KIER DISCUSSION PAPER SERIES

KYOTO INSTITUTE
OF
ECONOMIC RESEARCH

Discussion Paper No.843

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January 2013

KYOTO UNIVERSITY

KYOTO, JAPAN

Financial Dependence Analysis: Applications of Vine Copulae

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Abstract

This paper features the application of a novel and recently developed method of statistical and mathematical analysis to the assessment of financial risk: namely Regular Vine copulas. Dependence modelling using copulas is a popular tool in financial applications, but is usually applied to pairs of securities. Vine copulas offer greater flexibility and permit the modelling of complex dependency patterns using the rich variety of bivariate copulas which can be arranged and analysed in a tree structure to facilitate the analysis of multiple dependencies. We apply Regular Vine copula analysis to a sample of stocks comprising the Dow Jones Index to assess their interdependencies and to assess how their correlations change in different economic circumstances using three different sample periods: pre-GFC (Jan 2005- July 2007), GFC (July 2007-Sep 2009), and post-GFC periods (Sep 2009 - Dec 2011). The empirical results suggest that the dependencies change in a complex manner, and there is evidence of greater reliance on the Student t copula in the copula choice within the tree structures for the GFC period, which is consistent with the existence of larger tails in the distributions of returns for this period. One of the attractions of this approach to risk modelling is the flexibility in the choice of distributions used to model co-dependencies.

Keywords: Regular Vine Copulas, Tree structures, Co-dependence modelling.

JEL Codes: G11, C02.

1. Introduction

In the last decade copula modelling has become a frequently used tool in financial economics. Accounts of copula theory are available in Joe (1997) and Nelsen (2006). Hierarchical, copula-based structures have recently been used in some new developments in multivariate modelling; notable among these structures is the pair-copula construction (PCC). Joe (1996) originally proposed the PCC and further exploration of its properties has been undertaken by Bedford and Cooke (2001, 2002) and Kurowicka and Cooke (2006). Aas et al. (2009) provided key inferential insights which have stimulated the use of the PCC in various applications, (see, for example, Schirmacher and Schirmacher (2008), Chollete et al. (2009), Heinen and Valdesogo (2009), Berg and Aas (2009), Min and Czado (2010), and Smith et al. (2010).

There have also been some recent applications of copulas in the context of time series models (see the survey by Patton (2009), and the recently developed COPAR model of Breckmann and Czado (2012), which provides a vector autoregressive VAR model for analysing the non-linear and asymmetric co-dependencies between two series). Nevertheless, in this paper we focus on static modelling of dependencies based on R Vines in the context of modelling the co-dependencies of Dow Jone Index constituents for three different sample periods which include the GFC.

The paper is divided into five sections: the next section provides a review of the background theory and models applied, section 3 introduces the sample, section 4 presents the results and a brief conclusion follows in section 5.

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2. Background and models

Sklar (1959) provides the basic theorem describing the role of copulas for describing dependence in statistics, providing the link between multivariate distribution functions and their univariate margins. The argument proceeds as follows: let F be a d- dimensional distribution function with margins $F_1, ...F_d$. Then there exists a copula C such that, for all $x = (x_1, ..., x_d)' \in (\mathbb{R} \cup \{\infty, -\infty\})^d$,

$$F(x) = C(F_1(x_1), ..., F_d(x_d)). \quad (1)$$

C is unique if $F_1, ..., F_d$ are continuous. Conversely, if C is a copula and $F_1, ..., F_d$ are distribution functions, then the function F defined by (1) is a joint distribution with margins $F_1, ..., F_d$. In particular, C can be interpreted as the distribution function of a d-dimensional random variable on $[0, 1]^d$.

We can speak generally of the copula of continuous random variables $X = (X_1, ...X_d) \sim F$. The problem in practical applications is the identification of the appropriate copula.

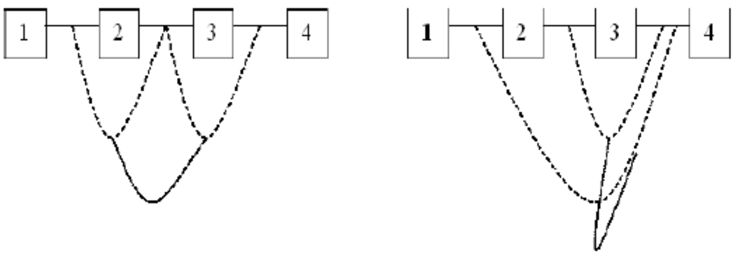
Standard multivariate copulas, such as the multivariate Gaussian or Student-t, as well as exchangeable Archimedean copulas, lack the exibility of accurately modelling the dependence among larger numbers of variables. Generalizations of these offer some improvement, but typically become rather intricate in their structure, and hence exhibit other limitations such as parameter restrictions. Vine copulas do not suffer from any of these problems.

Initially proposed by Joe (1996) and developed in greater detail in Bedford and Cooke (2001, 2002) and in Kurowicka and Cooke (2006), vines are a flexible graphical model for describing multivariate copulas built up using a cascade of bivariate copulas, so-called pair-copulas. Their statistical breakthrough was due to Aas, Czado, Frigessi, and Bakken (2009) who described statistical inference techniques for the two classes of canonical C-vines and D-vines. These belong to a general class of Regular Vines, or R-vines which can be depicted in a graphical theoretic model to determine which pairs are included in a pair-copula decomposition. Therefore a vine is a graphical tool for labelling constraints in high-dimensional distributions.

A regular vine is a special case for which all constraints are two-dimensional or conditional two-dimensional. Regular vines generalize trees, and are themselves specializations of Cantor trees. Combined with copulas, regular vines have proven to be a flexible tool in high-dimensional dependence modelling. Copulas are multivariate distributions with uniform univariate margins. Representing a joint distribution as univariate margins plus copulas allows the separation of the problems of estimating univariate distributions from problems of estimating dependence.

Figure 1 provides an example of two different vine structures, with a regular vine on the left and a non-regular vine on the right, both for four variables.

Figure 1: Vines



A vine V on n variables is a nested set of connected trees $V = \{T_1, ..., T_{n-1}\}$, where the edges of tree j are the nodes of tree $j + 1$, $j = 1, ..., n - 2$. A regular vine on n variables is a vine in which two edges in tree j are joined by an edge in tree $j = 1$ only if these edges share a common node, $j = 1, ..., n - 2$. Kurowicka and Cook (2003) provide the following definition of a Regular vine.

Definition 1. (Regular vine)

V is a regular vine on n elements with $E(V) = E_1 \cup ..... \cup E_{n-1}$ denoting the set of edges of V if

1. $V = \{T_1, ..., T_{n-1}\}$,
2. $T_1$is a connected tree with nodes $N_1 = \{1, ..., n\}$, plus edges E_1; for $i = 2, ..., n-1$,$T_i$is a tree with nodes $N_i = E_{i-1}$,

3. (proximity) for $i = 2, ..., n-1$, $\{a, b\} \in E_i \#(a \triangle b = 20$, where $\triangle$ denotes the symmetric difference operator and $\#$ denotes the cardinality of a set.

An edge in a tree T_j is an unordered pair of nodes of T_j or equivalently, an unordered pair of edges of T_{j-1}. By definition, the order of an edge in tree T_j is $j-1$, $j = 1, ..., n-1$. The degree of a node is determined by the number of edges attached to that node. A regular vine is called a *canonical vine*, or $C-$vine, if each tree T_i has a unique node of degree $n-1$ and therefore, has the maximum degree. A regular vine is termed a $D-$vine if all the nodes in T_1 have degrees no higher than 2.

Definition 2. (The following definition is taken from Cook et al. (2011)). For $e \in E_i$, $i \leq n-1$, the constraint set associated with e is the complete union of U_e^* of e, which is the subset of $\{1, ..., n\}$ reachable from e by the membership relation.

For $i = 1, ..., n-1$, $e \in E_i$, if $e = \{j, k\}$, then the conditioning set associated with e is

$$D_e = U_j^* \cap U_k^*$$

and the conditioned set associated with e is

$$\{C_{e,j}, C_{e,k}\} = \{U_j^* \setminus D_e, U_k^* \setminus D_e\}.$$

Figure 2 below shows a D-Vine with 5 dimensions.

Figure 2: D-Vine 5 Dimensions

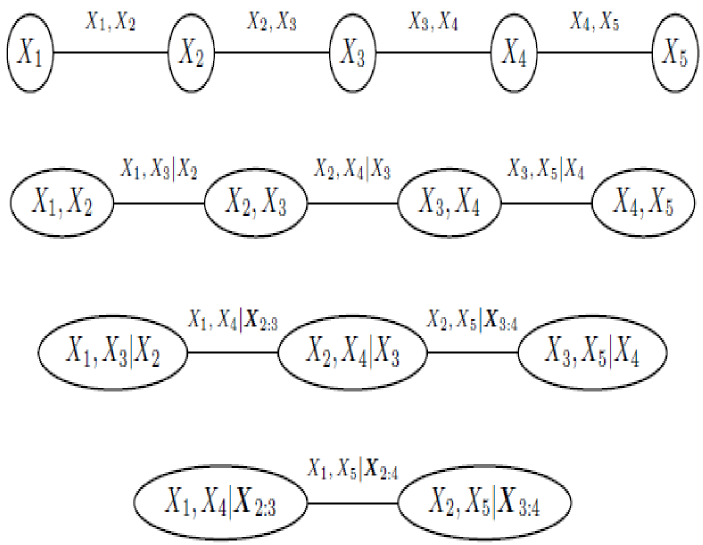


Figure 3 shows an R-Vine on 4 variables, and is sourced from Dissman (2010). The node names appear in the circles in the trees and the edge names appear below the edges in the trees. Given that an edge is a set of two nodes, an edge in the third tree is a set of a set. The proximity condition can be seen in tree T_2, where the first edge connects the nodes $\{1, 2\}$and $\{2, 3\}$, and both share node 2 in tree T_1.

