KIER DISCUSSION PAPER SERIES

KYOTO INSTITUTE
OF
ECONOMIC RESEARCH

Discussion Paper No.823

"A Numerical Evaluation on a Sustainable Size of Primary Deficit in Japan"

Real Arai, Junji Ueda

June 2012

KYOTO UNIVERSITY

KYOTO, JAPAN

A Numerical Evaluation on a Sustainable Size of Primary Deficit in Japan[*]

Real Arai[†]
Graduate School of Social Sciences,
Hiroshima University

Junji Ueda[‡]
Policy Research Institute,
Ministry of Finance Japan

June 18, 2012

Abstract

We investigate how large a size of primary deficit to GDP ratio the Japan's government can sustain. For this investigation, we construct an overlapping generations model, in which multi-generational households live and the government maintains a constant ratio of primary deficit to GDP. We numerically show that the primary deficit cannot be sustained unless the rate of economic growth is unrealistically high, which is more than five percent according to our settings. Our result implies that Japan's government needs to achieve a positive primary balance in the long-run in order to avoid the divergence of the public debt to GDP ratio.

[*]We would like to thank Ryo Hasumi, Kaoru Hosono, Hisakazu Kato, Masahiko Nakazawa, Ryosuke Okazawa, Manabu Shimasawa, Hiroshi Teruyama, Toshiki Tomita, Naoyuki Yoshino, and participants in the seminar of Human Economy Research Group at Chuo University and the Kansai Public Economics Seminar for their helpful comments. Real Arai acknowledges the Startup Grant-in-Aid for Young Scientists from Kyoto University, the Grant-in-Aid for Young Scientists (B) (No.23730298) from Japan Society for the Promotion of Science (JSPS), and the joint research project program of KIER. The views expressed in this paper do not necessarily reflect those of the Ministry of Finance, Japan. All remaining errors are ours.

[†]Graduate School of Social Sciences, Hiroshima University, 1-2-1 Kagamiyama, Higashi-hiroshima, Hiroshima, 739-8525, JAPAN, E-mail: real@hiroshima-u.ac.jp, TEL: +81-82-424-7297.

[‡]Policy Research Institute, Ministry of Finance Japan, 3-1-1 Kasumigaseki, Chiyoda-ku, Tokyo, 100-8940, JAPAN, E-mail: junji.ueda@mof.go.jp

JEL Classification Number: E62, H62, H63, H68

Key Words: fiscal sustainability, public debt, primary deficit, economic growth

1 Introduction

Since the 1980s, the amount of public debt has ballooned in Japan. The financial liabilities of the general government in Japan increased from 53.7 % of GDP in 1980 to 222.8 % in 2010. Other developed countries, such as EU member countries and the US, also face the problem of the accumulation of public debt. Under such circumstances, fiscal sustainability becomes one of the most important macroeconomic and policy issues, and many researchers have studied it. For example, Ihori et al. (2003) empirically investigate the sustainability of the Japanese fiscal stance by checking whether the intertemporal budget constraint of the government holds or not[1)]. They conclude that in Japan, fiscal consolidation is necessary to avoid the divergence of the ratio of public debt to GDP.

Meanwhile, a theoretical literature exists on the dynamics of public debt and fiscal sustainability with primary fiscal deficit. Previous studies in this literature provide implications about conditions preventing the ratio of public debt to GDP from diverging infinitely[2)]. Chalk (2000) constructs a simple overlapping generations model and shows that public debt to GDP ratio converges to some finite level with primary deficit if, and only if, (i) the rate of economic growth

1)This approach to evaluate fiscal sustainability has been developed and used in many previous studies, such as Hamilton and Flavin (1986) and Chalk and Hemming (2000).

2)For example, Bräuninger (2005), Yakita (2008), Arai and Kunieda (2011), and Arai (2011) analyze the dynamics of public debt to GDP ratio in endogenous growth settings.

is higher than the interest rate on public debt in steady states, (ii) the primary deficit is sufficiently small, and (iii) the initial amount of public debt is also sufficiently small compared with that of physical capital. In other words, Chalk (2000) points out that there exists an upper bound of the size of primary deficit permissible to achieve fiscal sustainability, even if the interest rate on public debt is below the rate of economic growth in steady states.

Based on the results obtained in previous empirical and theoretical studies, this paper aims to answer the following questions: *Under what conditions can the Japan's government run a primary deficit? How large a primary deficit to GDP ratio can Japan's government maintain?* Empirical studies on fiscal sustainability in Japan such as Ihori et al. (2003) imply that the government must reconstruct its public finance at some time. Furthermore, as Chalk (2000) shows, there exists an upper bound of the sustainable size of primary deficit, even if the economy enjoys a very high rate of economic growth. To reconstruct public finance, we must know the *set* of fiscal policies under which the ratio of public debt to GDP does not diverge infinitely.

In order to evaluate the sustainable size of primary deficit, we construct an overlapping generations model in which the economy is closed and the government keeps the primary deficit to GDP ratio constant. In our model, the engine of economic growth is assumed to be the growth of labor productivity and the growth rate is exogenously given[3)]. We introduce the exogenous difference between the interest on public debt and the return on physical capital in order to replicate the realistic gap in them[4)]. Given the level of physical capital per GDP, the difference decreases the interest rate on public debt, and thus, affects

[3)]In our model, economic growth happens due to growing population and labor productivity. However, we assume that the growth rate of population is zero in our numerical evaluations. In addition, it is easy to introduce technological progress though we do not introduce the growth of technological progress.

[4)]For example, Ueda (2012) explains that, in Japan, the returns on physical capital have been around 4.5–5.0 % since 2000, while the interest rates on public debt have been around 1.5 %.

the sustainable size of primary deficit[5)].

Using the model with parameters calibrated so as to match to the data for Japanese economy in 2005, we show that not even a small size of primary deficit can be sustained in the long-run, unless the economic growth rate continues to attain an unrealistically high level. In our benchmark case, the rate of economic growth must be more than 5.0 % for a steady state to exist. In other words, if the economic growth rate is below five percent, the ratio of public debt to GDP diverges infinitely, as long as the government keeps its primary deficit. We note that this condition is a necessary but not sufficient one to guarantee fiscal sustainability. As Chalk (2000) shows, the ratio of public debt to GDP does not diverge if, and only if, a steady state exists *and* the initial amount of public debt is sufficiently small. In fact, a higher growth rate than 5 % may be necessary to avoid the divergence of the ratio of public debt to GDP depending on the initial amount of public debt. Furthermore, we check the robustness of our results with respect to alternative intertemporal elasticity of substitution, since previous empirical studies estimate various values.

