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# Temporary Bubbles and Discount Window Policy\*

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## Abstract

This paper presents a monetary growth model where limited communication and random relocation create endogenous roles for money and banks. The economy can exhibit two different regimes. In the first, money is a dominated asset and banks economize cash reserves. In the second, money has the same return as capital and banks use the reserves as storage. I show that the economy can experience switching between the two regimes and that cyclical bubbles can occur. In addition, discount window lending is considered as a counter-bubble policy. I also show that the discount window can simultaneously lead the economy to the social optimum and stabilize bubbly fluctuations when the economy is dynamically inefficient.

**Key words:** overlapping generations, temporary bubbles, discount window.

**JEL Classification:** D90, E32, E44

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# 1 Introduction

There is a large body of theoretical literature on the interactions between monetary systems and economic volatility. Many economists attempt to explain economic volatility within general equilibrium frameworks. The overlapping generations model is one of the most useful methods to explore the existence of multiple equilibria and indeterminacy, which is the source of endogenous volatility.<sup>1</sup> Michel and Wigniolle (2003, 2005) study interesting cyclical bubbly equilibria in an overlapping generations model with a cash-in-advance constraint. They show that along an intertemporal equilibrium, the economy can experience both periods where money is a dominated asset, which are called *Hahn and Solow regimes*, and periods where money and capital have the same return, which are called *Tirole regimes*. Since periods in Tirole regimes are finite, they refer to these periods as *temporary bubbles*. Here, I address the following questions: Do temporary bubbles occur in a microfounded model of money? If so, when? How should the central bank deal with the bubbles? The goal of this paper is to develop a monetary growth model with frictions that give rise to a role for money and to answer these questions.

The theory presented here is an overlapping generations model with spatial separation and random relocation, which was popularized by Schreft and Smith (1997, 1998). The economy consists of two spatially separated islands, and some agents are randomly relocated to a different island from the one on which they were born. Since communication between the two islands is limited, the only asset that relocated agents can use is fiat money. Communication frictions prevent movers from transacting with privately issued liabilities in the new location. Thus, spatial separation and limited communication create endogenous roles for money and allow money to be held even when dominated

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<sup>1</sup>See Azariadis (1993), Cass and Shell (1983), and Grandmont (1985).

in the rate of return. In addition, the stochastic relocations act as shocks to the agents' liquidity preferences and create an opportunity for the banks to provide insurance against these shocks, as in Diamond and Dybvig (1983). Since banks have an active role in the model, it is possible to consider liquidity injections or discount window lending as a counter-bubble policy. The clear differences between the cash-in-advance model and mine are that money holding has a microfoundation and banks play endogenous roles in this paper.

The main results of this paper are as follows: (i) the equilibrium in which money and capital have the same return, which is called a *Tirole regime*, is a unique equilibrium when the relocation shock is below a threshold level, while the equilibrium in which money is a dominated asset, which is called a *Schreft and Smith regime*, exists when the shock is greater than the threshold level; (ii) the economy can experience regime switches only when the equilibrium in the Schreft and Smith regime exists; and (iii) if the economy is dynamically inefficient, the discount window policy can lead the equilibrium in the Schreft and Smith regime to the first-best allocation and can eliminate the cyclical bubbles. In this paper, cyclical bubbles are used interchangeably with temporary bubbles.

The result (i) shows that the level of the relocation shock is quite important in determining the equilibrium regime. Few studies mention this fact. Result (ii) confirms the robustness of the temporary bubbles studied by Michel and Wigniolle (2003) in different frameworks. The difference between Michel and Wigniolle's work and this paper is that the existence of an equilibrium in the Schreft and Smith regime is not a sufficient condition for a two-period cycle in this paper. They show that the existence of a two-period cycle is guaranteed when there exists an equilibrium in the Hahn and Solow regime. In contrast, I show that there exists an equilibrium in the Schreft and Smith regime that cannot experience any cycles. Note that the Hahn and Solow regime in their

model is equivalent to the Schreft and Smith regime in the model of this paper. Result (iii) states that the discount window policy not only restores efficiency but also stabilizes the economy. These results can be considered as theoretical contributions.