2.1. Modelling Vines

Vine structures are developed from pair-copula constructions, in which $d(d-1)/2$ pair-copulas are arranged in $d-1$ trees (in the form of connected acyclic graphs with nodes and edges). At the start of the first C-vine tree, the first root node models the dependence with respect to one particular variable, using bivariate copulas for each pair. Conditioned on this variable, pairwise dependencies with respect to a second variable are modelled, the second root node. The tree is thus expanded in this manner; a root node is chosen for each tree and all pairwise dependencies with respect to this node are modelled conditioned on all previous root nodes. It follows that C-vine trees have a star structure. Brechmann and Schepsmeier (2012) use the following decomposition in their account of the routines incorporated in the R Library CDVine, which was used for the empirical work in this paper. The multivariate density, the $C\,Vine\ density\ w.l.o.g.\ root\,nodes\, 1, ..., d$,

$$f(x)=\prod_{k=1}^{d} f_k(x_k)\times\prod_{i=1}^{d-1}\prod_{j=1}^{d-i} c_{i,i+j|1:(i-1)}(F(x_i\mid x_1,....,x_{i-1}),\, F(x_{i+j}\mid x_1,.....,x_{i-1})\mid \theta_{i,+j|1:(i-1)}) \tag{2}$$

where f_k, $k=1,.....,d$, denote the marginal densities and $c_{i,i+j|1:(i-1)}$bivariate copula densities with parameter(s)$\theta_{i,i+j|1:(i-1)}$ (in general, $i_k : i_m\, means\, i_k,....,i_m$). The outer product runs over the $d-1$ trees and root nodes i, while the inner product refers to the $d-i$ pair copulas in each tree $i=1,....,d-1$.

D-Vines follow a similar process of construction by choosing a specific order for the variables. The first tree models the dependence of the first and second variables, of the second and third, and so on,... using pair copulas. If we assume the order is $1,...,d$, then first the pairs (1,2), (2,3), (3,4) are modelled. In the second tree, the co-dependence analysis can proceed by modelling the conditional dependence of the first and the third variables, given the second variable; the pair $(2,4\mid 3)$, and so forth. This process can then be continued in the next tree, in which variables can be conditioned on those lying between entries $a\, and\, b$ in the first tree, for example, the pair $(1,5\mid 2,3,4)$. The D-Vine tree has a path structure which leads to the construction of the $D-vine$ density, which can be constructed as follows:

$$f(x)=\prod_{k=1}^{d} f_k(x_k)\times\prod_{i=1}^{d-1}\prod_{j=1}^{d-i} c_{j,j+i|(j+1):(j+i-1)}(F(x_j\mid x_{j+1},.....,x_{j+i-1}),\, F(x_{j+i}\mid x_{j+1},....,\, x_{j+i-1})\mid \theta_{j,j+i|(j+1):(j+i-1)}) \tag{3}$$

The outer product runs over $d-1$ trees, while the pairs in each tree are determined according to the inner product. The conditional distribution functions $F(x\mid\nu)$ can be obtained for an $m-dimensional$ vector ν. This can be done in a pair copula term in tree $m-1$, by using the pair-copulas of the previous trees $1,....,\, m$, and by sequentially applying the following relationship:

$$h(x\mid\nu,\,\theta):=F(x\mid\nu)=\frac{\partial C_{x\nu_j|\nu_{-j}}(F(x\mid\nu_{-j}),\,F(\nu_j\mid\nu_{-j})\mid\theta)}{\partial F(\nu_j\mid\nu_{-j})} \tag{4}$$

where ν_j is an arbitrary component of ν, and ν_{-j} denotes the $(m-1)$- dimensional vector ν excluding ν_j. The bivariate copula function is specified by $C_{x\nu_j|\mathbf{v}-j}$ with parameters θ specified in tree m.

The model of dependency can be constructed in a very flexible way because a variety of pair copula terms can be fitted between the various pairs of variables. In this manner, asymmetric dependence or strong tail behaviour can be accommodated. Figure 3 shows the various copulae available in the CDVine library in R.

Figure 3: Notation and Properties of Bivariate Elliptical and Archimedean Copula Families included in CDVine

No.	Elliptical distribution	Parameter range	Kendall's τ	Tail dependence
1	Gaussian	$\rho\in(-1,1)$	$\frac{2}{\pi}\arcsin(\rho)$	0
2	Student-t	$\rho\in(-1,1),\ \nu>2$	$\frac{2}{\pi}\arcsin(\rho)$	$2t_{\nu+1}\left(-\sqrt{\nu+1}\sqrt{\frac{1-\rho}{1+\rho}}\right)$

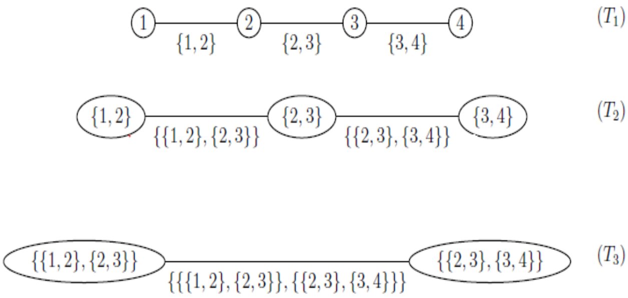
No.	Name	Generator function	Parameter range	Kendall's τ	Tail dependence (lower, upper)
3	Clayton	$\frac{1}{\theta}(t^{-\theta}-1)$	$\theta>0$	$\frac{\theta}{\theta+2}$	$(2^{-1/\theta},\ 0)$
4	Gumbel	$(-\log t)^{\theta}$	$\theta\geq 1$	$1-\frac{1}{\theta}$	$(0,\ 2-2^{1/\theta})$
5	Frank[a]	$-\log[\frac{e^{-\theta t}-1}{e^{-\theta}-1}]$	$\theta\in\mathbb{R}\setminus\{0\}$	$1-\frac{4}{\theta}+4\frac{D_1(\theta)}{\theta}$	$(0,\ 0)$
6	Joe	$-\log[1-(1-t)^{\theta}]$	$\theta>1$	$1+\frac{4}{\theta^2}\int_0^1 t\log(t)(1-t)^{\frac{2(1-\theta)}{\theta}}dt$	$(0,\ 2-2^{1/\theta})$
7	BB1	$(t^{-\theta}-1)^{\delta}$	$\theta>0,\ \delta\geq 1$	$1-\frac{2}{\delta(\theta+2)}$	$(2^{-1/(\theta\delta)},\ 2-2^{1/\delta})$
8	BB6	$(-\log[1-(1-t)^{\theta}])^{\delta}$	$\theta\geq 1, \delta\geq 1$	$1+4\int_0^1\big(-\log(-(1-t)^{\theta}+1)\times\frac{(1-t-(1-t)^{-\theta}+t(1-t)^{-\theta})}{\delta\theta}\big)dt$	$(0, 2-2^{1/(\theta\delta)})$
9	BB7[b]	$[1-(1-t)^{\theta})]^{-\delta}-1$	$\theta\geq 1,\ \delta>0$	$1-\frac{2}{\delta(2-\theta)}+\frac{4}{\theta^2\delta}B(\frac{2-\theta}{\theta},\delta+2)$	$(2^{-1/\delta},\ 2-2^{1/\theta})$
10	BB8	$-\log\left[\frac{1-(1-\delta t)^{\theta}}{1-(1-\delta)^{\theta}}\right]$	$\theta\geq 1, 0<\delta\leq 1$	$1+4\int_0^1\big(-\log\left(\frac{(1-t\delta)^{\theta}-1}{(1-\delta)^{\theta}-1}\right)\times\frac{1-t\delta-(1-t\delta)^{-\theta}+t\delta(1-t\delta)^{-\theta}}{\theta\delta}\big)dt$	$(0,0^{c})$

2.2. Regular vines

Until recently, the focus had been on modelling using C and D vines. However, Dissmann (2010) has pointed the direction for constructing regular vines using graph theoretical algorithms. This interest in pair-copula constructions/regular vines is doubtlessly linked to their high flexibility as they can model a wide range of complex dependencies.

Figure 4 shows an R-Vine on 4 variables, and is sourced from Dissman (2010). The node names appear in the circles in the trees and the edge names appear below the edges in the trees. Given that an edge is a set of two nodes, an edge in the third tree is a set of a set. The proximity condition can be seen in tree T_2, where the first edge connects the nodes $\{1,2\}$and $\{2,3\}$, plus both share the node 2 in tree T_1.

Figure 4: Example of R-Vine on 4 Variables. (Source Dissman (2010))



The drawback is the curse of dimensionality: the computational effort required to estimate all parameters grows exponentially with the dimension. Morales-Nápoles et al (2009) demonstrate that there are $\frac{n!}{2} \times 2^{\binom{n-2}{2}}$ possible R-Vines on n nodes. The key to the problem is whether the regular vine can be either truncated or simplified. Brechmann et al, (p2, 2012) discuss such simplification methods. They explain that: "by a pairwisely truncated regular vine at level K, we mean a regular vine where all pair-copulas with conditioning set equal to or larger than K are replaced by independence copulas". They pairwise simplify a regular vine at level K by replacing the same pair-copulas with Gaussian copulas. Gaussian copulas mean a simplification since they are easier to specify than other copulas, easy to interpret in terms of the correlation parameter, and quicker to estimate.

They identify the most appropriate truncation/simplification level by means of statistical model selection methods; specifically, the AIC, BIC and the likelihood-ratio based test proposed by Vuong (1989). For R-vines, in general, there are no expressions like equations (2) and (3). This means that an efficient method for storing the indices of the pair copulas required in the joint density function, as depicted in equation (5), is required; (5) is a more general case of (2) and (3).

$$f(x_1, ..., x_d) = \left[\prod_{k=1}^{d} f_x(x_k)\right] \times \left[\prod_{i=1}^{d-1} \prod_{e \in E_i} c_{j(e),k(e)|D(e)}(F(x_{j(e)} \mid \mathbf{x}_{D(e)}), F(x_{k(e)} \mid \mathbf{x}_{D(e)}))\right] \tag{5}$$

Kurowicka (2011) and Dissman (2010) have recently suggested a method of proceeding which involves specifying a lower triangular matrix $M = (m_{i,j} \mid i, j = 1, ..., d) \in \{0, ..., d\}^{d \times d}$, with $m_{i,i} = d - i + 1$. This means that the diagonal entries of M are the numbers $1, ..., d$ in descending order. In this matrix, each row proceeding from the bottom represents a tree, the diagonal entry represents the conditioned set and by the corresponding column entry of the row under consideration. The conditioning set is given by the column entries below this row. The corresponding parameters and types of copula can be stored in matrices relating to M. The following example in Figure 5 is taken from Dissman (2010).