Our results imply that, in Japan, the government needs to achieve a positive primary balance in order to avoid the divergence of the ratio of public debt to GDP. In other words, we consider that it is overly optimistic that Japan's fiscal sustainability can be guaranteed by a significant economic boost without eliminating primary deficit.

A few previous studies have a similar motivation to ours. In particular, this paper is related to Ihori et al. (2006) and İmrohoroğlu and Sudo (2011). Ihori et al. (2006) quantitatively find the tax rates and the contribution rate that are needed to maintain the current systems of public pension and medical insurance, as well as to achieve an exogenously given target ratio of public debt

[5)]Sakuragawa and Hosono (2010) emphasize the effect of the difference between interest rates. In their paper, the difference is considered by introducing the financial intermediation cost.

to GDP in the future. We do not levy any constraints on the level of public debt and give the *set* of the size of primary balance that prevents the public debt to GDP ratio from diverging infinitely. Furthermore, we also consider a labor-leisure choice and the endogenous retirement of households, which are not considered in Ihori et al. (2006).

İmrohoroğlu and Sudo (2011) show the necessary economic growth rate to prevent Japan's public debt to GDP ratio from diverging by using a standard neoclassical growth model. While their motivations are similar to ours, our approach is qualitatively different from theirs. They assume a level of future government expenditures, including that of interest payment on public debt, as exogenous variables. In such a setting, economic growth has an effect in reducing the relative size of government expenditures and fiscal deficit to GDP. In this paper, we adopt an overlapping generations setting in which the interest rate on public debt is endogenously determined and the government's primary deficit relative to GDP is kept constant under different economic growth rates.

The remaining part of this paper is constructed as follows. In section 2, we analyze a simple overlapping generations model with multiple generations in which households live for only two periods, in order to understand the properties of the dynamics of public debt. The discussion in section 2 is based on that of Chalk (2000). In section 3, we construct an overlapping generations model with multiple generations to calculate the maximum sustainable primary deficit to GDP ratio. Section 4 gives the results of our numerical calculations and discusses them. Section 5 is the conclusion.

2 A Simple Overlapping Generations Model

Before our numerical evaluation on the sustainable size of primary deficit, we review the theoretical results shown in Chalk (2000). The purpose of this section is that we understand the properties of the dynamics of public debt in our model.

We consider an overlapping generations economy in which households live for two periods (young and old). There is no uncertainty and intra-generational heterogeneity. The size of the population of each generation grows at a rate of $1 + n$. When households are young, they supply their labor inelastically and consume and/or save their wage income. When old, they deplete and consume their savings. Households born at t have an identical utility function given by[6]:

$$\frac{c_t^{1-\sigma}}{1-\sigma} + \beta \frac{d_{t+1}^{1-\sigma}}{1-\sigma}, \tag{1}$$

where $\beta \in (0, 1)$ is the subjective discount rate, σ is the inverse of the intertemporal elasticity of substitution, c_t and d_{t+1} are their consumption when young and old, respectively. Households face intertemporal budget constraints as follows,

$$c_t + \frac{d_{t+1}}{R_{t+1}} = w_t, \tag{2}$$

where R_{t+1} is interest rate and w_t is wage rate, respectively. Each household maximizes the lifetime utility (1) subject to intertemporal budget constraint (2). By solving the maximization problem, we obtain the amount of saving, s_t, as follows,

$$s_t = \frac{1}{1 + \beta^{-1/\sigma} R_{t+1}^{1-1/\sigma}} w_t. \tag{3}$$

A representative firm produces final goods by using labor and physical capital in a perfectly competitive market. A production function is given by $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where Y_t is output, A is a scaling parameter, K_t is an aggregate amount of physical capital, L_t is an aggregate amount of labor supply, and $\alpha \in$

[6]Even if household's utility function is assumed to be a more general form, qualitative results do not change. See Chalk (2000).

$(0,1)$ is an exogenous parameter, respectively. Then, the profit maximization conditions are:

$$r_t = A\alpha K_t^{\alpha-1} L_t^{1-\alpha}, \tag{4}$$

$$w_t = A(1-\alpha) K_t^{\alpha} L_t^{-\alpha}. \tag{5}$$

A government keeps the ratio of primary deficit, D_t, to GDP constant forever. That is, for all t,

$$\frac{D_t}{Y_t} = D. \tag{6}$$

Note that D is exogenously given. Government expenditure is supposed to be wasteful: public spending does not directly affect households' utility and productivity of final good production. A government follows the budget constraints in every period,

$$B_{t+1} = R_t B_t + D_t, \tag{7}$$

where B_t is an amount of public debt in the beginning of period t, and R_t is interest rate on public debt, respectively.

Because we consider a closed economy, all markets must clear in all periods: the labor market clearing condition is $L_t = N_t$ and the capital market clearing condition is $K_{t+1} = s_t N_t - B_{t+1}$. Furthermore, we suppose that a no-arbitrage condition holds in a competitive equilibrium: for all t, $R_t = 1 + r_t$.

Lastly, we define a competitive equilibrium and a balanced growth path steady state. Given the initial level of public debt, B_0, the initial level of physical capital, K_0, and the primary deficit to GDP ratio, D, a set of sequences of predetermined variables $\{K_t, B_t\}_{t=1}^{\infty}$ and of price system $\{R_t, w_t, r_t\}_{t=0}^{\infty}$ constitutes a *competitive equilibrium* if they satisfy the conditions as explained above for all t. Furthermore, given the primary deficit to GDP ratio, D, a set of sequences of predetermined variables $\{K_t, B_t\}_{t=1}^{\infty}$ and of price system $\{R_t, w_t, r_t\}_{t=0}^{\infty}$ constitutes