Some papers study economic volatility using the overlapping generations model with random relocation. Bhattacharya et al. (1997) and Schreft and Smith (1997, 1998) produce a monetary growth model in which banks provide liquidity and the government issues both money and bonds. They show how multiple steady states, endogenous volatility, and indeterminacies can arise in such a framework. Gomis-Porqueras (2000) considers their model but without bonds, and shows that the equilibrium path experiences endogenous volatility when agents are sufficiently risk-averse and when the elasticity of substitution between capital and labor is relatively low. He points out that the results of Schreft and Smith depend on the design of monetary policy. Paal and Smith (2004) construct an endogenous growth model with a collateral constraint and show the threshold effect in the relationship between inflation and growth rate. In their model, there also exist dynamic equilibrium paths that display oscillation when agents are sufficiently risk-averse. The main difference between all of these analyses and mine is that they focus on different types of equilibrium indeterminacy. This paper shows that another endogenous volatility, which is inspired by Michel and Wigniolle (2003), can arise in this framework.

This paper is also related to Antinolfi et al. (2001) and Antinolfi and Keister (2006), where the central bank, as a lender of last resort, prints money and injects it into banks that face high liquidity demands. In their model, the discount window policy relaxes the liquidity constraints of banks and allows banks to provide better insurance for depositors. This paper points out that the discount window has a role not only in relaxing the liquidity constraints but also in stabilizing bubbly fluctuations. This policy can also allow banks

to economize cash reserves and to increase investments in capital. I show that when the economy is dynamically inefficient, this policy can simultaneously lead the economy to the social optimum and eliminate cyclical bubbles.<sup>2</sup>

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 analyses the equilibria of the two different regimes, and Section 4 studies equilibria with regime switching and economic welfare. Section 5 considers the discount window policy that reduces bubbly volatility. Section 6 concludes.

## 2 The Model

The economy has the same basic structure as in Schreft and Smith (1997, 1998). Time is discrete and denoted by  $t = 0, 1, 2, \dots$ . The world is divided into two spatially separated locations. Each location is populated by a continuum of agents of unit mass. There is no population growth. Agents live for two periods and are endowed with one unit of labor that they supply inelastically. At  $t = 0$ , there is a continuum of old agents with unit mass in each location. Each of these agents is endowed with  $M > 0$  units of fiat money, which I will refer to as “base money.” The stock of base money is constant over time. As is standard in such literature, I assume that the agents derive utility from consuming the good only when old. The utility function is given by  $u(c) = \ln c$ .

The consumption good is produced by perfectly competitive firms that rent capital,  $K$ , and hire labor,  $L$ , from the young agents. I assume that the production technology is of the Cobb-Douglas form,  $f(k) = Ak^\alpha$ , where  $k \equiv K/L$  is the capital-labor ratio. For simplicity, the depreciation of capital is complete in each period.

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<sup>2</sup>Haslag and Martin (2007) show that if the central bank implements discount window lending and the Friedman rule, the economy can achieve the social optimum. However, since the production technology is linear in their model, they consider only the case of dynamic efficiency.

After receiving the wage and depositing it into a bank, the agents learn whether they must move to the other location. Let  $\pi$  denote the probability that an individual will be relocated. The law of large numbers holds and hence  $\pi$  also represents the measure of movers. The movers redeem their deposits in the form of money, as this is the only way for them to acquire goods in the new location. Spatial separation and limited communication generate a transactions role for money. Money can be valued even if it is dominated in return. In contrast, the non-movers redeem their deposits in the form of goods. The stochastic relocations act as shocks to portfolio preferences. Hence, they motivate banks to insure agents against random liquidity needs, as in Diamond and Dybvig (1983).

## 2.1 The Social Optimum

I begin with the first best solution in this environment.<sup>3</sup> The social planner can directly control investment and allocation decisions in both locations and is essentially unaffected by communication friction. Let  $c^m$  and  $c^n$  denote the consumption of movers and non-movers, respectively. Efficient allocation maximizes the steady-state expected utility of a representative agent subject to a feasibility constraint. The planner's problem is:

$$\begin{aligned} \max_{c^m, c^n, k} \quad & \pi u(c^m) + (1 - \pi)u(c^n) \\ \text{s.t.} \quad & \pi c^m + (1 - \pi)c^n + k = f(k). \end{aligned}$$

The efficient allocation in the steady state, denoted by  $c^{m*}$ ,  $c^{n*}$ ,  $k^*$ , satisfies the following conditions:

$$\begin{aligned} f'(k^*) &= 1, \\ c^{m*} &= c^{n*} = f(k^*) - k^*. \end{aligned}$$

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<sup>3</sup>Haslag and Martin (2007) consider the problem of a social planner in this environment with linear storage technology.