Figure 5: Matrix Mapping of vine copulas (source Dissman (2010))

ID	Copula
1	Gaussian
2	t
3	Clayton
4	Gumbel
5	Frank

We define

$$T_1 = \begin{pmatrix} 1 & & & & & \\ 5 & 5 & & & & \\ 4 & 4 & 4 & & & \\ 3 & 3 & 3 & 3 & & \\ 2 & 2 & 2 & 2 & 2 & \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix},$$

$$P_1^1 = \begin{pmatrix} 0.0 & & & & & \\ -1.3 & 0.9 & & & & \\ 1.5 & 1.3 & 4.6 & & & \\ 1.5 & 0.5 & 0.2 & 1.3 & & \\ -0.3 & 0.9 & 0.2 & 0.4 & 0.2 & \\ 0 & 0.1 & -0.6 & 0.3 & -0.9 & 0.3 \end{pmatrix}, \text{ and}$$

$$P_1^2 = \begin{pmatrix} - & & & & & \\ - & - & & & & \\ - & - & - & & & \\ - & - & - & - & & \\ 10 & 4 & 6 & 12 & 4 & \\ - & - & - & - & - & - \end{pmatrix}$$

The first section of Figure 5 provides a key to indicate the 5 different types of copulas used in this example, ranging from Gaussian (1) to Frank (5). The second lower triangular matrix T_1 shows the application of particular types of copulas in the trees, P_1^1shows the parameters estimated, and P_1^2provides the extra parameters needed when we apply the t copula.

Figure 6: Use of Matrices to Store R-vine Information (source: Dissman (2010))

$$M_1 = \begin{matrix} 4 & & & & & & \\ 7 & 5 & & & & & \\ 6 & 7 & 1 & & & & \\ 5 & 6 & 7 & 7 & & & \\ 1 & 1 & 6 & 2 & 6 & & \\ 2 & 3 & 3 & 3 & 2 & 2 & \\ 3 & 2 & 2 & 6 & 3 & 3 & 3 \end{matrix} \qquad T_1 = \begin{matrix} 1 & & & & & \\ 5 & 5 & & & & \\ 4 & 4 & 4 & & & \\ 3 & 3 & 3 & 3 & & \\ 2 & 2 & 2 & 2 & 2 & \\ 1 & 1 & 1 & 1 & 1 & 1 \end{matrix} \qquad P_1^1 = \begin{matrix} 0.0 & & & & & \\ -1.3 & 0.9 & & & & \\ 1.5 & 1.3 & 4.6 & & & \\ 1.5 & 0.5 & 0.2 & 1.3 & & \\ -0.3 & 0.9 & 0.2 & 0.4 & 0.2 & \\ 0 & 0.1 & -0.6 & 0.3 & -0.9 & 0.3 \end{matrix}$$

: The edge e_1 with $C_{e_1} = \{4,5\}$ and $D_{e_1} = \{1,2,3\}$ has a copula of type $t_{4,1} = 4$ = Gumbel and the parameter $p^1_{4,1} = 1.5$. The edge e_2 with $C_{e_2} = \{7,6\}$ and $D_{e_2} = \{\}$ has a copula of type $t_{7,4} = 1$ = Gauss and the parameter $p^1_{7,4} = 0.3$.

In Figure 6 the bottom row of M_1corresponds to T_1, the second row to T_2, and so on. In order to determine the edges in T_1, we combine the numbers in the bottom row with the diagonal elements in the corresponding columns, for example the edges are (4,3), (5,2), (1,2) and so on. In order to determine the edges in T_2, we combine the numbers in the second row from the bottom with the diagonal elements in the corresponding columns and condition on the elements in the bottom row. This would give edges $(4,2 \mid 3)$, $(5,3 \mid 2)$, $(1,3 \mid 2)$, and so on The final entry is given by the upper entries to the left of the matrix $(4,7 \mid 65123)$.

2.3. Prior work with R-Vines

The literature was initially mainly concerned with illustrative examples, (see, for example, Aas et al. (2009), Berg and Aas (2009), Min and Czado (2010) and Czado et al. (2011)). Mendes et al. (2010) use a D-Vine copula model to a six-dimensional data set and consider its use for portfolio management. Dissman (2010) uses R-Vines to analyse dependencies between 16 financial indices covering different European regions and different asset classes, including five equity, nine fixed income (bonds), and two commodity indices. He assesses the relative effectiveness of the use of copulas, based on mixed distributions, t distributions and Gaussian distributions, and explores the loss of information from truncating the R-Vine at earlier stages of the analysis and the substitution of independence copula. He also analyses exchange rates and windspeed data sets with fewer variables.

There have been other studies on European stock return series: Heinen and Valdesogo (2009) constructed a CAPM extension using their *Canonical Vine Autoregressive* (CAVA) model using marginal GARCH models and a canonical vine copula structure. Breckmann and Czado (2011) develop a regular vine market sector factor model for asset returns that uses GARCH models for margins, and which is similarly developed in a CAPM framework. They explore systematic and unsystematic risk for individual stocks, and consider how vine copula models can be used for active and passive portfolio management and VaR forecasting.

3. Sample

We use a data set of daily returns, which runs from 1 January 2005 to 31 January 2011 for the DOW Jones Index and its component 30 stocks. We divide our sample into returns for the pre-GFC (Jan 2005-July 2007), GFC (July 2007-Sep 2009) and post-GFC (Sep 2009 - Dec 2011) periods. The sample for the three periods is shown in Table 1. We analyse the behaviour of the stocks that remain constituents of the DOW Jones index throughout the three periods. Not all Dow Jones stocks are included in each period.

Table 1: Dow Jones Stocks used in Each Period

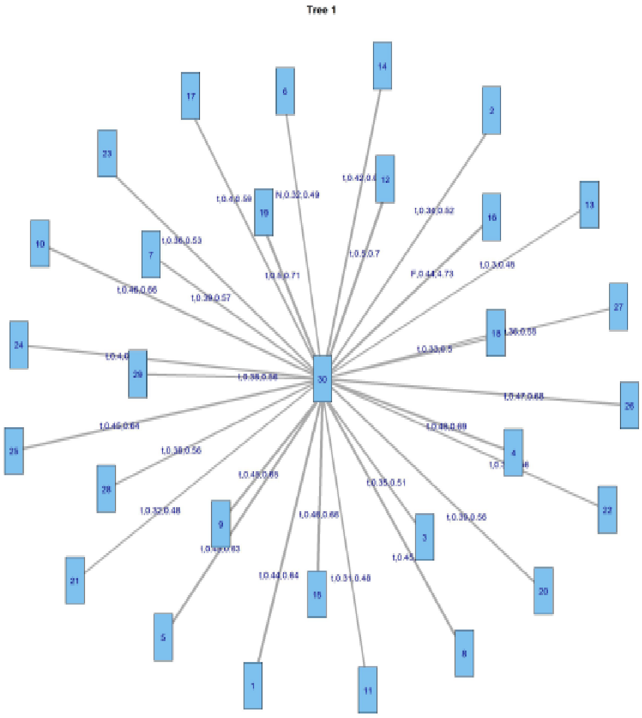
	Pre-GFC	GFC	Post-GFC
V1	3M	3M	3M
V2	ALCOA	ALCOA	ALCOA
V3	ALTRIA GRP	AMERICAN EXPRESS	AMERICAN EXPRESS
V4	AMERICAN EXPRESS	AT&T	AT&T
V5	AMERICAN INTL GRP	BOEING	BANK OF AMERICA
V6	AT&T	CATERPILLAR	BOEING
V7	BOEING	E I DU PONT DE NEMOURS	CATERPILLAR
V8	CATERPILLAR	EXXON MOBIL	CHEVRON
V9	CITIBANK	GENERAL ELECTRIC	CISCO SYSTEMS
V10	E I DU PONT DE NEMOURS	HEWLETT-PACKARD	E I DU PONT DE NEMOURS
V11	EXXON MOBIL	HOME DEPOT	EXXON MOBIL
V12	GENERAL ELECTRIC	INTEL	GENERAL ELECTRIC
V13	HEWLETT-PACKARD	INTERNATIONAL BUS.MCHS.	HEWLETT-PACKARD
V14	HOME DEPOT	JOHNSON & JOHNSON	HOME DEPOT
V15	HONEYWELL	JP MORGAN CHASE & CO.	INTEL
V16	INTEL	MCDONALDS	INTERNATIONAL BUS.MCHS.
V17	INTERNATIONAL BUS.MCHS.	MERCK & CO.	JOHNSON & JOHNSON
V18	JOHNSON & JOHNSON	MICROSOFT	JP MORGAN CHASE & CO.
V19	JP MORGAN CHASE & CO.	PFIZER	KRAFT FOODS
V20	MCDONALDS	PROCTER & GAMBLE	MCDONALDS
V21	MERCK & CO.	COCA COLA	MERCK & CO.
V22	MICROSOFT	UNITED TECHNOLOGIES	MICROSOFT
V23	PFIZER	VERIZON COMMUNICATIONS	PFIZER
V24	PROCTER & GAMBLE	WAL MART STORES	PROCTER & GAMBLE
V25	COCA COLA	WALT DISNEY	COCA COLA
V26	UNITED TECHNOLOGIES	DOW JONES	TRAVELERS COS.
V27	VERIZON COMMUNICATIONS		UNITED TECHNOLOGIES
V28	WAL MART STORES		VERIZON COMMUNICATIONS
V29	WALT DISNEY		WAL MART STORES
V30	DOW JONES		WALT DISNEY
V31			DOW JONES

4. Results

We divide the data into three time periods covering the pre-GFC (Jan 2005- July 2007), GFC (July 2007-Sep 2009), and post-GFC periods (Sep 2009 - Dec 2011) to run the C-Vine and R-Vine dependence analysis in the stocks of Dow Jones Index. Before we can do this we require appropriately standardised marginal distributions for the basic company return series. Appropriate marginal time series models for the Dow Jones data have to be found in the first step of our two step estimation approach. The following time series models are selected in a stepwise procedure: GARCH (1,1), ARMA (1,1), AR(1), GARCH(1,1), MA(1)-GARCH(1,1). These are applied to the return data series and we select the model with the highest p-value, so that the residuals can be taken to be i.i.d. The residuals are standardized and the marginals are obtained from the standardized residuals using the Ranks method. These marginals are then used as inputs to the Copula selection routine. The copula are selected using the AIC criterion. We first discuss the results obtained from the pre-GFC period data followed by the GFC and post-GFC periods.

