\cdot)}{\partial K}\left(A - \sum_{j\neq i}\frac{\partial \phi_j^*}{\partial K}\right)$$
$$+ \frac{\partial^2 V_i(\cdot)}{\partial K\partial h_i}\left(Bh_i + \phi_i^* - \psi_i^*\right) + \sum_{j\neq i}\frac{\partial V_i(\cdot)}{\partial h_j}\left(\frac{\partial \phi_j^*}{\partial K} - \frac{\partial \psi_j^*}{\partial K}\right)$$
$$+ \sum_{j\neq i}\frac{\partial^2 V_i(\cdot)}{\partial K\partial h_j}\left(Bh_j + \phi_j^* - \psi_j^*\right), \tag{9}$$

$$\rho\frac{\partial V_i(\cdot)}{\partial h_i} =\frac{\partial^2 V_i(\cdot)}{\partial h_i \partial K}\left(AK-\phi_i^*-\sum_{j\neq i}\phi_j^*\right)+\frac{\partial V_i(\cdot)}{\partial K}\left(-\sum_{j\neq i}\frac{\partial \phi_j^*}{\partial h_i}\right)$$
$$+\sum_{j\neq i}\frac{\partial^2 V_i(\cdot)}{\partial h_i\partial h_j}\left(Bh_j+\phi_j^*-\psi_j^*\right)+\sum_{j\neq i}\frac{\partial V_i(\cdot)}{\partial h_j}\left(\frac{\partial \phi_j^*}{\partial h_i}-\frac{\partial \psi_j^*}{\partial h_i}\right)$$
$$+\frac{\partial^2 V_i(\cdot)}{\partial h_i^2}\left(Bh_i+\phi_i^*-\psi_i^*\right)+\frac{\partial V_i(\cdot)}{\partial h_i}B, \tag{10}$$

and

$$\rho\frac{\partial V_i(\cdot)}{\partial h_j} =\frac{\partial^2 V_i(\cdot)}{\partial h_j \partial K}\left(AK-\phi_i^*-\phi_j^*-\sum_{k\neq i,j}\phi_k^*\right)+\frac{\partial V_i(\cdot)}{\partial K}\left(-\frac{\partial \phi_j^*}{\partial h_j}-\sum_{k\neq i,j}\frac{\partial \phi_k^*}{\partial h_j}\right)$$
$$+\frac{\partial^2 V_i(\cdot)}{\partial h_j\partial h_i}\left(Bh_i+\phi_i^*-\psi_i^*\right)+\frac{\partial^2 V_i(\cdot)}{\partial h_j^2}\left(Bh_j+\phi_j^*-\psi_j^*\right)$$
$$+\frac{\partial V_i(\cdot)}{\partial h_j}\left(B+\frac{\partial \phi_j^*}{\partial h_j}-\frac{\partial \psi_j^*}{\partial h_j}\right)+\sum_{k\neq i,j}\frac{\partial V_i(\cdot)}{\partial h_k}\left(\frac{\partial \phi_k^*}{\partial h_j}-\frac{\partial \psi_k^*}{\partial h_j}\right)$$
$$+\sum_{k\neq i,j}\frac{\partial^2 V_i(\cdot)}{\partial h_j\partial h_k}\left(Bh_k+\phi_k^*-\psi_k^*\right). \tag{11}$$

For Markov perfect equilibrium to be derived, we should know the forms of consumption and appropriation strategies and the value function. In what follows, we specify them and then derive the equilibrium of the game.

3 Equilibrium

Following the existing literature, we focus on linear Markovian strategies. Since the main purpose of the present paper is to relax the assumption of consumption strategy in the Tornell and Velasco (1992) model, we suppose group i believes that all other agents $j(\neq i)$ use a linear Markovian consumption strategy, $\psi_i(K,\vec{h})=a'+aK+eh_i+bZ_i$. As for a appropriation strategy, we use the same linear Markovian appropriation strategy, $\phi_i(K,\vec{h})=\gamma K$, as it is used in the existing literature. The unknown parameters a', a, b, e, and γ are constant. For notational simplicity, we define the aggregate private capital of the opponents' group $\sum_{j\neq i} h_i$ as Z_i. Since we focus on the symmetric case, it is plausible that parameter b with respect to the opponents' capital stock is assumed to be the same among all the opponents' private capital stocks, h_j for all $j \neq i$. The appropriation strategy is assumed to depend on the aggregate capital in the formal sector, K, as in the existing literature.