At the optimum, the consumption levels of movers and non-movers are equalized, and the net production is maximized. If the production function has the Cobb-Douglas form  $f(k) = Ak^\alpha$ , the values of  $c^{m*}$ ,  $c^{n*}$  and  $k^*$  are given by

$$k^* = (\alpha A)^{\frac{1}{1-\alpha}}, \quad (1)$$

$$c^{m*} = c^{n*} = \frac{1-\alpha}{\alpha} (\alpha A)^{\frac{1}{1-\alpha}}. \quad (2)$$

## 2.2 A Banking Economy

The banks take deposits from the young agents and choose how much to invest in capital,  $s_t$ , and real money balances,  $m_t$ . The banks promise a gross real return to pay the movers,  $d^m$ , and to pay the non-movers,  $d$ . Because of free entry, in equilibrium, the banks choose their portfolio in a way that maximizes the expected utility of the representative agents,  $\pi \ln(d_t^m w_t) + (1 - \pi) \ln(d_t w_t)$ , subject to the following constraints:

$$m_t + s_t = w_t, \quad (3)$$

$$\pi d_t^m w_t = \theta_t \frac{p_t}{p_{t+1}} m_t, \quad (4)$$

$$(1 - \pi) d_t w_t = (1 - \theta_t) \frac{p_t}{p_{t+1}} m_t + R_{t+1} s_t. \quad (5)$$

Equation (3) is the banks' balance sheet constraint. Equation (4) states that the real money balances held by the banks must be sufficient to satisfy the liquidity demand from the movers. Equation (5) states that the remaining money and goods go to the non-movers. Of course,  $0 \leq \theta_t \leq 1$  and the non-negativity constraint must hold.

Let  $\gamma_t \equiv m_t/w_t$  represent the reserve-deposit ratio. Then, the banks' problem can be rewritten as

$$\max_{\gamma_t, \theta_t \in [0,1]} \ln w_t + \pi \ln \left[ \theta_t \frac{\gamma_t}{\pi} \frac{p_t}{p_{t+1}} \right] + (1 - \pi) \ln \left[ \frac{1 - \theta_t}{1 - \pi} \gamma_t \frac{p_t}{p_{t+1}} + \frac{1 - \gamma_t}{1 - \pi} R_{t+1} \right].$$

The optimal choices are given by

$$\pi = \theta_t \gamma_t, \quad (6)$$

$$\frac{\pi}{\theta_t} \leq \frac{(1 - \pi)\gamma_t}{1 - \theta_t \gamma_t} = \text{if } \theta_t < 1. \quad (7)$$

The factor markets are perfectly competitive in that the factors of production are paid as per their marginal product. The rental rate for capital,  $R_t$ , and the real wage at period  $t$ ,  $w_t$ , are, respectively,

$$R_t \equiv R(k_t) = \alpha A k_t^{\alpha-1}, \quad (8)$$

$$w_t \equiv w(k_t) = (1 - \alpha) A k_t^\alpha. \quad (9)$$

### 3 Equilibria

An equilibrium of this economy is characterized by the market clearing conditions for money and capital and the optimization of the firms and banks. Because the supply of real balances is equal to  $M/p_t$  and the demand for real balances is given by  $\gamma_t w_t$ , the market clearing for real balances is

$$\frac{M}{p_t} = \gamma_t w_t. \quad (10)$$

Next, the capital stock at period  $t + 1$  must equal the level of investments at period  $t$ . From the banks' balance sheet constraint (3), this requires that

$$k_{t+1} = s_t = (1 - \gamma_t) w_t. \quad (11)$$

#### 3.1 Schreft and Smith Regime

I first consider the case where money is a dominated asset. In this case, money does not serve as the storage tool, and banks finance the consumption of the non-movers only by the capital return. That is,  $\theta_t = 1$  holds, and

conditions (6) and (7) imply that  $\gamma_t^{SS} = \pi$ . In addition, from (9) and (11), I obtain the dynamics of  $k_t$  as follows:

$$k_{t+1} = (1 - \alpha)(1 - \pi)Ak_t^\alpha. \quad (12)$$

The dynamic properties of this equation are the same as the properties of the standard Diamond model. The model has a unique positive steady state that is given by<sup>4</sup>

$$k^{SS} = [(1 - \alpha)(1 - \pi)A]^{-\frac{1}{1-\alpha}}. \quad (13)$$

The arbitrage condition between the two assets implies that the real return on money is not larger than the return on investment in capital:

$$\frac{p_t}{p_{t+1}} \leq R_{t+1}. \quad (14)$$

From conditions (9) and (10), I obtain the equilibrium inflation rate:

$$\frac{p_{t+1}}{p_t} = \frac{\pi w_t}{\pi w_{t+1}} = \frac{k_t^\alpha}{k_{t+1}^\alpha}. \quad (15)$$

Given (8) and (15), the arbitrage condition (14) becomes

$$k_{t+1} \leq \alpha Ak_t^\alpha. \quad (16)$$

Conditions (12) and (16) yield

$$\pi \geq \frac{1 - 2\alpha}{1 - \alpha}. \quad (17)$$

I summarize the result of this subsection as the following proposition.