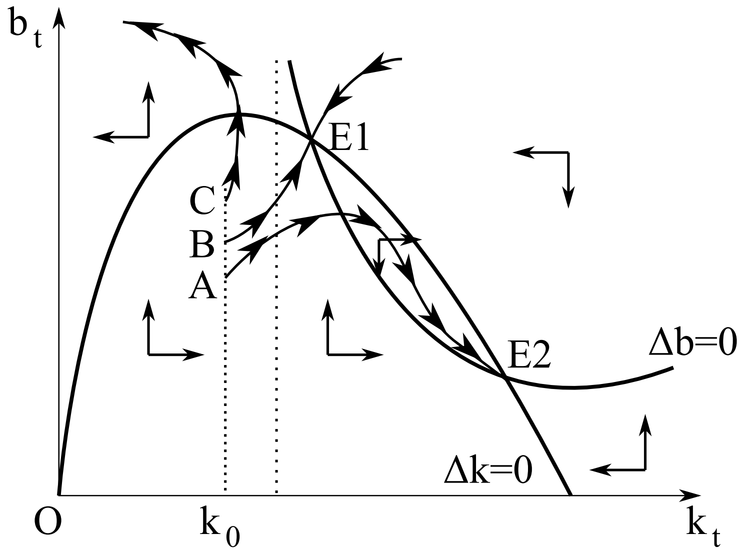

Figure 1: A Phase Diagram (A Case of Sufficiently Small D).

a *balanced growth path steady state (steady state, henceforth)* if they constitute a competitive equilibrium and if the predetermined variables grow at the same rate of $1 + n$ for any t.

In the overlapping generations economy considered here, it is shown that there exists a critical value regarding the ratio of primary deficit to GDP such that, if the ratio of primary deficit to GDP, D, is lower than the critical value, there exist two steady states. Meanwhile, if the ratio of primary deficit to GDP, D, is higher than the critical value, there exists no steady state. In order to show the properties, we use phase diagrams with respect to the levels of public debt per capita and of physical capital per capita.

Figure 1 illustrates the dynamics of the levels of physical capital per capita, k_t, and of public debt per capita, b_t, when the ratio of primary deficit to GDP, D, is smaller than the critical value. In figure 1, there exist two steady states, E1 and E2. We obtain that if the initial level of public debt is low relative to that of physical capital, the economy converges to the locally-stable steady state, E2.

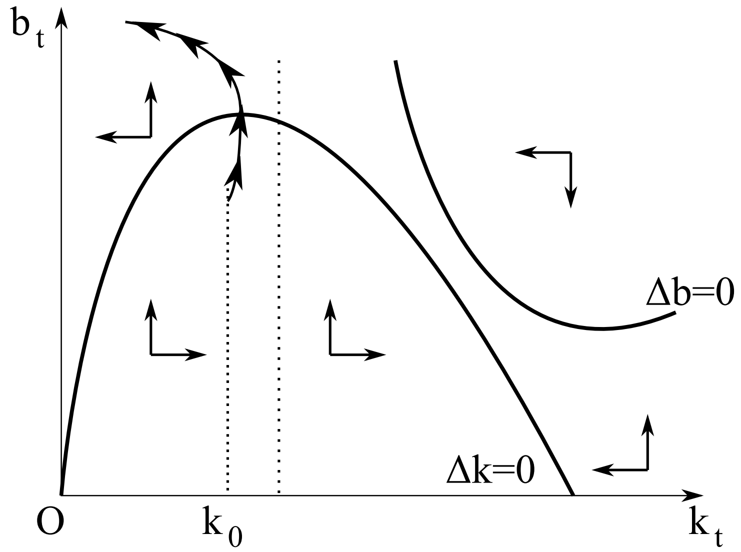

Figure 2: A Phase Diagram (A Case of Too Large D).

Meanwhile, if the initial level of public debt is high relative to that of physical capital, the ratio of public debt to GDP diverges and the given primary deficit to GDP ratio is not sustainable because physical capital is crowded out by the rapid accumulation of public debt[7)]. For instance, suppose that an initial level of physical capital is given by k_0 in figure 1. If the initial level of public debt is smaller than the level at point B (for example, let the initial point be A), this economy converges to E2, which is the locally-stable steady state. Meanwhile, if the initial level of public debt is larger than the level at point B (for example, let the initial point be C), public debt continues to accumulate and the physical capital will be crowded out, which leads to the divergence of the ratio of public debt to GDP.

However, if the ratio of primary deficit to GDP is higher than the critical value, the properties of the dynamics change, which is illustrated in figure

[7)]In this case, the level of public debt per capita grows and that of physical capital per capita decreases to zero in finite time. Therefore, the ratio of public debt to GDP increases infinitely in finite time.

2. In this case, there exists no steady state: starting from any level of initial public debt, the ratio of public debt to GDP diverges infinitely. In other words, regardless of the level of initial public debt, the constant primary deficit to GDP ratio cannot be sustained and the government must improve its fiscal balance.

In sum, we obtain the following two results. First, if the ratio of primary deficit to GDP is lower than the critical value of the ratio, there exists a steady state. Furthermore, if the initial amount of public debt is also sufficiently small, the ratio of public debt to GDP will converge to some finite level. Second, if the ratio of primary deficit to GDP is higher than the critical value, there exists no steady state. In this case, the ratio of public debt to GDP diverges infinitely, whatever the initial amount of public debt.

3 The Overlapping Generations Model with Multiple Generations

In this section, we construct an overlapping generations model used to find the maximum sustainable level of primary deficit. Unlike the model explained in the previous section, we assume that households live for multiple periods in order to calibrate model parameters so as to match to yearly data for the Japanese economy. We explain the outline of the overlapping generations model used in our numerical evaluations and the details are demonstrated in Appendix.

3.1 Households

In every period, households are born and live for T periods. The population of households born in period t is denoted by $N(t)$ and $N(t)$ grows at the rate of $1+n$, that is, $N(t+1) = (1+n)N(t)$. Households obtain utility from consumption and leisure. A periodic utility function is given by

$$u(c_t(t+j), l_t(t+j)) = \frac{1}{1-\gamma}[c_t(t+j)^{\theta}(1-l_t(t+j))^{1-\theta}]^{1-\gamma}, \tag{8}$$

where $j = 0, 1, \cdots, T$ is their age, $c_t(t+j)$ and $1 - l_t(t+j)$ are amounts of consumption and leisure of households born in period t when their age is j, respectively. γ and θ are preference parameters. Households face flow budget constraints as follows:

$$c_t(t+j) + z_t(t+j) = w_{t+j}e_t(t+j)l_t(t+j) + R(t+j)z_t(t+j-1), \tag{9}$$

$$l_t(t+j) \geq 0 \;\; \forall j = 0, \cdots, T, \tag{10}$$

where $z_t(t+j)$ is assets holdings of generation t in period $t+j$. $\{e_t(t+j)\}_{j=0}^{T}$ represents profiles of labor productivity. The productivity depends on their generation and age. In this paper, we assume that $e_{t+1}(t+1+j) = (1+h)e_t(t+j)$ for any t and j. h is the growth rate of labor productivity, which is exogenously given. Households maximize their lifetime utility which is defined as:

$$\sum_{j=0}^{T} \beta^j u(c_t(t+j), l_t(t+j)), \tag{11}$$

subject to their lifetime budget constraints.