The following figure presents the structure of the C-Vines.

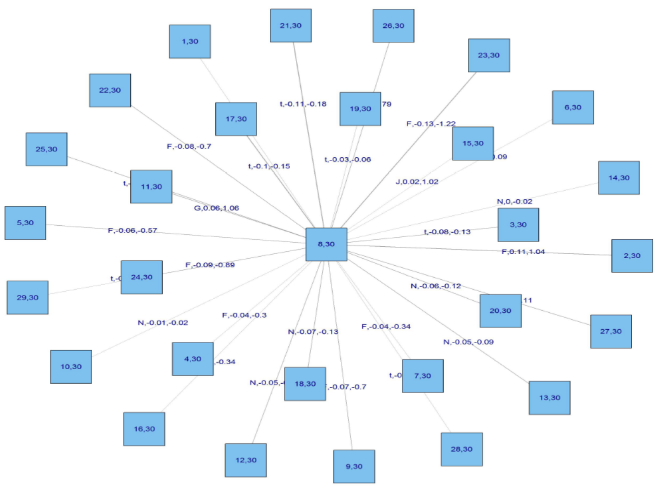
Figure 7: Results-C-Vine Tree-1 Pre-GFC



For this C Vine selection, we choose as root node the node that maximizes the sum of pairwise dependencies to this node.We commence by linking all the stocks to the DowJones 30 index which is at the centre of this diagram. We use a range of Copulas from which it is selected, the range being (1:6). We apply AIC as the selection criterion to select from the following menu of copulae: 1 = Gaussian copula, 2 = Student t copula (t-copula), 3 = Clayton copula, 4 = Gumbel copula, 5 = Frank copula, 6 = Joe copula.

We then compute transformed observations from the estimated pair copulas and these are used as input parameters for the next trees, which are obtained similarly by constructing a graph according to the above C-Vine construction principles (proximity conditions), and finding a maximum dependence tree. The C-Vine tree for period 2 is shown below.

Figure 8: C-Vine Tree 2 Pre-GFC



The pre-GFC C-Vine copula specification matrix is displayed in Table 2 below.

Table 2: Pre-GFC C-Vine Copula Specification Matrix

24																													
29	19																												
19	29	25																											
25	25	29	10																										
10	10	10	29	22																									
22	22	22	22	29	21																								
21	21	21	21	21	29	14																							
14	14	14	14	14	14	29	20																						
20	20	20	20	20	20	20	29	6																					
6	6	6	6	6	6	6	6	29	2																				
2	2	2	2	2	2	2	2	2	29	15																			
15	15	15	15	15	15	15	15	15	15	29	16																		
16	16	16	16	16	16	16	16	16	16	16	29	4																	
4	4	4	4	4	4	4	4	4	4	4	4	29	13																
13	13	13	13	13	13	13	13	13	13	13	13	13	29	26															
26	26	26	26	26	26	26	26	26	26	26	26	26	26	29	12														
12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	29	5													
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	29	28												
28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	29	17											
17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	29	18										
18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	29	1									
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	29	3								
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	29	27							
27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	29	23						
23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	29	11					
11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	29	7				
7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	29	9			
9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	29	8		
8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	29	30	
30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	29	29

From Table 2, it can be seen that the strongest individual correlations in the pre-GFC period, are with the Dow Jones Index, security 30 in the final row, and the individual diagonal entries starting with security 24 at the top of the first column, representing Proctor and Gamble, which define the edges. Proctor and Gamble (security 24) is correlated with Caterpillar (security 8), then conditioned by its relationship with Citibank (security 9), then Boeing (security 7), Exxon mobil (security 11), and so on. It can also be seen in Table 2 that C Vines are less flexible in that the same security number can always be seen to appear across the rows. This means that it is always appearing in the nodes at that level in the tree. R Vines are more flexible and do not have this requirement. Henceforth, we will concentrate on the results of the R Vine analysis.

Figure 9: R Vine, Trees 1 and 2 , Pre-GFC Period.

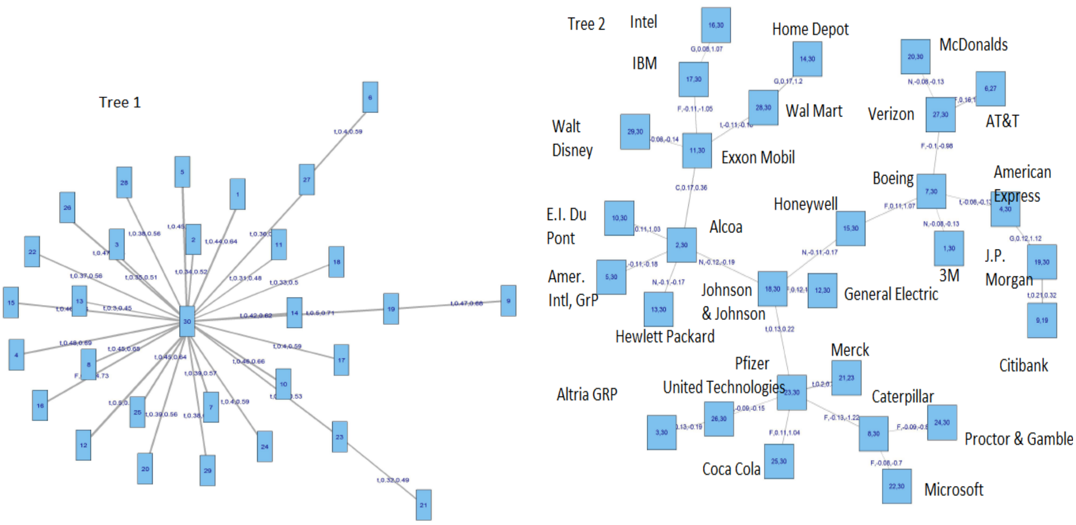


Table 3: R Vine Pre-GFC Copula Specification Matrix

29																													
3	9																												
26	3	3																											
14	26	19	26																										
22	14	20	19	14																									
24	22	14	20	19	22																								
21	24	6	14	20	19	24																							
8	21	27	6	22	20	19	21																						
9	8	1	27	24	6	20	19	19																					
19	16	4	1	21	27	6	20	8	20																				
16	17	7	4	8	1	27	6	16	8	6																			
17	28	15	7	6	4	1	27	17	16	8	27																		
28	11	16	15	27	7	4	1	28	17	16	8	1																	
20	10	22	16	1	15	7	4	11	28	17	16	8	8																
6	5	24	22	4	16	15	7	10	11	28	17	16	4	16															
27	13	17	24	7	17	16	15	5	10	11	28	17	7	4	17														
1	2	28	17	15	28	17	16	13	5	10	11	28	15	7	4	28													
4	25	11	28	25	11	28	17	2	13	5	10	11	16	15	7	4	11												
7	23	10	11	23	10	11	28	25	2	13	5	10	17	25	15	7	4	10											
15	12	5	10	12	5	10	11	23	25	2	13	5	28	23	25	15	7	4	5										
25	18	13	5	18	13	5	10	12	23	25	2	13	11	12	23	25	15	7	4	13									
23	15	2	13	13	2	13	5	18	12	23	25	2	10	18	12	23	25	15	7	4	2								
12	20	12	2	5	21	2	13	15	18	12	23	25	5	13	18	12	23	25	15	7	4	4							
18	6	18	12	10	12	21	2	20	15	18	12	23	13	5	13	18	12	23	25	15	7	25	7						
13	27	21	18	2	18	12	12	6	4	15	18	12	2	10	5	13	18	12	23	25	15	23	25	15					
5	1	25	21	16	25	18	18	27	1	4	15	18	12	2	10	5	13	18	12	23	25	12	23	25	12				
10	7	8	25	17	23	25	25	1	7	1	4	15	18	28	2	10	5	13	18	12	23	18	12	23	25	18			
2	4	23	8	11	24	23	8	7	6	7	1	4	25	11	28	2	10	5	13	18	12	15	18	12	23	25	23		
11	30	26	23	28	8	8	30	4	27	30	7	7	23	17	11	11	2	2	2	2	18	7	15	18	18	23	25	30	
30	19	30	30	30	30	30	23	30	30	27	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	25	25

It can be seen in Figure 9 and in Table 2 above that the R Vine structure is more flexible. The same company numbers no longer appear across the rows. By and large, the greatest dependency is between the individual constituents of the Dow Jones Index and the Dow Jones itself. This is evident in the fact that the majority of the entries in the last row are for security number 30 which represents Dow Jones in the Pre-GFC period. However, at the bottom of the second column security 19, representing J.P. Morgan Chase appears, while at the top of this column sits security number 9, Citibank. Hence, the greatest dependency for this pair of financial securities was between themselves, and this out-weighed their dependency on the index. Similarly, in column 8, headed by security 21, Merck and Co, the entry at the bottom of the column is for security 23 Pfizer, so, in the Pre-GFC period, this pair of securities showed relatively high joint dependency. At the top of column 11 sits security 6, AT&T, while at the bottom is security 26, Verizon communications.