In addition, we conjecture the following value function.[3]

$$V_i(K, \vec{h}) = \frac{\xi(K + \alpha h_i + \beta Z_i)^{1-\theta}}{1-\theta} \tag{12}$$

where ξ, α, and β are constant. For the same reason above, β is assumed to be the same among h_j for all $j \neq i$.

We use these strategies, the value function, first-order conditions, and the envelope theorem to solve unknown parameters. After some manipulations, we obtain the following:

$$\beta\{(1-\beta)(n-1)\gamma - [A - B + a(1-\beta)]\} = 0, \tag{13}$$

$$(1-\beta)(n-1)\gamma - [A - B - a\beta(n-1)(1-\beta)] = 0. \tag{14}$$

A detailed derivation of these equations is given in appendix. Using these equations, the following equation is obtained.

$$a\beta(1-\beta)[\beta(n-1)+1] = 0. \tag{15}$$

This implies that there are three solution candidates of the model, which are

$$\beta = 0, 1, \text{ or } -\frac{1}{n-1}. \tag{16}$$

In the subsequent subsections, we consider whether or not these candidates are equilibrium solution.

3.1 The candidate 1: $\beta = 1$

We first consider the case in which $\beta = 1$. If $\beta = 1$, the above conditions require that $A = B$. This contradicts, however, the assumption; thus, $\beta = 1$ is not a solution.

[3]It is known that a value function for the CRRA utility function is the CRRA form.

3.2 The candidate 2: $\beta = 0$

Second, we consider the case in which $\beta = 0$. In this case, the conditions above are all satisfied; thus, we obtain

$$\bar{\gamma} = \frac{A - B}{n - 1}. \tag{17}$$

Substituting this into (8) yields

$$\bar{a} = \frac{\rho + (\theta - 1)B}{\theta}. \tag{18}$$

The remaining unknown parameters may be solved by using these parameters, the first-order conditions, and the value function,

$$\bar{a}' = 0, \bar{e} = \bar{a}, \alpha = 1, \bar{\xi} = \bar{a}^{-\theta}, \text{ and } \bar{b} = \bar{a}\bar{\beta}.$$

These solutions correspond to those obtained in Tornell and Velasco (1992).[4]

3.3 The candidate 3: $\beta = -1/(n-1)$

Finally, we consider the case in which $\beta = -1/(n-1)$. In this case, the conditions above are satisfied if and only if

$$\gamma^* = \frac{A - B}{n} + \frac{a^*}{n - 1}. \tag{19}$$

Substituting this into (8), we get

$$a^* = \frac{(n-1)[\rho + (\theta - 1)B]}{\theta n - 1}. \tag{20}$$

As before, we can solve for the remaining unknown parameters.

$$(a^*)' = 0, e^* = a^*, \alpha^* = 1, \xi^* = (a^*)^{-\theta}, \text{ and } b^* = a^*\beta^*. \tag{21}$$

As in Tornell and Velasco (1992), we call a^* the marginal propensities to consume and γ^* the appropriation rate. We summarize the results above in the following lemma.

[4]In this case, the balanced growth rate of the common capital is obtained by $\bar{g} = \frac{nB-A}{n-1}$. See Tonell and Velasco (1992) for a detailed derivation.

Lemma 1. *If group i can observe and has interest in the opponents' private capital stocks, h_j $(j \neq i)$, parameter β is $-\frac{1}{n-1}$. In this case, other optimal parameters are obtained by* (18) − (20).

In the following analysis, we focus on the equilibrium obtained under the case in which each group can observe and has interest in the opponents' private capital. We call this the interest equilibrium. On the other hand, the equilibrium obtained in Tornell and Velasco (1992) is called the non-interest equilibrium.