3.2 Firms

A representative firm produces final goods from labor and physical capital. The final goods market is perfectly competitive. A firm's production technology is represented by a Cobb-Douglas production function as $y(t) = f(k(t)) = Ak(t)^{\alpha}$, where $y(t)$ and $k(t)$ are output per capita and capital per capita, respectively. A and α are exogenous parameters. Thus, the firm's profit maximization conditions are as follows:

$$r(t) = A\alpha k(t)^{\alpha-1}, \tag{12}$$

$$w(t) = A(1-\alpha)k(t)^{1-\alpha}. \tag{13}$$

Lastly, δ is the depreciation rate of physical capital.

3.3 Government

The government is assumed to maintain a constant ratio of primary deficit, $D(t)$, to GDP, $Y(t)$: D is constant for all t, where $D := D(t)/Y(t)$. The deficit is a wasteful one and thus does not directly affect on the economy. The primary deficit and the interest payment on public debt are financed by issuance of public debt. The government has to conform to the flow budget constraints in every period,

$$B(t+1) = R(t)B(t) + D(t), \tag{14}$$

where $B(t)$ is an amount of public debt in period t and $R(t)$ is interest rate on public debt.

3.4 Competitive Equilibrium

Here, we define a competitive equilibrium as follows.

Definition 1 (Competitive equilibrium)**.** *Given a constant ratio of public deficit to GDP, D, given initial amounts of physical capital, K_0 and public debt, B_0, a set of sequences of state variables $\{K(t+1), B(t+1)\}_{t=0}^{\infty}$, allocations $\{\{c_t(t+j)\}_{j=0}^{T}, \{l_t(t+j)\}_{j=0}^{T}\}_{t=0}^{\infty}$ and price system $\{R(t), w(t), r(t)\}_{t=0}^{\infty}$ is a competitive equilibrium if, for all t, the sequences satisfy the following conditions:*

1. *Given the price system, the allocations maximize the households' lifetime utility*

subject to their lifetime budget constraints;

2. *Given the price system, the allocations and the state variables maximize the firm's profit;*

3. *They satisfy the government's flow budget constraints and the constant primary deficit per GDP rule;*

4. *They satisfy the no-arbitrage condition,* $R(t) = 1 + r(t) - \delta$*; and*

5. *They clear all markets.*

In a competitive equilibrium, the dynamical system can be transcribed [8)] as

$$Z({}_{t-T}R^{t+T}) = K(t+1) + B(t+1), \tag{15}$$

$$B(t+1) = R(t)B(t) + D(t) \tag{16}$$

where ${}_{t-T}R^{t+T} := \{R_{t-T}, R_{t-T+1}, \cdots, R_{t+T-1}, R_{t+T}\}$ means the sequence of interest rates of public debt in from period $t-T$ to $t+T$, and $Z({}_{t-T}R^{t+T})$ is aggregate asset holdings in period t.

3.5 Balanced Growth Path Steady State

Furthermore, we focus on a balanced growth path steady state in order to investigate what types of fiscal policies are sustainable. A balanced growth path steady state is defined as follows:

Definition 2. *A set of sequences of state variables* $\{K(t+1), B(t+1)\}_{t=0}^{\infty}$*, allocations* $\{\{c_t(t+j)\}_{j=0}^{T}, \{l_t(t+j)\}_{j=0}^{T}\}_{t=0}^{\infty}$ *and price system* $\{R(t), w(t), r(t)\}_{t=0}^{\infty}$ *is a balanced growth path steady state ("steady state") if the set is a competitive equilibrium and the interest rate,* $R(t)$*, is constant for all* t*,* $R(t) = R$*.*

[8)] The derivation of the dynamical system, equations (15) and (16), is demonstrated in Appendix.

If there exists a steady state, the following conditions are satisfied in the steady state,

$$Z(t, R) = K(t + 1, R) + B(t + 1, R), \tag{17}$$

$$B(t + 1, R) = RB(t, R) + D(t, R). \tag{18}$$

Eliminating B, we obtain

$$\Phi(R) = \Theta(R; D). \tag{19}$$

The derivation of equation (19) is explained in Appendix. However, we can intuitively understand equation (19). $\Phi(R)$ corresponds to the ratio of aggregate assets holdings to GDP in the steady state. $\Theta(R; D)$ means the sum of the levels of physical capital to GDP ratio and of public debt to GDP ratio.

We can show that there is a critical value of primary deficit $D^* \geq 0$ such that

- if $D \leq D^*$, equation (19) has a solution with respect to R, and
- if $D > D^*$, equation (19) has no solution with respect to R.

In other words, when the size of primary deficit is too large, the dynamical system has no steady state. Therefore, we can consider that there exists a maximum sustainable level of primary deficit per GDP. Furthermore, the theoretical results obtained in the previous section imply that, if there is no steady state, the ratio of public debt to GDP necessarily diverge infinitely for *any* initial amounts of public debt. In the following section, we calibrate the maximum sustainable size of primary deficit per GDP and find the necessary condition for preventing the ratio of public debt to GDP from diverging.

4 Numerical Evaluation

We numerically evaluate the maximum sustainable level of primary deficit in this section: this section is the main part of our paper. We take three steps to obtain quantitative findings. First, we calibrate the preference parameters, β, θ, and γ, so as to match the steady state data for the Japanese economy. Second, we also calibrate the other parameters, using the economic data for Japan in 2005 and the preference parameters calibrated. Lastly, the sustainable size of primary deficit to GDP ratio is numerically calculated.

4.1 Calibrating Preference Parameters

We calibrate the preference parameters by using data for the Japanese economy in 1985. The reason we use the Calendar Year (CY) 1985 data is that the relationship between the size of primary deficit and the amount of public debt can be described as a steady state in our model. Concretely speaking, the ratio of Japanese primary surplus to GDP was 0.71 % and that of net financial liabilities of Japan's general government to GDP was 30.8 % in CY 1985. This relationship can be sustained in the long-run. Therefore, we assume that the Japanese economy was in a steady state in 1985 and calibrate the preference parameters by using 1985 data.