Once these initial nodes and edges have been taken into acount, the second row is conditioned on their dependency relationship, and is the next branch in the tree. In the first entry at the top of the first column, we have security 29, Walt Disney, but when its relationship with the Dow Jones is taken into account, the penultimate entry in that column is security 11, Exxon Mobil, indicating, quite surprisingly, that there is a co-dependency relationship between these two pairs of securities, after conditioning on the Index. Thus, an intricate picture of co-dependencies can be created.

Table 4 shows the types of copulas fitted in the empirical analysis.

Table 4: Pre-GFC Period Types of Copulas Fitted

2																												
2	6																											
5	1	5																										
5	5	1	5																									
1	5	1	2	1																								
1	2	1	2	1	5																							
1	5	1	1	5	1	2																						
3	5	1	1	2	1	5	1																					
1	5	2	2	2	5	5	5	1																				
1	5	2	1	1	1	5	2	1	1																			
3	5	1	6	1	5	1	6	1	1	1																		
6	5	2	2	1	1	1	1	2	5	5	2																	
3	5	1	1	1	1	1	1	1	2	1	3	1																
1	3	5	5	1	4	1	2	6	2	5	2	1	5															
4	5	1	5	5	3	2	5	4	4	1	2	2	2	5														
1	5	1	1	5	1	1	1	1	1	2	1	1	1	1	1													
3	5	1	1	2	1	1	5	5	2	1	5	1	5	2	2	2												
2	5	1	1	5	2	5	5	5	1	1	5	3	2	4	2	2	1											
5	6	5	2	3	1	3	2	2	5	4	2	1	1	2	4	2	1	5										
3	1	1	1	1	2	6	5	1	5	6	2	2	6	1	2	5	3	2	5									
6	1	5	1	1	2	1	5	2	5	1	5	6	1	2	3	6	2	5	1	1								
5	5	2	3	5	6	1	1	2	2	1	5	2	5	1	5	5	1	5	5	1	1							
1	1	3	5	1	3	3	1	3	2	6	5	5	1	5	3	5	2	6	1	3	5	2						
2	3	6	2	1	1	5	5	3	2	2	2	5	5	5	1	1	1	1	1	1	2	2	1					
2	5	3	1	1	3	4	4	5	5	5	2	1	1	1	1	1	1	3	5	2	1	3	1	2				
1	1	2	5	3	2	5	2	2	5	2	5	1	1	6	1	1	1	1	5	5	5	2	5	2	1			
2	5	1	5	1	3	4	2	1	5	5	2	1	2	1	5	1	5	1	5	1	2	5	1	5	5	5		
1	2	2	2	4	5	5	2	4	1	5	5	1	5	4	5	2	3	5	1	1	1	2	5	1	5	2	5	
2	2	2	2	2	2	2	2	2	2	2	2	2	2	5	2	2	2	2	2	2	2	2	2	2	2	2	2	2

The advantage of the use of R Vines is apparent in Table 4. Complex patterns of dependency can be readily captured. It can be seen that at different dependencies conditioned across the same node six different copulas are used. For example, in column 1 the first copula used is the Student t copula (no 2), followed by the Frank copula (no 5) for a couple of levels, then the Gaussian (no 1), Clayton (no 3), Joe (no 6) and further down the column the Gumbel (no 4) makes one appearance. This variety of usage is apparent across Table 4 at various levels in the Tree structures used to capture dependencies.

The actual parameters estimated for the Pre-GFC period are shown in Table 5.

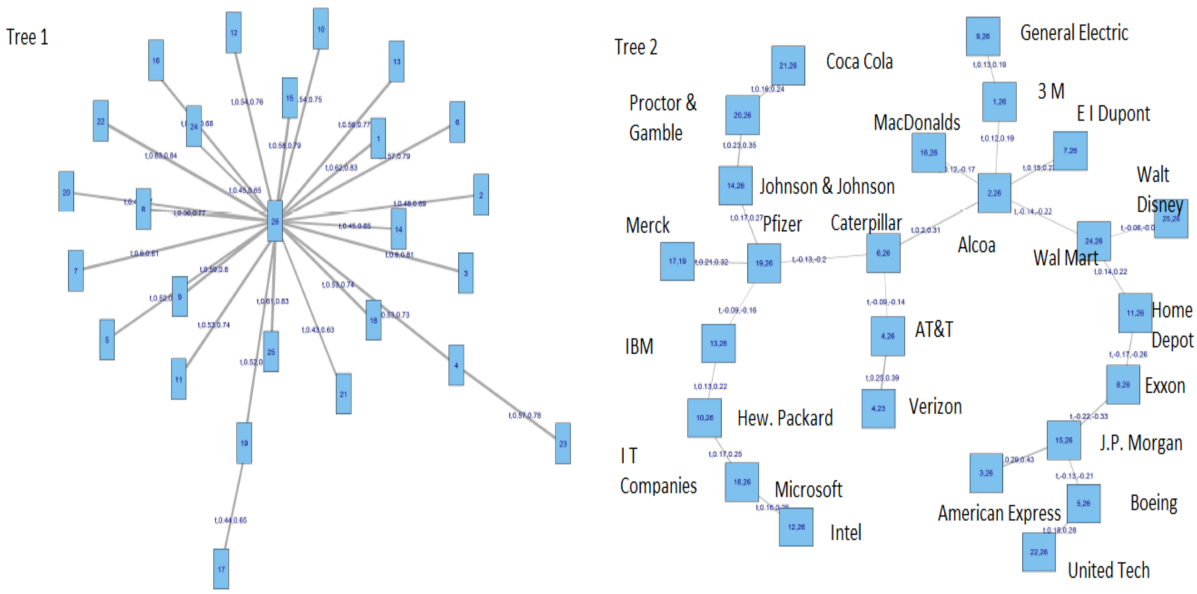
Table 5: Pre-GFC Copula Parameter Estimates

-0.02																												
-0.09	1.01																											
-0.15	-0.07	-0.25																										
0.33	0.16	-0.11	0.58																									
-0.04	0.36	-0.11	-0.02	0.01																								
-0.05	-0.03	-0.05	-0.04	-0.03	0.23																							
-0.03	-0.67	-0.12	-0.08	-0.36	-0.07	-0.05																						
0.06	-0.66	-0.05	-0.06	-0.06	-0.05	-0.24	-0.08																					
-0.02	-0.23	-0.10	0.00	-0.04	0.18	0.25	-0.49	-0.10																				
-0.03	-0.29	-0.12	-0.14	-0.03	-0.12	0.26	0.04	-0.02	-0.15																			
0.05	0.17	-0.12	1.02	0.13	0.23	-0.12	1.03	-0.06	-0.07	-0.07																		
1.03	-0.28	-0.04	0.11	-0.04	-0.03	-0.09	-0.18	-0.06	0.13	-0.06	-0.10																	
0.09	0.08	-0.06	-0.01	-0.06	-0.02	-0.08	-0.10	-0.06	-0.05	-0.04	0.02	-0.07																
-0.02	0.09	0.22	-0.13	-0.02	1.06	-0.09	-0.12	1.05	-0.07	0.09	-0.05	-0.01	-0.54															
1.05	-0.72	-0.12	-0.14	-0.13	0.10	-0.07	0.38	1.04	1.02	-0.07	0.00	-0.03	-0.10	-0.37														
-0.09	-0.32	-0.18	-0.17	0.25	-0.01	-0.12	-0.01	-0.10	-0.02	-0.01	-0.09	-0.07	-0.02	-0.09	-0.08													
0.11	0.15	-0.03	-0.09	-0.04	-0.02	-0.05	-0.70	0.06	0.00	-0.04	-0.79	-0.01	-0.09	-0.10	-0.18	-0.07												
0.01	0.33	-0.06	-0.04	-0.63	0.01	-0.16	-0.12	-0.08	-0.06	-0.03	0.11	0.11	-0.13	1.03	0.00	-0.04	-0.06											
-0.16	1.17	-0.35	-0.01	0.06	-0.02	0.02	-0.12	0.00	-0.20	1.01	0.07	-0.07	-0.04	0.06	1.04	-0.08	-0.04	-0.45										
0.04	-0.01	-0.07	-0.07	-0.03	0.07	1.02	-0.30	0.02	0.11	1.05	-0.06	-0.09	1.04	-0.02	-0.04	-0.28	0.04	-0.01	0.46									
1.05	-0.12	-0.60	-0.03	-0.03	-0.02	-0.06	-0.40	-0.02	-0.38	-0.04	0.52	1.03	-0.05	-0.01	0.09	1.04	-0.07	0.07	-0.08	-0.08								
0.53	-0.23	-0.06	0.01	-0.22	1.02	-0.03	-0.04	-0.01	-0.07	-0.02	0.56	-0.04	-0.55	0.01	-0.22	-0.32	-0.06	-0.22	-0.62	-0.08	-0.05							
-0.08	-0.13	0.04	0.23	-0.03	0.07	0.09	-0.06	0.04	-0.02	1.02	-0.06	-0.48	-0.09	-0.27	0.13	-0.19	-0.13	1.03	-0.04	0.03	-0.50	0.03						
-0.01	0.10	1.04	-0.03	-0.01	0.01	0.73	-0.72	0.06	-0.01	-0.05	0.03	0.33	0.88	-0.14	-0.09	-0.03	-0.05	-0.01	-0.08	-0.09	0.00	-0.11	-0.07					
0.01	-0.23	0.10	-0.02	-0.02	0.08	1.05	1.03	-0.13	-0.34	-0.20	-0.04	0.01	-0.05	-0.04	-0.04	0.01	-0.04	0.04	0.46	-0.01	-0.04	0.07	-0.05	-0.07				
-0.04	-0.03	-0.12	-0.29	0.04	0.01	0.30	0.09	-0.07	-0.13	-0.12	0.20	-0.07	-0.08	1.04	-0.07	-0.03	-0.05	-0.04	0.25	-0.42	-0.28	-0.07	-0.10	-0.03	-0.03			
-0.07	1.42	-0.03	0.67	-0.06	0.07	1.04	-0.15	-0.10	0.31	-0.51	-0.10	-0.09	-0.12	-0.09	0.27	-0.10	-0.40	-0.06	-0.68	-0.10	-0.09	-0.38	-0.06	0.51	0.57	0.66		
-0.14	0.32	-0.19	-0.15	1.20	-0.70	-0.89	0.30	1.12	-0.13	1.46	-0.98	-0.13	-1.22	1.07	-1.05	-0.16	0.36	1.03	-0.18	-0.17	-0.19	-0.13	1.07	-0.17	1.16	0.22	1.04	
0.56	0.68	0.51	0.68	0.62	0.56	0.59	0.49	0.71	0.56	0.59	0.55	0.64	0.65	4.73	0.59	0.56	0.48	0.66	0.63	0.45	0.52	0.69	0.57	0.66	0.70	0.50	0.53	0.64

If we return to a consideration of the banks in column 2 of Table 5, the strong positive dependencies can be seen in the values of the entries at the very top and bottom three coefficients in the rows of column 2.