4 The Interest Equilibrium

In this section, we analyze the dynamical system and characterize the balanced growth rate in the case of the interest equilibrium.

4.1 Dynamical System

Since we focus on a symmetric equilibrium, in equilibrium h_i is equal to h_j for all $j \neq i$. Therefore, we can describe the dynamical system of our model by only K and h_i. These are as follows.

$$\dot{K} = \frac{(n-1)B - \rho n}{\theta n - 1} K, \tag{22}$$

$$\dot{h}_i = (\gamma^* - a^*)K + Bh_i. \tag{23}$$

It is easily verified that the dynamical system of our model is unstable, as it is in a standard AK model.

We denote the growth rate of the common capital by g. It is noteworthy that g does not depend on the technology level of the common capital, A. This implies that under the situation in which each group can observe the opponents' private capital there is no voracity effect; that is the increase in appropriation of all groups offsets the effect of an increase in the technology level, A.

4.2 The Voracity Effect

If each group cannot observe or, even if it can observe, it has no interest in the opponents' private capital, the increase in appropriation surpasses the increase in the technology level, A; as a result, over-appropriation occurs, and the voracity effect is observed. On the other hand, if each group can observe and has interest in the opponents' private capital, there is no over-appropriation. This means that since all groups can monitor the private capital stocks of other groups and are homogenous, they know that if one group appropriates more someone else also appropriate more and the increase in the appropriation by the opponents decreases the consumption of the group. Therefore, all groups have no incentive to extract the common capital excessively.

Proposition 1. *If the opponents' private capital can be observed and each group has interest in the capital, h_j, i.e., $\beta \neq 0$, there is no voracity effect.*

Proof. Differentiating (21) with respect to A, we get $\frac{\partial g}{\partial A} = 0$. This implies that there is no voracity effect in the economy. □

4.3 Balanced Growth Rates

In this subsection, we derive the balanced growth rates of common capital, K, private capital, h_i, consumption, c_i, and appropriation, d_i. We first characterize a symmetric Markov perfect equilibrium of the interest equilibrium.

Proposition 2. *Let us assume that the following conditions*

$$(1-\theta)B < \rho < \frac{n-1}{n}B \text{ and } \frac{1}{n} < \theta$$

are satisfied. The strategy profile $\{(\phi_i, \psi_i)\}_{i=1}^n$ defined by $\phi_i(K, \vec{h}) = a' + aK + eh_i + bZ_i$ and $\phi_i(K, \vec{h}) = \gamma K$ forms a symmetric Markov perfect equilibrium. The consumption strategy is the Markov control-state substitutive, $\partial\psi_i^*/\partial h_j < 0$.

Proof. For the first part, we know that, in the equilibrium, parameters are given by Lemma 1. These constitute an Markov perfect equilibrium. We check that the boundary condition is satisfied and that parameters a^* and γ^* are positive. For the boundary condition to be satisfied, it is required that $\rho + (\theta - 1)B > 0$. In this situation, if $\theta n > 1$, it is easily verified that both parameters, a^* and γ^*, are positive.

For the second part, we know that the equilibrium consumption strategy of group i is

$$\psi_i^* = \frac{(n-1)[\rho + (\theta - 1)B]}{\theta n - 1}(K + h_i) - \frac{\rho + (\theta - 1)B}{\theta n - 1} Z_i,$$

where $Z_i = \sum_{j \neq i} h_j$. Differentiating this with respect to h_j, we get $\partial \psi_i^* / \partial h_j < 0$. □

The proposition illustrates that the opponents' private capital, h_j, has a negative effect on group i's consumption strategy. Furthermore, from the first-order condition (7), although group i's private capital has same effect on its value as the common capital, in the symmetric Markov perfect equilibrium this effect is offset by the negative effect through the sum of the opponents' private capital, Z_i. This implies that none of the private capital affect its value. As a result, the growth rate of consumption is obtained as

$$\frac{\dot{c}_i}{c_i} = g = \frac{(n-1)B - \rho n}{\theta n - 1}. \tag{24}$$

This corresponds to the growth rate of the common capital.