We set the model parameters except for the preference parameters in order to calibrate them. The length of lifetime, T, is set to 61, which means that households live for 62 periods[9]. The weight parameter in the production function, α, is set to 0.354, so as to match the ratio of capital income to GDP in CY 1985[10]. The growth rate of population, n and that of labor productivity, h, are set

[9] We suppose that individuals enter the economy when they are 20 years old. Thus, households live from 20 to 81 years old.

[10] *Capital income* is made from the sum of (i) operating surplus, (ii) 40 percent of mixed income, and (iii) consumption of fixed capital in CY 1985. These data are obtained from Japan's SNA national accounts data.

to $n = 0$ and $h = 0.0175$, respectively, in the steady state[11]. The profiles of labor productivity of households depend on their generations and age. Following Ishikawa et al. (2012), we identify the profiles of labor productivity as follows.

$$\frac{\exp(0.691591 + 0.044425 \cdot j - 0.00086 \cdot j^2)}{\exp(0.691591)} \times (1+h)^{t-1}, \tag{20}$$

where j is household's age ($j = 0, 1, \cdots, T$), and h is the growth rate of labor productivity. Lastly, the intertemporal elasticity of substitution is assumed to be 0.8.

Using our overlapping generations model and the parameters calibrated as above, the preference parameters, β, θ, and γ, are also calibrated matching the data for the Japanese economy in 1985. The target ratios of capital to GDP and of public debt to GDP in the steady state are set to $K/Y = 1.9088$ and $B/Y = 0.3084$, respectively[12]. The target ratio of the average working time to the discretionary time of households is given as 0.5767[13]. Lastly, the target real interest rate on public debt is supposed to be $R - 1 = 0.04217$, which is consistent with the Japan's nominal interest rate on public debt, 6.61 %, and the inflation rate of Japan, 2.30 % in 1985[14]. We calibrate the preference parameters to maintain consistency with the data explained above. As a result, we obtain the calibrated parameters, $\beta = 0.9669$, $\theta = 0.5338$, and $\gamma = 1.468$.

[11] The growth rates set here, h and n, are lower than the actual ones in 1985. We consider that the actual growth rates cannot be kept in the long-run and thus the steady state growth rates must be lower.

[12] They are obtained from SNA national accounts data. The amount of physical capital, K, is made from the sum of the stocks of tangible and intangible fixed asset. The amount of public debt, B, is the net financial liabilities of Japan's general government.

[13] The target ratio is set in accordance with 1986 data from the "Survey of Time Use and Leisure Activities" by the Ministry of Internal Affairs and Communications.

[14] The nominal interest rate is obtained as the average of the interest rates of Japanese government bonds whose remaining duration was 9 years in 1985. The longest remaining duration is 9 years in those of available interest rates reported in the dataset from the Ministry of Finance Japan, and thus, we take the average as the long-run interest rate. The inflation rate is made from the GDP deflators of 1984 and 1985 in Japan.

4.2 Calibrating the Remaining Parameters

Next, we calibrate the other model parameters, using recent data for the Japanese economy and the preference parameters obtained in the previous subsection.

The length of lifetime, T, remains as 61. The parameter in the production function is set to $\alpha = 0.370$, based on the capital income share in 2005. The depreciation rate of physical capital is calibrated as $\delta = 0.078$, based on the ratio of the size of consumption of fixed capital to the amount of physical capital in 2005[15)]. The profiles of labor productivity are estimated in a similar way to those in section 4.1 and we obtain

$$\frac{\exp(0.971 + 0.0485 \cdot j - 0.000915 \cdot j^2)}{\exp(0.971)} \times (1 + h)^{t-1}, \tag{21}$$

where j is household's age. The growth rate of population is assumed to be zero. We note that the growth rate of labor productivity, h, is given afterwards.

In addition, we assume that an exogenous difference exists between the interest rate on public debt and the return of physical capital. The reason we introduce the difference is that we replicate the actual gap between the interest rates and increase the accuracy of our evaluations on the sustainable size of primary deficit. Ueda (2012) explains that the difference can be observed, as the interest rates of public debt in Japan have been around 1.5 % since 2000, while the rates of return of physical capital have been 4.5–5.0 %. Moreover, Sakuragawa and Hosono (2010) emphasize that the difference between the interest rates may have a significant effect on fiscal sustainability. Based on these studies, we introduce the exogenously given gap in the interest rates, s,

[15)]The size of consumption of fixed capital is obtained from the SNA national accounts data. The amount of physical capital is made from the sum of the stocks of tangible and intangible fixed assets.

T	length of lifetime	61
α	capital income share	0.370
δ	depreciation rate of physical capital	0.091
s	difference between the interest rates	0.036
h	growth rate of labor productivity	(exogenously given)
n	growth rate of population	0.000
$e_t(t+j)$	profile of labor productivity	$\frac{\exp(0.971 + 0.0485 \cdot j - 0.000915 \cdot j^2)}{\exp(0.971)} \times (1+h)^{t-1}$
γ	parameter corresponding to RRA	1.468
θ	weight parameter between c and $1-l$	0.5338
β	subjective discount rate	0.9669

Table 1: List of Calibrated Parameters

and thus, the no-arbitrage condition is rewritten as

$$R(t) = 1 + r(t) - \delta - s. \tag{22}$$

In fact, there are several reasons why the interest rate on public debt differs from the rate of return of capital: uncertainty on changes in volume of physical capital, financial intermediation costs, risk premiums, and so on. In our paper, the difference between the interest rates is set to $s = 0.036$ based on Ueda (2012), which means that the interest rate on public debt continues to be lower than the rate of return of capital by 3.6 %. Table 1 is the list of the calibrated model parameters. Lastly, we define a *benchmark case* as one in which the model parameters are set as those in table 1.