The key issue for the current analysis is whether these dependencies changed during the GFC and this is the focus of the next stage of our analysis. Figure 10 shows Trees 1 and 2 for the GFC period.

Figure 10: GFC Period R Vines Trees 1 and 2



There is a change in the groupings in the tree structures produced by the impact of the GFC. Citibank is absent from the list because it had to be rescued by the US Government under plans agreed for Citi-Group, following large losses in the value of its subprime mortgage assets. The remaining major financial services companies are grouped together, J.P. Morgan and American Express, together with the aviation and defence sector companies United Technologies and Boeing. Similarly, the IT companies, Intel, Microsoft, Hewlett Packard and IBM, are grouped together, as are the main-stream consumer products and industrial groupings, Coca-Cola, Proctor and Gamble, Johnson and Johnson. Drug companies Merck and Pfizer, and communications giants Verizon and AT&T are linked. A final chain is provided by General Electric, 3M and Alcoa.

The details of the linkages in the tree structures and the nature of the dependencies in the GFC period are provided in Table 6.

Table 6: R Vine GFC Copula Specification Matrix

22																									
9	21																								
1	9	23																							
12	1	9	20																						
18	12	1	9	14																					
25	18	12	1	9	4																				
10	25	18	12	1	9	9																			
21	10	25	18	12	1	5	1																		
23	5	10	25	18	12	25	5	12																	
20	3	5	10	25	18	3	25	5	18																
14	15	3	5	10	25	15	3	25	5	25															
4	8	15	3	5	10	8	15	3	25	5	10														
17	11	8	15	3	5	11	8	15	3	10	5	17													
13	16	11	8	15	3	16	11	8	15	17	3	5	13												
19	24	16	11	8	15	17	16	11	8	13	15	3	5	19											
6	23	24	16	11	8	24	17	16	11	19	8	15	3	5	6										
16	7	7	24	16	11	12	24	17	16	6	11	8	15	3	5	16									
3	2	2	7	24	16	18	12	24	17	3	16	11	8	15	3	5	3								
7	13	13	2	7	24	10	18	7	24	15	17	16	11	8	15	3	5	7							
2	4	20	13	2	7	13	10	2	7	16	24	24	16	11	8	15	7	5	2						
24	6	14	4	13	2	19	13	6	2	7	7	7	24	16	11	8	2	15	5	5					
11	17	17	6	4	13	6	19	19	6	8	2	2	7	24	16	11	24	8	15	24	15				
8	19	19	17	6	17	7	6	13	19	2	6	13	2	7	24	7	11	11	8	11	24	8			
15	14	6	19	17	19	2	7	10	13	11	19	6	6	2	7	24	8	24	11	8	11	24	11		
5	20	26	14	19	6	1	2	18	10	24	13	26	19	6	2	2	15	2	24	15	8	11	24	26	
26	26	4	26	26	26	26	26	26	26	26	26	19	26	26	26	26	26	26	26	26	26	26	26	24	24

The picture and tree structures are changed dramatically by the GFC. At the top of the first column sits United Technologies (no 22 in the GFC set), paired with the Dow Jones (no 26). The next link in the tree is with Boeing (no 5) sitting next to the bottom of the column, followed by J.P. Morgan Chase (no 15). This segment of Tree 2 can be seen in the bottom right of Figure 10. Column 3 is dominated by the links between Verizon (no 23) and AT&T (no 4). The other column in which the strongest dependency is not on the index is column 13, in which Merck (no 17) sits at the top with Pfizer (no 19) at the bottom. This linkage is shown in the middle of the right-hand side of the diagram for Tree 2 in Figure 10. All the other remaining dominating dependencies are with the Dow Jones Index.

The specification of the copula types fitted during the GFC are presented in Table 7.

Table 7: GFC Period Types of Copulas Fitted

5																								
5	5																							
2	2	2																						
2	1	5	1																					
1	5	1	2	5																				
1	6	5	1	2	1																			
2	2	1	2	2	2	5																		
5	5	1	6	2	5	3	2																	
5	1	2	5	2	5	2	5	2																
5	5	2	1	5	1	2	1	5	1															
1	2	5	5	1	2	2	1	2	3	2														
2	2	1	1	5	2	5	5	1	2	6	1													
5	4	6	2	5	1	5	6	2	2	2	5	1												
2	1	6	5	2	5	5	2	2	2	5	2	2	1											
2	5	6	2	5	5	1	2	3	2	2	2	1	5	2										
6	5	2	1	1	2	3	1	3	5	4	1	2	2	2	6									
2	2	1	3	5	2	1	1	1	1	3	2	1	2	1	3	3								
2	2	1	2	2	5	2	2	4	1	1	1	1	5	2	2	2	6							
2	4	5	5	2	1	2	2	5	5	2	2	2	2	2	2	2	2	2						
1	2	2	2	2	1	2	2	4	5	2	1	2	2	1	2	5	2	2	3					
2	2	3	2	2	2	2	2	2	5	2	2	2	2	3	1	6	2	2	2	5				
2	2	5	4	2	2	2	3	2	2	3	2	2	2	2	2	2	2	4	2	5	1			
2	2	2	2	2	6	2	2	2	2	2	2	2	1	2	2	2	1	5	2	2	2	2		
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

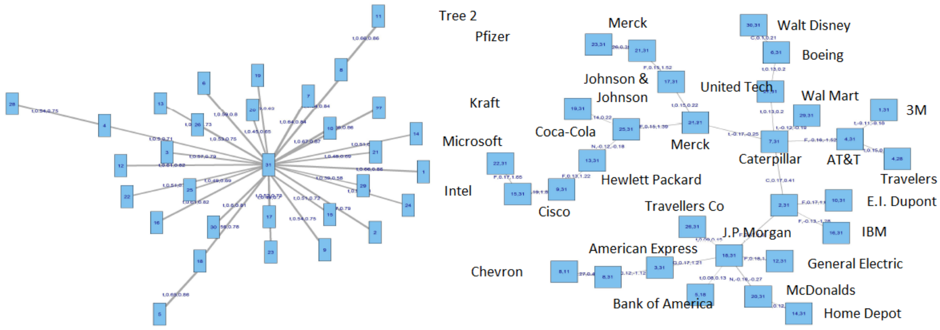
What is apparent in Table 7 is the much greater application of Copula type 2, the Student t copula, and a much lower usage of the Gaussian copula. This is not surprising, as we would expect the tails of the distributions to increase during periods of financial distress. The parameters fitted to the copulas are shown in Table 8.

Table 8: GFC Period Copula Parameter Estimates

-0.11																								
0.70	-0.37																							
0.04	-0.05	0.02																						
0.06	-0.02	-0.07	-0.05																					
-0.03	0.92	-0.04	0.06	0.48																				
-0.01	1.03	0.54	-0.02	0.02	-0.02																			
-0.04	-0.07	-0.06	-0.03	-0.05	-0.05	0.10																		
-0.49	-0.31	-0.04	1.07	0.01	-0.03	0.05	-0.05																	
0.43	-0.06	-0.08	-0.12	0.00	0.11	0.15	0.30	-0.11																
0.25	-0.36	0.00	-0.07	-0.57	-0.01	0.16	-0.09	-0.64	-0.02															
-0.09	0.02	-0.78	-0.46	-0.07	0.00	-0.14	-0.14	0.06	0.04	0.00														
-0.10	-0.02	-0.07	-0.08	-0.80	-0.06	0.19	-0.72	-0.01	0.07	1.06	-0.06													
-0.81	1.04	1.03	0.05	-1.00	-0.11	-0.37	1.04	0.01	-0.08	0.08	-0.20	-0.09												
-0.01	-0.04	1.02	-0.79	-0.01	-0.58	0.10	-0.02	0.10	-0.01	0.14	-0.04	-0.04	-0.16											
0.05	-0.07	1.02	0.11	-0.41	0.24	-0.09	-0.04	0.07	-0.03	0.01	-0.02	-0.15	-0.90	-0.08										
1.02	-0.16	-0.04	-0.01	0.02	-0.02	0.03	-0.03	0.01	0.38	1.01	-0.10	-0.04	-0.18	-0.04	1.02									
-0.09	-0.02	-0.09	0.11	0.76	-0.08	-0.10	-0.01	-0.03	-0.01	0.05	-0.03	-0.09	-0.06	-0.07	0.06	0.04								
0.05	0.00	-0.01	-0.11	0.00	0.16	-0.09	0.03	1.07	-0.09	-0.05	-0.04	-0.05	0.63	0.02	-0.10	-0.01	1.01							
-0.06	1.01	0.34	-0.27	-0.11	-0.06	-0.09	0.04	0.24	-0.24	0.00	-0.05	-0.05	-0.10	-0.07	-0.07	-0.12	0.06	-0.11						
-0.05	-0.05	0.03	0.05	0.01	-0.02	0.01	-0.09	1.03	-0.03	0.04	-0.06	-0.06	0.00	-0.02	-0.01	-0.40	0.00	-0.07	0.11					
-0.08	0.11	0.06	-0.15	0.09	-0.01	0.06	0.03	-0.03	-0.15	-0.07	0.08	-0.05	-0.02	0.11	-0.11	1.01	-0.09	-0.04	-0.14	-0.95				
-0.13	0.08	0.38	1.03	-0.06	0.04	0.00	0.08	0.14	0.00	0.09	0.03	-0.12	-0.02	0.00	-0.05	0.02	0.03	1.06	0.11	-0.51	-0.06			
-0.09	0.16	-0.07	0.01	0.28	1.06	0.08	0.13	0.16	0.08	0.11	-0.09	-0.18	-0.11	-0.03	0.14	0.08	-0.15	-0.60	-0.09	-0.08	0.12	-0.07		
0.28	0.24	0.39	0.35	0.27	-0.14	0.19	0.19	0.25	0.25	-0.09	0.22	0.32	-0.16	-0.20	0.31	-0.17	0.43	0.23	-0.22	-0.21	-0.33	-0.26	0.22	
0.84	0.63	0.78	0.70	0.65	0.73	0.80	0.83	0.76	0.74	0.83	0.75	0.65	0.77	0.73	0.79	0.68	0.81	0.81	0.69	0.73	0.79	0.77	0.74	0.65

The final analysis undertaken is for the post-GFC period, September 2009 to December 2011.