Next, from the assumption of the appropriation strategy of group i, it is easy to verify that the growth rate of appropriation is the same as that of the common capital. Finally, using (21) and (22), we can derive the growth rate of the private capital. After some manipulation, it is verified that the growth rate of the private capital is B in the long run; i.e., $\lim_{t \to \infty} g_{h_i} = B$, where g_{h_i} is the growth rate of the private capital of i. [5]

5 Discussion

In this section, we discuss the balanced growth rates and a welfare comparison of the two equilibria and the comparative statics of the interest equilibrium.

[5] From (21) and (22), we obtain

$$h_i(t) = \frac{(\gamma^* - a^*)(\theta n - 1)}{n(\rho + (\theta - 1)B)} K_0 \left[e^{Bt} - e^{\frac{(n-1)B - \rho n}{\theta n - 1} t} \right].$$

Since $B > \frac{(n-1)B - \rho n}{\theta n - 1}$, for h_i to be non-negative, it requires $\gamma^* \geq a^*$. If $\gamma^* > a^*$, $h_i(t)$ is positive for all $t > 0$ and it is verified that $\lim_{t \to \infty} g_{h_i} = B$. Otherwise, for all t, $h_i(t) = 0$.

5.1 Balanced Growth Rate Comparison

First, to determine whether each group's interest in the opponents' private capital stocks induces more investment in the common capital, we compare the growth rates under both the interest and non-interest equilibria. From the Footnote 5 and (21), we obtain the following:

$$\bar{g} - g = \frac{nB - A}{n-1} - \frac{(n-1)B - \rho n}{\theta n - 1}$$
$$= \frac{n(n-1)\{\rho + (\theta - 1)B\} - (\theta n - 1)(A - B)}{(n-1)(\theta n - 1)}. \tag{25}$$

The effect on investment depends on the sign in the numerator. The numerator is rewritten as follows.

$$\text{the numerator} = n(\theta n - 1)\left\{a^* - \frac{A-B}{n}\right\}.$$

Therefore, if the difference between both technology levels is relatively small, that is if the marginal propensity to consume, a^*, is larger than the term $\frac{A-B}{n}$, each group's interest in the opponents' private capital stocks induces less investment in the common capital. If the difference is relatively large, that is, if $a^* < \frac{A-B}{n}$, the interest induces more investment. We summarize the above results in the following proposition.

Proposition 3. *Each group's interest in the opponents' private capital stocks leads to*

$$\begin{cases} \bar{g} \geq g & \text{if } a^* \geq \frac{A-B}{n} \\ \bar{g} < g & \text{if } a^* < \frac{A-B}{n}. \end{cases}$$

5.2 Welfare Comparison

Next, let us discuss the welfare implication of the interest equilibrium. We define the lifetime utility of group i in the non-interest equilibrium as $\bar{U}_i$ and that in the interest equilibrium as U_i^*. From Tornell and Velasco (1992), the lifetime utility of group i in the non-interest equilibrium is

$$\bar{U}_i = \frac{1}{1-\theta} K_0^{1-\theta} z^{-\theta}, \tag{26}$$

where $z = \frac{\rho+(\theta-1)B}{\theta}$. The lifetime utility of group i in the interest equilibrium is obtained by using (1) and (23).

$$U_i^* = \frac{1}{1-\theta} K_0^{1-\theta} y^{-\theta}, \tag{27}$$

where $y = \frac{(n-1)\{\rho+(\theta-1)B\}}{\theta n-1}$.

Using these lifetime utility functions (25) and (26), we get the following proposition.