4.3 Results of Numerical Calculations

In this section, we calculate the maximum sustainable size of the primary deficit to GDP ratio in the benchmark case. Our procedure of calculating the sustainable primary deficit to GDP ratio is as follows. First, we exogenously give various growth rates of labor productivity, h. We next calculate the maximum

h	Maximum P.D. (%)	Debt Interest Rate (%)	K/Y	B/Y
$h = 0.050$	N.A.	—	—	—
$h = 0.055$	0.0007	5.3091	2.0454	0.3456
$h = 0.060$	0.2118	5.5399	2.0285	0.4604
$h = 0.075$	1.0513	6.4787	1.9292	1.0294
$h = 0.080$	1.4064	6.9992	1.8782	1.4053

Table 2: Maximum Sustainable Levels of Primary Deficit per GDP

Note: h is the growth rate of labor productivity and "Maximum P.D." means the maximum sustainable level of the ratio of primary deficit to GDP (%). "N.A." means that there exists no steady state for any levels of primary deficit per GDP. "Debt Interest Rate", K/Y, and B/Y are the real interest rate on public debt (%), the ratio of capital to GDP, and the public debt per GDP in the steady state, respectively.

sustainable level of the ratio of primary deficit to GDP.

Our results of numerical evaluations are described in table 2. We show that, to sustain the primary deficit, the rate of economic growth needs to be more than 5 %. Furthermore, for example, if the level of primary deficit is maintained at one percent of GDP, the economic growth rate must be kept to around 7.5 %. We note that the condition on the size of primary deficit is a necessary but not sufficient one to guarantee fiscal sustainability. As shown in Chalk (2000), the ratio of public debt to GDP does not diverge if, and only if, there exists a steady state *and* the initial amount of public debt is sufficiently small. In fact, depending on the initial amount of public debt, a much higher growth rate than 5 % may be necessary to prevent the ratio of public debt to GDP from diverging.

Our results imply that a primary surplus is necessary for Japan in the long-run in order to avoid divergence of the public debt to GDP ratio under realistic assumptions of economic growth rate. In other words, it is unrealistic to attain fiscal sustainability only by boosting economic growth without eliminating primary deficit in Japan.

IES (target)	θ	γ	β
IES = 0.25	0.5771	6.1983	0.9968
IES = 0.5	0.5658	2.7674	0.9734
IES = 0.8 (benchmark)	0.5338	1.4683	0.9669
IES = 1.2	0.3710	0.5508	0.9652

Table 3: List of Alternative Preference Parameters under Various IES

Note: "IES" means the intertemporal elasticity of substitution. This table describes the calibrated values of preference parameters, θ, γ, and β, so as to match to the target level of IES, by using the data for the Japanese economy in 1985.

4.4 Comparative Analysis

Next, we do a comparative analysis. In particular, we focus on the intertemporal elasticity of substitution. The intertemporal elasticity of substitution in Japan has been estimated in many previous studies, such as Hamori (1996) and Fuse (2004). The values estimated by them are diverse, and thus, our calibration based on the single number of the intertemporal elasticity of substitution may not be robust.

The procedure of our comparative analysis is as follows. First, we assume another value of the intertemporal elasticity of substitution. Second, we again calibrate the preference parameters, β, γ, and θ, using the data for the Japanese economy in 1985. Third, we calibrate the other parameters to match to the data for 2005 and recalculate the maximum sustainable level of primary deficit per GDP. Here, we consider three cases in which the values of the intertemporal elasticity of substitution are assumed to be 0.25, 0.5, and 1.2, respectively[16)]. In each case, we obtain the calibrated preference parameters as table 3.

Using the preference parameters, we calculate the maximum sustainable level of primary deficit per GDP under various intertemporal elasticity of substitution. Our numerical results of the comparative analysis are described in

[16)] We note that if we assume a too large value of the intertemporal elasticity of substitution, there is no preference parameter to match to the data for Japanese economy in 1985.

	h = 0.05	h = 0.06	h = 0.08	h = 0.10
IES = 0.25	N.A.	N.A.	N.A.	N.A.
IES = 0.5	N.A.	N.A.	0.0002	0.1637
IES = 0.8 (benchmark)	N.A.	0.2118	1.4064	3.0686
IES = 1.2	0.0002	1.4379	8.0904	13.473

Table 4: Maximum Sustainable Levels of Primary Deficit under Various IES

Note: "IES" and h are the intertemporal elasticity of substitution and the growth rate of labor productivity, respectively. This table describes the maximum sustainable levels of the ratio of primary deficit to GDP (%). "N.A." means there exists no steady state for any levels of primary deficit to GDP.

table 4. From our results, the higher (lower) the intertemporal elasticity of substitution is, the larger (smaller) the primary deficit contrast to GDP can be kept. The intuitive explanation for the results is as follows. Suppose that households' intertemporal elasticity of substitution is high. When the interest rate rises, households would like to increase their labor supply and asset holdings because of the stronger intertemporal substitution effect, which increases the demand for public debt and lowers the interest rate on public debt.

However, under various assumptions, we consider that unrealistically high rates of economic growth remain necessary to sustain the primary deficit, although the difference in the intertemporal elasticity of substitution affects the sustainable size of primary deficit. Even if the intertemporal elasticity of substitution is increased to 1.2, we would still need economic growth of five percent in order for Japan's government to maintain the primary deficit.

5 Conclusion

We have investigated how large a primary deficit to GDP ratio Japan's government can sustain. We constructed an overlapping generations model in which multi-generational households live and the government maintains a constant ratio of primary deficit to GDP. We have numerically shown that the primary deficit cannot be sustained unless the rate of economic growth is unrealistically high. Our result implies that Japan's government needs to achieve a positive primary balance in order to avoid the divergence of the ratio of public debt to GDP.

However, there are still some problems with our analysis which need to be solved. One of the important problems is that we have not obtained the result on the sustainable size of initial public debt. As shown by Chalk (2000), a sufficiently small amount of initial public debt is also necessary to avoid the divergence of the ratio of public debt to GDP infinitely. Nevertheless, we have not investigated the conditions on the sustainable size of initial public debt. One of the reasons is that we would have to calculate transition paths of public debt and of physical capital in order to obtain the initial condition, and the calculation of transition paths is very complex. Another future task is that economic institutions and fiscal systems need to be considered when we investigate the set of sustainable fiscal policies. For example, the imperfections of financial markets, the systems of social pension and of medical insurance must be introduced, because they greatly affect fiscal sustainability through the actions of agents.

Although problems still remain, as explained above, we consider that our research is valuable for suggesting future directions of the Japan's fiscal stance. We believe that this paper contributes not only to macroeconomic and public finance literatures, but also to deciding on future fiscal policies for Japan.