**Proposition 1** *When  $\pi \geq (1 - 2\alpha)/(1 - \alpha)$ , there exists an intertemporal equilibrium in the Schreft and Smith regime. The dynamics of  $k_t$  monotonically converge to the steady state  $k^{SS}$ .*

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<sup>4</sup>In contrast to the model with a cash-in-advance constraint, the equilibrium of this model does not require an upper limit for relocation shocks. In a model, the cash-in-advance constraint should not be too restrictive for investment in capital to be positive. For details, see Crettez et al. (1999).

Note that condition (17) is equivalent to  $k^{SS} \leq k^*$ . In addition, the consumption of movers and non-movers is given by, respectively,

$$d^m w(k^{SS}) = [(1 - \pi)^\alpha (1 - \alpha) A]^{\frac{1}{1-\alpha}}, \quad (18)$$

$$dw(k^{SS}) = \frac{\alpha}{(1 - \pi)(1 - \alpha)} [(1 - \pi)^\alpha (1 - \alpha) A]^{\frac{1}{1-\alpha}}. \quad (19)$$

Since the returns on money and capital are not equalized, a “wedge” between the return received by movers and non-movers exists. Clearly, the allocation given by (13), (18), and (19) is not the social optimum.

### 3.2 Tirole Regime

Next, I consider the case where money has the same return as capital, i.e.,  $p_t/p_{t+1} = R_{t+1}$ . Money can be viewed as a rational bubble. In this case, money can serve as the storage tool and banks can finance the consumption of the non-movers using money and the capital return. That is,  $\theta_t < 1$  holds, and conditions (6) and (7) imply that  $\pi = \theta_t \gamma_t < \gamma_t$ . Since the equilibrium inflation rate is  $\gamma_t w_t / \gamma_{t+1} w_{t+1}$ , the arbitrage condition is rewritten as

$$\frac{\gamma_{t+1} w_{t+1}}{\gamma_t w_t} = R_{t+1}. \quad (20)$$

Combining (11) and (20) with conditions (8) and (9), I obtain the dynamics of  $\gamma_t$  as follows:

$$\gamma_{t+1} = \frac{\alpha}{1 - \alpha} \frac{\gamma_t}{1 - \gamma_t} \equiv \phi(\gamma_t). \quad (21)$$

The sequence  $\{\gamma_t\}_{t=0}^\infty$  satisfying (21) is characterized by the equilibrium in the Tirole regime, and the corresponding sequence of  $k_t$  is given by

$$k_{t+1} = (1 - \alpha)(1 - \gamma_t) A k_t^\alpha. \quad (22)$$

Then, the positive constant solutions of (21) and (22) are<sup>5</sup>

$$\gamma^T = \frac{1 - 2\alpha}{1 - \alpha} > \pi, \quad (23)$$

$$k^T = k^* = (\alpha A)^{\frac{1}{1-\alpha}}. \quad (24)$$

As Tirole (1985) points out, this steady state exists only if the economy is dynamically inefficient, such that  $\alpha < 1/2$  that is equivalent to  $(1 - 2\alpha)/(1 - \alpha) > 0$ , and achieves the golden rule. The only difference between Tirole's condition and mine is that the bubbly steady state in this model requires a sufficiently low relocation shock, such that  $\pi < (1 - 2\alpha)/(1 - \alpha)$ . The following proposition summarizes the result of this subsection.

**Proposition 2** *When  $\pi < (1 - 2\alpha)/(1 - \alpha)$ , there exists an intertemporal equilibrium in the Tirole regime. The dynamics of  $k_t$  monotonically converge to the golden rule  $k^*$ .*

In the steady state of this regime, consumption of movers and non-movers are equalized as follows:

$$d^m w(k^*) = dw(k^*) = \frac{1 - \alpha}{\alpha} (\alpha A)^{\frac{1}{1-\alpha}}. \quad (25)$$

Clearly, the steady-state allocation of the Tirole regime given by (24) and (25) achieves the social optimum.