Proposition 4. *If each group has an interest in the opponents' private capital stocks, its welfare becomes worse.*

Proof. Subtracting (26) from (25), we compare the welfare in the non-interest equilibrium with that in the interest equilibrium.

$$\bar{U}_i - U_i^* = \frac{1}{1-\theta} K_0^{1-\theta} \left\{ \frac{y^\theta - z^\theta}{(yz)^\theta} \right\}$$

For the sign to be determined, we need to know the sign of the term, $y^\theta - z^\theta$. First, we check the sign, $y - z$.

$$y - z = \frac{\{\rho + (\theta-1)B\}(1-\theta)}{\theta(\theta n - 1)}. \tag{28}$$

Using this equation, we find the following relations. If $\theta > 1$, $y < z$ and thus $y^\theta < z^\theta$. On the other hand, if $1/n < \theta < 1$, $y > z$ and thus $y^\theta > z^\theta$. Therefore, we can confirm that, for all $\theta > 1/n$, $\bar{U}_i$ is always larger than U_i^*. □

Propositions 3 and 4 imply that, when each group can observe and has an interest in its opponents' private capital stocks, its welfare cannot be better off. The reason is that, in symmetric equilibrium, the positive effect of its private capital stock on consumption is offset by the sum of the negative effect of its opponents' private capital stocks.

5.3 Comparative Statics

Finally, we investigate the properties of the balanced growth rate of common capital and the welfare of group i in the interest equilibrium with respect to the technology level of private capital, B, and the number of groups, n. First, we analyze the effects of the changes in B and n on the balanced growth rate of the common capital, g.

Proposition 5. *The equilibrium growth rate of the common capital, g, is increasing with respect to the technology level of private capital, B, and the number of groups, n.*

Proof. Differentiating g with respect to B and n, we can easily get

$$\frac{\partial g}{\partial B} = \frac{n-1}{\theta n - 1} > 0, \text{ and } \frac{\partial g}{\partial n} = \frac{B(\theta - 1) + \rho}{(\theta n - 1)^2} > 0. \tag{29}$$

□

The results of this proposition are the same as those in Tornell and Velasco (1992) and Tornell and Lane (1999). An increase in B leads to all groups having a weaker incentive to extract the common capital, that is a decrease in the appropriation rate, γ^*. This implies that much more common capital stock remains and is used to invest in the common sector, and thus the balanced growth rate of common capital increases. The same can be obtained for an increase in n.[6]

Next, we analyze how the lifetime utility, U_i^*, changes as parameters B and n change. Differentiating (26) with respect to B and n respectively, we get the following result.

Proposition 6. *The lifetime utility of group i, U_i^*, is increasing with respect to the technology level of private capital, B, and the number of groups, n.*

Proof. It is easily confirmed that

$$\frac{\partial U_i^*}{\partial B} = \frac{\chi(n-1)}{\theta n - 1} > 0, \text{and} \frac{\partial U_i^*}{\partial n} = \frac{\chi\{\rho + (\theta - 1)B\}}{(\theta n - 1)^2} > 0, \tag{30}$$

where $\chi = \theta K_0^{1-\theta} y^{-(\theta+1)} > 0$. □

It must be remarked that, in Tornell and Velasco (1992) and Tornell and Lane (1999), the lifetime utility has a positive relation with the technology level of the private sector but there is no effect of the increasing in the

[6] These can be easily demonstrated. Differentiating γ^* with respect to B and n, we get

$$\frac{\partial \gamma^*}{\partial B} = -\frac{n-1}{n(\theta n - 1)} < 0, \quad \text{and} \quad \frac{\partial \gamma^*}{\partial n} = -\frac{A - B}{n^2} - \frac{\theta\{\rho + (\theta - 1)B\}}{(\theta n - 1)^2} < 0.$$

number of groups. [7] This is because, when the number of groups increases, the growth rate of the common capital stock increases but that of the private capital stocks decreases. This opposite action is canceled out, and, as a result, the lifetime utility is not affected. On the other hand, in the case that each group can observe and has interest in the opponents' private capital, since its utility is derived from only the common capital stock the reduction in the growth rate of the private capital stock has no effect on each group's utility.