Figure 11: Post-GFC Period R Vines Trees 1 and 2



The industry groupings are apparent in Tree 2, shown in Figure 11. There is an IT cluster featuring Microsoft, Intel, Cisco and Hewlett Packard, and a financial services cluster which includes Bank of America, American Express, Travellers Co and J.P. Morgan. The drug companies group together in the dependencies shown between Pfizer, Merck and Johnson and Johnson. Oil, retail companies and manufacturing companies are spread about.

Table 9 shows the copulas specification matrix for the Post-GFC period.

Table 9: Post-GFC R Vine Copula Specification Matrix

11																														
30	8																													
6	30	14																												
27	6	30	3																											
1	27	6	30	22																										
28	1	27	6	30	5																									
4	28	1	27	6	30	30																								
23	4	28	1	27	6	20	6																							
22	23	4	28	1	27	12	20	27																						
15	22	23	4	28	1	26	12	20	1																					
9	15	22	23	4	28	18	26	12	20	28																				
21	9	15	22	23	4	16	18	26	12	20	20																			
19	21	9	15	5	23	15	16	18	26	12	4	12																		
17	19	21	9	20	15	9	15	16	18	26	23	4	26																	
13	17	19	21	12	9	13	9	15	16	18	15	23	4	18																
25	13	17	19	26	21	10	13	9	15	16	9	15	23	4	16															
29	25	13	17	18	19	19	10	13	9	15	21	9	15	23	4	15														
24	29	25	13	16	17	2	19	10	13	9	19	21	9	15	23	4	9													
7	24	29	25	21	13	29	2	19	10	13	17	19	21	9	15	23	4	13												
10	7	24	29	19	25	25	29	2	19	10	13	17	19	21	9	21	23	4	10											
14	10	7	24	17	29	23	25	29	2	19	25	13	17	19	21	19	21	23	4	19										
16	14	10	7	10	24	21	23	25	29	2	29	25	13	17	19	17	19	21	23	4	2									
2	16	16	10	2	7	17	21	23	25	29	24	29	25	13	17	10	17	19	21	23	4	4								
26	2	2	16	29	10	24	17	21	23	25	7	24	29	25	13	2	10	17	19	21	23	29	29							
20	26	26	2	7	16	1	24	17	21	23	10	7	24	29	25	29	2	10	17	2	21	25	23	25						
12	20	3	26	24	2	28	1	24	17	21	16	10	7	24	29	7	29	2	25	29	17	23	21	23	7					
5	12	5	20	25	26	4	28	1	24	17	2	16	10	7	24	24	7	29	29	17	25	21	17	21	23	24				
18	5	12	12	13	20	7	4	28	7	24	26	2	16	10	7	25	24	7	24	7	29	17	25	17	21	23	17			
3	18	18	5	9	12	27	7	4	28	7	12	26	2	16	10	13	25	24	7	24	24	24	24	7	17	21	23	21		
31	3	20	18	15	31	6	27	7	4	31	18	18	18	2	2	9	13	25	2	25	7	7	7	24	24	17	21	23	31	
8	31	31	31	31	18	31	31	31	31	4	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	23	23

If we look first at the cases of strong dependencies that are not initially partnered with the Dow Jones Index at the top of column one in Table 9, we have Exxon Mobil (no 11) at the top and Chevron (no 8) at the bottom, revealing strong co-dependencies between these two major oil companies. Bank of America (no 5) is at the top of column six and J.P. Morgan Chase (no 18), is at the bottom. Verizon (no 28) is at the top of column eleven and AT&T (no 4), is at the bottom, revealing the linkages between these two communications companies. All the other companies are linked via their relationship with the Dow Jones Index (no 31), which appears as the bottom entry in most of the columns.

The copulas fitted in the Post-GFC period are shown in Table 10.

Table 10: Post-GFC Period Types of Copulas Fitted

1
1	1
1	1	3
1	5	6	6
1	1	2	1	5
2	1	1	1	2	5
4	6	5	1	1	4	5
6	4	2	2	1	5	3	1
1	5	2	1	5	5	1	3	1
5	5	2	1	1	5	2	1	2	1
2	2	3	2	6	6	2	5	1	5	2
1	2	5	4	3	1	6	1	1	5	2	3
5	5	2	1	5	1	5	1	2	5	2	3	1
5	1	2	1	1	3	4	1	2	1	5	1	5	5
5	2	5	5	1	5	6	1	5	1	5	5	4	2	1
2	1	1	1	1	5	5	2	6	5	5	2	5	6	2	2
2	2	5	2	2	6	3	6	1	2	1	5	1	6	2	5	5
1	1	2	1	2	6	1	5	1	2	1	2	1	1	6	2	1	2
2	3	2	1	2	5	1	1	5	1	5	1	1	1	5	2	3	2	5
1	5	2	1	5	5	5	1	3	3	1	3	5	2	2	1	1	5	6	5
1	5	6	3	2	1	6	5	2	6	1	4	1	5	5	2	2	1	5	1	5
2	2	2	2	3	5	1	5	2	2	3	5	5	3	2	2	1	1	5	5	3	2
1	5	1	1	4	2	1	1	2	1	5	2	2	6	1	2	3	2	5	1	2	5	5
2	2	5	2	1	5	2	1	2	2	2	6	2	5	1	2	2	1	5	2	1	2	4	2
5	5	3	2	2	4	1	1	2	5	3	2	1	1	2	2	1	1	3	5	1	5	2	5	1
2	3	2	6	2	2	1	1	2	5	4	2	2	1	2	2	1	1	2	5	5	2	5	2	5	5
5	1	5	6	5	2	2	1	2	2	3	2	2	1	1	1	5	1	5	2	1	5	2	2	2	1	2
2	5	2	4	1	5	1	2	5	6	2	2	2	5	5	1	2	5	5	5	2	2	4	4	2	2	5	4
2	5	5	4	5	2	3	2	2	2	2	1	5	2	5	5	5	5	1	5	2	3	5	2	5	2	2	5	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

Table 10 shows that the reliance on Student t copulas, which was apparent in the GFC period, is reduced in the post-GFC period. As in the other two periods considered, the bottom row in Table 10 consists of Student t (no 2) copulas. Thus, the predominant modelling of dependencies in all three periods (the first steps in the tree), uses a distribution with fat tails. However, once this primary dependency is taken into account, subsequent links in the tree make less use of the Student t copula than in the GFC period. The Gaussian copula features more prominently in the contingent dependencies than in the GFC period.

The parameters fitted to the copulas in the post-GFC period are shown in Table 11.