References

Arai, R. (2011) "Productive Government Expenditure and Fiscal Sustainability," *FinanzArchiv (Public Finance Analysis)*, Vol. 67, No. 4, pp. 327–351.

Arai, R. and T. Kunieda (2011) “Sustainability of Fiscal Policy Rules and Credit Constraints: Theory and Evidence,” Department of Economics and Finance Working Paper No.2011001, City University of Hong Kong.

Bräuninger, M. (2005) “The Budget Deficit, Public Debt, and Endogenous Growth,” *Journal of Public Economic Theory*, Vol. 7, No. 5, pp. 827–840.

Chalk, N.A. (2000) “The Sustainability of Bond-financed Deficits: An Overlapping Generations Approach,” *Journal of Monetary Economics*, Vol. 45, No. 2, pp. 293–328.

Chalk, N.A. and R. Hemming (2000) “Assessing Fiscal Sustainability in Theory and Practice,” IMF Working Paper No. 00/81.

Fuse, M. (2004) “Estimating Intertemporal Substitution in Japan,” *Applied Economics Letters*, Vol. 11, pp. 267–269.

Hamilton, J.D. and M.A. Flavin (1986) “On the Limitations of Government Borrowing: A Framework for Empirical Testing,” *American Economic Review*, pp. 808–819.

Hamori, S. (1996) “Consumption Growth and the Intertemporal Elasticity of Substitution: Some Evidence from Income Quintile Groups in Japan,” *Applied Economics Letters*, Vol. 3, pp. 529–532.

Ihori, T., T. Nakazato, and M. Kawade (2003) “Japan’s Fiscal Policies in the 1990s,” *World Economy*, Vol. 26, No. 3, pp. 325–338.

Ihori, T., R. Kato, M. Kawade, and S. Bessho (2006) “Public Debt and Economic Growth in an Aging Japan,” in Kaizuka, K. and A. Krueger eds. *Tackling Japan’s Fiscal Challenges*: Palgrave Macmillan.

İmrohoroğlu, S. and N. Sudo (2011) “Will a Growth Miracle Reduce Debt in Japan?” IMES Discussion Paper No. 2011-E-1.

Ishikawa, D., J. Ueda, and R. Arai (2012) "Future Changes of the Industrial Structure due to Aging and Soaring Demands for Health Care Services in Japan? An Analysis Using a Multi-Sector OLG Model in an Open Economy." mimeo.

Sakuragawa, M. and K. Hosono (2010) "Fiscal Sustainability of Japan: A Dynamic Stochastic General Equilibrium Approach," *Japanese Economic Review*, Vol. 61, pp. 517–537.

Yakita, A. (2008) "Sustainability of Public Debt, Public Capital Formation, and Endogenous Growth in an Overlapping Generations Setting," *Journal of Public Economics*, Vol. 92, No. 3-4, pp. 897–914.

Ueda, J. (2012) *Fiscal Policies under Dynamic Controls — A Future Perspective of Social Security System —*: Iwanami Shoten. (in Japanese).

Appendix

A Details of Overlapping Generations Model with Multi Generations

A.1 Household's Problem

The utility maximization problem of household born at period t is given by:

$$\max \sum_{i=0}^{T} \beta^i \frac{[c_t(t+i)^\theta (1-l_t(t+i))^{1-\theta}]^{1-\gamma}}{1-\gamma} \tag{23}$$

$$\begin{aligned}
\text{s.t. } & c_t(t) + z_t(t) = w(t)e_t(t)l_t(t) \\
& c_t(t+1) + z_t(t+1) = w(t+1)e_t(t+1)l_t(t+1) + R(t+1)z_t(t) \\
& \vdots \\
& c_t(T) = w(T)e_t(T)l_t(T) + R(T)z_t(T-1), \\
& l_t(t+i) \geq 0 \;\; \forall i = 0, 1, \cdots, T.
\end{aligned} \tag{24}$$

Combining the flow-budget constraints, we obtain the lifetime budget constraint. To solve the problem, we define the Lagrangian as:

$$\begin{aligned}
\mathcal{L} = & \sum_{i=0}^{T} \beta^i \frac{[c_t(t+i)^\theta (1-l_t(t+i))^{1-\theta}]^{1-\gamma}}{1-\gamma} \\
+ & \mu \left[\sum_{i=0}^{T} w(t+i)e_t(t+i)l_t(t+i) \times \left(\prod_{j=1}^{i} R(t+j)^{-1} \right) - \sum_{i=0}^{T} c_t(t+i) \times \left(\prod_{j=1}^{i} R(t+j)^{-1} \right) \right] + \sum_{i=0}^{T} \lambda_i l_t(t+i).
\end{aligned} \tag{25}$$

The first-order conditions are

$$\beta^i \theta c_t(t+i)^{\theta-1}(1-l_t(t+i))^{1-\theta} \times [c_t(t+i)^{\theta}(1-l_t(t+i))^{1-\theta}]^{-\gamma} - \mu\left(\prod_{j=1}^{i} R(t+j)^{-1}\right) = 0 \tag{26}$$

$$\Rightarrow \beta^i \theta c_t(t+i)^{\theta(1-\gamma)-1}(1-l_t(t+i))^{(1-\theta)(1-\gamma)} - \mu\left(\prod_{j=1}^{i} R(t+j)^{-1}\right) = 0, \tag{27}$$

$$-\beta^i(1-\theta)c_t(t+i)^{\theta}(1-l_t(t+i))^{-\theta} \times [c_t(t+i)^{\theta}(1-l_t(t+i))^{1-\theta}]^{-\gamma} + \mu w_t(t+i)e_t(t+i)\left(\prod_{j=1}^{i} R(t+j)^{-1}\right) + \lambda_i = 0 \tag{28}$$

$$\Rightarrow \beta^i(1-\theta)c_t(t+i)^{\theta(1-\gamma)}(1-l_t(t+i))^{(1-\theta)(1-\gamma)-1} - \mu w_t(t+i)e_t(t+i)\left(\prod_{j=1}^{i} R(t+j)^{-1}\right) + \lambda_i = 0. \tag{29}$$