6 Conclusions

In the present paper, we have analyzed the effect of another consumption strategy by considering the situation in which each interest group can observe and has interest in the opponents' private capital stocks. We have shown that the consumption strategy of each interest group becomes Markov control-state substitutability, that is the capital stock has a negative effect on the consumption. This is a very intuitive result because in the economy each interest group only extracts resources from the common capital stock but cannot invest in the common sector using its own private capital stock. When one group extracts more the common capital stock, the remaining common capital stock becomes small in amount and the other groups must appropriate resources from fewer capital stocks. Therefore, each interest group has a disrelish for the situation in which other groups except for itself extract more and, thus, the opponents' private capital stocks have a negative effect on the consumption of each interest group.

Next, we have shown that the voracity effect is not obtained, which means that the growth rate of the common capital stock does not depend on the technology level of the common sector. This means that since all groups can monitor the private capital stocks of other groups and are homogenous, they know that if one group appropriate more the opponents also appropriate more and that an increase in the appropriation by the opponents decreases the consumption of the group. Therefore, no group will have an incentive to excessively extract its common capital.

Finally, it has been shown that, in the situation that the consumption strategy of each group only depends on the common capital stock and on its own capital stock, its welfare is always higher than it is in the case that the consumption strategy rate depends on all the capital stock. The result

[7] See Equation (25).

implies that, in the linear strategy class, we need not impose the assumption that each interest group can observe and/or has no interest in the opponents' private capital stocks. Even if the interest groups can actually observe the opponents' private capital stocks, they will not take them into account when choosing their consumption strategies.

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Appendix A. The derivation of equations (13) and (14)

This appendix provides a detail derivation of equations (13) and (14). In section 3, we assumed a linear consumption strategy, $\psi_i(K, \vec{h}) = a' + aK + eh_i + bZ_i$, and a linear appropriation strategy $\phi_i(K, \vec{h}) = \gamma K$, where a', a, b, e, and γ are constant.

Using (7), (12), and the consumption strategy, we obtain the following:

$$(a' + aK + eh_i + bZ_i)^{-\theta} = (\xi^{-\frac{1}{\theta}} K + \xi^{-\frac{1}{\theta}} \alpha h_i + \xi^{-\frac{1}{\theta}} \beta Z_i)^{-\theta}.$$

Furthermore, using (8) and (12), we obtain $\alpha = 1$. These lead to

$$a' = 0, \quad a = e = \xi^{-\frac{1}{\theta}}, \quad b = \alpha\beta. \tag{31}$$

Differentiating (12) with respect to respective capital, K, h_i, and h_j, yields the relation,

$$\frac{\partial V_i(K, \vec{h})}{\partial h_j} = \beta \frac{\partial V_i(K, \vec{h})}{\partial K} = \beta \frac{\partial V_i(K, \vec{h})}{\partial h_i} = \beta\xi(K + \alpha h_i + \beta Z_i)^{-\theta}. \tag{32}$$

Substituting Markovian strategies, (32), and (33) into (9)−(11), we obtain

$$(\rho - A + (1-\beta)(n-1)\gamma + a\beta(n-1)) \frac{\partial V_i(\cdot)}{\partial K} = \frac{\partial^2 V_i(\cdot)}{\partial K^2} F(K, \vec{h}), \tag{33}$$

$$(\rho - B + a\beta^2(n-1)) \frac{\partial V_i(\cdot)}{\partial K} = \frac{\partial^2 V_i(\cdot)}{\partial K^2} F(K, \vec{h}), \tag{34}$$

and

$$(\beta\rho - \beta B + a\beta^2(n-2) + a\beta) \frac{\partial V_i(\cdot)}{\partial K} = \beta \frac{\partial^2 V_i(\cdot)}{\partial K^2} F(K, \vec{h}), \tag{35}$$

where

$$\begin{aligned} F(K, \vec{h}) = {} & (A - a - (1-\beta)(n-1)\gamma - a\beta(n-1))K \\ & + (B - a - a\beta^2(n-1))h_i + (\beta B - 2a\beta - a\beta^2(n-2))Z_i. \end{aligned}$$

From (33) and (34) we obtain (14) and, from (33) and (35), we obtain (13).