Table 11: Post-GFC Period Copula Parameter Estimates

-0.07
-0.07	-0.09
-0.03	-0.29	0.05
-0.09	-1.04	1.01	1.05
-0.03	-0.10	-0.03	-0.02	-0.22
0.07	0.01	-0.08	-0.04	-0.07	-0.21
1.03	1.04	-0.24	-0.05	-0.02	1.01	0.15
1.08	1.04	-0.04	-0.02	-0.05	-0.09	0.03	-0.06
-0.02	-0.54	0.04	-0.05	-0.32	0.10	-0.01	0.06	-0.03
-0.29	-0.59	-0.07	-0.03	-0.01	-0.59	0.06	-0.05	0.06	-0.08
-0.01	-0.03	0.13	0.04	1.03	1.06	-0.11	-0.35	-0.14	0.37	-0.05
-0.10	-0.04	0.13	1.05	0.04	0.05	1.01	-0.15	-0.13	0.44	0.09	0.04
-0.21	-0.11	-0.03	-0.06	-0.26	-0.04	0.14	-0.06	-0.03	-0.68	0.07	0.06	-0.02
-0.48	-0.02	-0.05	0.01	-0.07	0.11	1.04	-0.03	-0.07	-0.11	-0.09	-0.06	0.45	-0.29
0.32	-0.15	-0.31	0.39	-0.11	0.10	1.02	-0.06	0.48	-0.05	0.05	-0.07	1.04	0.02	-0.02
-0.02	0.03	-0.04	-0.08	-0.05	-0.26	0.42	0.04	1.03	0.22	-0.19	0.00	0.13	1.03	-0.05	0.05
0.02	-0.07	-0.33	-0.09	0.07	1.03	0.05	1.07	-0.05	-0.07	-0.02	0.25	-0.09	1.01	0.00	0.03	-0.10
-0.17	-0.10	0.15	-0.06	-0.10	1.07	-0.02	0.12	-0.05	-0.06	-0.07	-0.06	-0.03	-0.03	1.01	0.10	-0.03	-0.05
-0.06	0.05	0.02	-0.09	-0.01	-0.08	-0.07	-0.11	-0.27	-0.04	-0.30	-0.13	-0.03	-0.07	-0.35	0.01	0.05	-0.02	-0.27
-0.10	0.11	-0.06	-0.05	-0.26	-0.28	-0.28	-0.07	0.03	0.05	0.06	0.09	-0.47	0.01	0.01	-0.08	-0.06	-0.47	1.03	0.50
-0.07	-0.08	1.04	0.02	-0.03	-0.08	1.04	0.23	0.02	1.02	0.02	1.05	-0.04	-0.57	-0.63	0.01	0.00	-0.04	-0.22	0.03	-0.54
-0.03	-0.18	-0.05	0.06	0.04	-0.36	-0.04	0.07	-0.06	-0.10	0.16	0.25	-0.41	0.11	0.00	-0.02	-0.07	-0.08	0.09	0.02	0.05	0.01
-0.01	0.87	-0.01	-0.07	1.04	0.00	-0.07	-0.05	-0.10	-0.07	0.49	-0.15	-0.01	1.07	-0.18	0.00	0.02	-0.03	-0.12	0.02	-0.01	0.75	-0.22
-0.01	-0.11	0.04	0.00	-0.12	-0.04	0.05	-0.09	-0.05	0.00	0.02	1.04	-0.01	0.31	-0.12	0.04	-0.04	-0.03	-0.77	-0.05	-0.01	0.01	1.02	-0.01
0.20	-0.47	0.12	-0.03	0.02	1.04	-0.01	-0.13	0.04	-0.13	0.10	-0.04	0.03	-0.12	-0.09	-0.16	-0.04	0.08	0.03	-0.27	-0.06	-0.22	-0.07	0.31	-0.08
-0.06	0.07	0.04	1.09	0.00	0.04	-0.03	-0.12	0.13	-0.70	1.07	-0.03	-0.11	-0.02	-0.10	-0.06	-0.04	-0.04	-0.06	-0.10	0.63	-0.06	0.55	0.01	0.45	-0.66
0.26	-0.01	0.66	1.03	0.39	0.01	-0.02	-0.06	-0.03	0.01	0.09	0.04	0.05	-0.04	-0.06	-0.09	0.18	-0.01	-0.46	-0.08	-0.07	-0.44	0.03	0.06	0.10	-0.07	-0.06
-0.05	-0.09	0.02	1.05	0.15	1.02	0.10	0.00	-0.88	1.06	-0.07	-0.07	-0.13	-0.45	-0.71	-0.13	0.09	-0.73	0.37	1.24	0.14	-0.16	1.10	1.08	-0.14	-0.18	0.53	1.04
0.40	-1.12	1.20	1.21	1.65	0.13	0.21	0.20	0.20	-0.18	0.22	-0.27	1.67	0.15	1.60	-1.28	1.81	1.22	-0.18	1.62	0.22	0.41	-1.52	-0.19	1.39	-0.25	0.22	1.52	0.39
0.86	0.84	0.72	0.79	0.72	0.86	0.81	0.80	0.86	0.86	0.75	0.65	0.82	0.75	0.78	0.82	0.72	0.75	0.73	0.87	0.62	0.79	0.71	0.58	0.69	0.84	0.71	0.73	0.69

5. Conclusion

In this paper we have used the recently developed R Vine copula methods (see Aas et al. (2009), Berg and Aas (2009), Min and Czado (2010) and Czado et al. (2011)) to analyse the changes in the co-dependencies of Dow Jones constituent stocks for three periods spanning the GFC: pre-GFC (Jan 2005- July 2007), GFC (July 2007-Sep 2009) and post-GFC periods (Sep 2009 - Dec 2011). The results suggest that the dependencies change in a complex manner and there is evidence of greater reliance on the Student t copula in the copula choice within the tree structures for the GFC period which is consistent with the existence of larger tails to the distributions of returns. One of the attractions of this approach to risk-modelling is the flexibility available in the choice of distributions used to model co-dependencies.

The main limitation is the static nature of the approach and dynamic applications are in the process of development. Breckmann and Czado (2012) have recently proposed a COPAR model which provides a vector autoregressive VAR model for analysing the non-linear and asymmetric co-dependencies between two series. A more dynamic approach will be the subject of future work.

Acknowledgement: For financial support, the authors wish to thank the Australian Research Council. The third author would also like to acknowledge the National Science Council, Taiwan, and the Japan Society for the Promotion of Science.

References

[1] Aas, K., Czado, C., Frigessi, A., and Bakken, H. (2009) "Pair-copula constructions of multiple dependence". Insurance, Mathematics and Economics, 44:182–198.

[2] Bedford, T. and R. M. Cooke (2001) "Probability density decomposition for conditionally dependent random variables modeled by vines", Annals of Mathematics and Articial Intelligence 32, 245-268.

[3] Bedford, T. and R. M. Cooke (2002) "Vines - a new graphical model for dependent random variables", Annals of Statistics 30, 1031-1068.

[4] Berg, D. (2009) "Copula goodness-of-fit testing: an overview and power comparison". The European Journal of Finance, 15:675–701.

[5] Berg, D. and Aas, K. (2009) "Models for construction of higher-dimensional dependence: A comparison study". European Journal of Finance, 15:639–659.

[6] Brechmann, E. C., C. Czado, and K. Aas (2012) "Truncated regular vines and their applications" Canadian Journal of Statistics 40 (1), 68-85.

[7] Brechmann, E.C., and U. Schepsmeier, (2012) "Modeling dependence with C- and D-vine copulas" The R-package CDVine, http://cran.r-project.org/web/packages/CDVine/vignettes/CDVine-package.pdf

[8] Brechmann, E. C., and C. Czado (2012) "COPAR - multivariate time-series modelling using the COPula AutoRegressive model", Working Paper, Faculty of Mathematics, Technical University of Munich.

[9] Chollete, L., Heinen, A., and Valdesogo, A. (2009) "Modeling international financial returns with a multivariate regime switching copula". Journal of Financial Econometrics, 7:437–480.

[10] Cooke, R.M., H. Joe and K. Aas, (2011) "Vines Arise", chapter 3 in DEPENDENCE MODELING Vine Copula Handbook, Ed. D. Kurowicka and H. Joe, World Scientific Publishing Co, Singapore.

[11] Czado, C., U. Schepsmeier, and A. Min (2011) "Maximum likelihood estimation of mixed C-vines with application to exchange rates". To appear in Statistical Modelling.

[12] Dimann, J., E. C. Brechmann, C. Czado, and D. Kurowicka (2012) "Selecting and estimating regular vine copulae and application to financial returns", Submitted preprint. http://arxiv.org/abs/1202.2002.

[13] Dissman, J.F. (2010) Statistical Inference for Regular Vines and Application, Thesis, Technische Universitat Munchen, Zentrum Mathematik.

[14] Heinen, A. and A. Valdesogo (2009) "Asymmetric CAPM dependence for large dimensions: the canonical vine autoregressive model". CORE discussion papers 2009069, Universite catholique de Louvain, Center for Operations Research and Econometrics (CORE).

[15] Joe, H. (1996) "Families of m-variate distributions with given margins and m(m- 1)/2 bivariate dependence parameters". In L. Rüschendorf and B. Schweizer and M. D. Taylor, editor, Distributions with Fixed Marginals and Related Topics.

[16] Joe, H. (1997) Multivariate Models and Dependence Concepts. Chapman & Hall, London.

[17] Joe, H., Li, H., and Nikoloulopoulos, A. (2010) "Tail dependence functions and vine copulas". Journal of Multivariate Analysis, 101(1):252–270.

[18] Kullback, S. and Leibler, R. A. (1951) "On information and sufficiency". The Annals of Mathematical Statistics, 22(1):79–86.

[19] Kurowicka, D. (2011) "Optimal truncation of vines", In D. Kurowicka and H. Joe (Eds.), Dependence Modeling: Handbook on Vine Copulae. Singapore: World Scientific Publishing Co.

[20] Kurowicka D. and Cooke R.M. (2003) “A parametrization of positive definite matrices in terms of partial correlation vines”, Linear Algebra and its Applications, 372: 225–251.

[21] Kurowicka, D. and R. M. Cooke, (2006) “Uncertainty Analysis with High Dimensional Dependence Modelling”. Chichester: John Wiley.

[22] Mendes, B. V. d. M., M. M. Semeraro, and R. P. C. Leal (2010) “Pair-copulas modeling in finance”. Financial Markets and Portfolio Management 24 (2), 193-213.

[23] Min, A. and Czado, C. (2010) “Bayesian inference for multivariate copulas using pair-copula constructions”. Accepted for publication in Journal of Financial Econometrics.

[24] Morales-N´apoles, O., R. Cooke, and D. Kurowicka (2009) “About the number of vines and regular vines on n nodes”, Submitted to Linear Algebra and its Applications.

[25] Nelsen, R. (2006). An Introduction to Copulas. Springer, New York, 2nd edition.

[26] Patton, A. J., (2009) “Copula based models for financial time series”. In: Handbook of Financial Time Series. pp. 767-785

[27] Prim, R. C. (1957) “Shortest connection networks and some generalizations”. Bell System Technical Journal, 36:1389–1401.

[28] Schirmacher, D. and Schirmacher, E. (2008) “Multivariate dependence modeling using pair-copulas”. Technical report, Society of Acturaries: 2008 Enterprise Risk Management Symposium, April 14-16, Chicago. Available from: http://www.soa.org/library/monographs/other-monographs/2008/april/ 2008-erm-toc.aspx

[29] Sklar, A. (1959) “Fonctions de repartition a n dimensions et leurs marges”, Publications de l’Institut de Statistique de L’Universite de Paris 8, 229-231.

[30] Smith, M., Min, A., Czado, C., and Almeida, C. (2010) “Modeling longitudinal data using a pair-copula decomposition of serial dependence”. In revision for the Journal of the American Statistical Association.

[31] Vuong, Q. H. (1989) “Likelihood ratio tests for model selection and non-nested hypotheses”. Econometrica, 57:307–333.