If $\lambda_i = 0$, we obtain the following equations from the two first-order conditions:

$$c_t(t+i) = \left(\frac{\theta}{\mu}\right)^{1/\gamma}\left(\prod_{j=1}^{i}(\beta R(t+j))\right)^{1/\gamma}\left[w(t+i)e_t(t+i)\frac{\theta}{1-\theta}\right]^{-(1-\theta)(1-\gamma)/\gamma}, \tag{30}$$

$$\frac{1-\theta}{\theta}c_t(t+i) = (1-l_t(t+i))w_{t+i}e_t(t+i). \tag{31}$$

Meanwhile, if $\lambda_i > 0$, we have $l_t(t+i) = 0$ from the complementarity condition. Combining the first order condition and $l_t(t+i) = 0$,

$$c_t(t+i) = \left[\frac{\theta}{\mu}\left(\prod_{j=1}^{i}(\beta R(t+j))\right)\right]^{\frac{1}{(1-\theta)(1-\gamma)+\gamma}} \tag{32}$$

$$\beta^i(1-\theta)c_t(t+i)^{\theta(1-\gamma)} - \mu w_t(t+i)e_t(t+i)\left(\prod_{j=1}^{i} R(t+j)^{-1}\right) < 0 \tag{33}$$

Then, given $\{e_t(t+i)\}_{i=0}^{T}$ and $\{w_{t+i}, R_{t+i}\}_{i=0}^{T}$,

- if $\mu \geq \tilde{\mu}(i)$,

$$c_t(t+i) = \left(\frac{\theta}{\mu}\right)^{1/\gamma} \left(\prod_{j=1}^{i} (\beta R(t+j))\right)^{1/\gamma} \left[w(t+i)e_t(t+i)\frac{\theta}{1-\theta}\right]^{-(1-\theta)(1-\gamma)/\gamma}, \tag{34}$$

$$l_t(t+i) = 1 - \frac{1-\theta}{\theta} \frac{c_t(t+i)}{w(t+i)e_t(t+i)}, \tag{35}$$

- and if $\mu < \tilde{\mu}(i)$,

$$c_t(t+i) = \left[\frac{\theta}{\mu} \left(\prod_{j=1}^{i} (\beta R(t+j))\right)\right]^{\frac{1}{(1-\theta)(1-\gamma)+\gamma}}, \tag{36}$$

$$l_t(t+i) = 0, \tag{37}$$

where $\tilde{\mu}(i)$ is defined as

$$\tilde{\mu}(i) = [w(t+i)e_t(t+i)]^{-(1-\theta)(1-\gamma)-\gamma}\ \theta^{\theta(1-\gamma)} \left(\prod_{j=1}^{i} (\beta R(t+j))\right). \tag{38}$$

To derive the profiles of consumption and leisure of households, we need to obtain the value of μ. μ can be obtained by substituting equations (34)-(37) into the household's intertemporal budget constraint.

From the profiles of consumption and leisure of household $\{c_t(t+i), l_t(t+i)\}_{i=0}^{T}$, we can recursively calculate the profile of asset holdings $\{z_t(t+i)\}_{i=0}^{T}$ using the flow budget constraints.

Finally, we derive the aggregate asset holdings $Z(t)$ as

$$Z(t) = \sum_{i=0}^{T} z_{t-i}(t)N_{t-i} = N(1)\sum_{i=0}^{T} z_{t-i}(t)(1+n)^{t-i-1}. \tag{39}$$

A.2 Firm's Problem

The firm solves the following profit-maximization problem.

$$\max A(t)K(t)^{\alpha}L(t)^{1-\alpha} - r(t)K(t) - w(t)L(t). \tag{40}$$

where $L(t) = \sum_{i=0}^{T} e_{t-i}(t)l_{t-i}(t)N_{t-i}$. F.O.C.s are

$$r(t) = A(t)\alpha K(t)^{\alpha-1}L(t)^{1-\alpha} \tag{41}$$

$$w(t) = A(t)(1-\alpha)K(t)^{\alpha}L(t)^{-\alpha} \tag{42}$$

A.3 Government

A Government finances the primary deficit, $D(t)$, and the rollover plus the interest payment of the existing public debt, $R(t)B(t)$, by issuance of public debt. Then, the flow budget constraint of the government is following:

$$B(t+1) = R(t)B(t) + D(t). \tag{43}$$

The government is assumed to keep primary deficit per GDP constant. That is, for all t,

$$D = \frac{D(t)}{Y(t)}. \tag{44}$$

A.4 Equilibrium

A competitive equilibrium of this economy is defined as a set of sequences of state variables, allocations, and price system which satisfy the following conditions in all period t:

1. Given the price system, the allocations solve the household's utility maximization problem;

2. Given the price system, the allocations and state variables solve the firm's profit maximization problem;

3. They satisfy the flow budget constraints of government;

4. They satisfy the following no arbitrage condition on interest rates,

$$1 + f'(k(t)) - \delta = \alpha K(t)^{\alpha-1} L(t)^{1-\alpha} + 1 - \delta = R(t); \tag{45}$$

5. they clear all markets.

A.5 Balanced Growth Path Steady State

Next, we define a balanced growth path steady state (referred to as steady state). We focus on existence of steady states in the numerical analysis. A set of sequences of state variables, allocations, and price system is a balanced growth path steady state if

1. the set of the sequences of the state variables, of allocations, and of price system is a competitive equilibrium, and

2. the gross interest rate is constant forever, $R(t) = R$ for any t.

From the capital market clearing condition and the government's budget constraints, we have

$$\tilde{Z}(t) = \frac{Y(t+1)}{Y(t)}[\tilde{B}(t+1) + \tilde{K}(t+1)], \tag{46}$$

$$\tilde{B}(t+1)\frac{Y(t+1)}{Y(t)} = R(t)\tilde{B}(t) + D, \tag{47}$$

where $\tilde{X}$ is denoted by the ratio of X to GDP. Eliminating $\tilde{B}(t)$ from (46) and (47), we obtain

$$\tilde{Z}(t) - \frac{Y(t+1)}{Y(t)}\tilde{K}(t+1) = R(t)\left[\frac{Y(t-1)}{Y(t)}\tilde{Z}(t-1) - \tilde{K}(t)\right] + D. \tag{48}$$

Finally, in the steady state, (48) can be rewritten as

$$\tilde{Z}(R) = (1+h)(1+n)\left[\frac{\alpha}{R-(1-\delta)}\right] + (1+h)(1+n)\frac{D}{(1+h)(1+n)-R}. \tag{49}$$

R in the steady state must satisfy equation (49). Thus, we obtain equation (19), replacing the left-hand side and the right-hand side of (49) with $\Phi(R)$ and $\Theta(R; D)$, respectively.