## 4 Equilibria with Regime Switching

Like Michel and Wigniolle (2003), I address the following question: Can the economy switch from one regime to another along an equilibrium trajectory? A change in regime implies that  $\pi \geq (1 - 2\alpha)/(1 - \alpha)$ . When  $\pi < (1 - 2\alpha)/(1 - \alpha)$ ,

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<sup>5</sup>Of course, there also exists a continuum of inflationary equilibria that are not Pareto efficient. Along these equilibria, the reserves of banks go to zero and the consumption level of movers becomes zero. For simplicity, I ignore these equilibria here.

a regime change cannot occur, and the economy will remain in the Tirole regime throughout.<sup>6</sup> This is the unique intertemporal equilibrium.

## 4.1 The Two-Period Cycle

I first consider a two-period cycle. Let us assume that the economy is in the Schreft and Smith regime in period  $t - 1$ , shifts into the Tirole regime in period  $t$ , shifts back to the Schreft and Smith regime in period  $t + 1$ , and again shifts into the Tirole regime in period  $t + 2 \dots$ . The two-period cycle is represented as  $(\pi, \phi(\pi))$ . Starting from  $\gamma_t = \pi$ , the Tirole regime in period  $t$  and the Schreft and Smith regime in period  $t + 1$  imply

$$\gamma_{t-1} = \pi, \quad \gamma_t = \phi(\pi), \quad \text{and} \quad \gamma_{t+1} = \pi. \quad (26)$$

For equilibrium in the Tirole regime to exist,  $\phi(\pi) < 1$  is required. Otherwise, the wage income of generation  $t + 1$  is zero and no one receives any money. This is inconsistent with the optimal solution of generation  $t$ .

The condition for a two-period cycle can be summarized as follows:

$$\max \left\{ 0, \frac{1 - 2\alpha}{1 - \alpha} \right\} < \pi < 1 - \alpha. \quad (27)$$

According to Michel and Wigniolle (2003), if there exists an equilibrium such that the cash-in-advance constraint is binding, a two-period cycle is always guaranteed. The reason is that the two-period cycle condition is equivalent to the initial parameter restriction of the cash-in-advance constraint. In contrast to the model with the cash-in-advance constraint, when  $\pi \geq 1 - \alpha$  that is equivalent to  $\phi(\pi) \geq 1$ , the equilibrium in the Schreft and Smith regime cannot experience a regime switch. In the model of this paper, the existence of equilibrium in the Schreft and Smith regime is not a sufficient condition

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<sup>6</sup>Suppose that the economy shifts into the Schreft and Smith regime in period  $t$ . Then,  $\gamma_t = \pi$  holds. Under the condition  $\pi < (1 - 2\alpha)/(1 - \alpha)$ , however, the rate of return of money strictly dominates that of capital, which is a contradiction to the optimal solution.

for the two-period cycle; this is the clear difference between the Michel and Wigniolle model and mine.

## 4.2 The $(p + n)$ -Cycle

Let us now consider the more general cyclical bubbly equilibria. Specifically, I consider a  $(p + n)$ -cycle such that for  $p$  consecutive periods, the economy is in a Schreft and Smith regime, and for  $n$  consecutive periods, in a Tirole regime, with  $p \geq 1$  and  $n \geq 1$ . This cycle is represented by the following form:

$$\underbrace{(\pi, \dots, \pi)}_{p \text{ times}}, \underbrace{(\phi(\pi), \phi^2(\pi), \dots, \phi^n(\pi))}_{n \text{ times}}.$$

Since  $\phi^n(\pi)$  is the positive solution of difference equation (21) with the initial value  $\pi$ , I obtain

$$\phi^n(\pi) = \frac{1}{\left(\frac{1-\alpha}{2\alpha-1} + \frac{1}{\pi}\right)\left(\frac{1-\alpha}{\alpha}\right)^n - \frac{1-\alpha}{2\alpha-1}}.$$

The condition  $\phi^n(\pi) < 1$  is equivalent to  $\pi < \pi_n$ , where

$$\pi_n = \frac{(2\alpha - 1)(1 - \alpha)^n}{\alpha^{n+1} - (1 - \alpha)^{n+1}}.$$

Since  $\phi^n(\pi)$  is increasing in  $n$ ,  $\phi^n(\pi) < 1$  implies that  $\phi^i(\pi) < 1$  for all  $i \leq n$ .

Then, the existence of the  $(p + n)$ -cycle is obtained for

$$\max \left\{ 0, \frac{1 - 2\alpha}{1 - \alpha} \right\} < \pi < \pi_n. \quad (28)$$

Since  $\pi_n$  is decreasing in  $n$ , a  $(p + n)$ -cycle with a large  $n$  is a rare equilibrium for the given  $\pi$ . Note also that (28) is equivalent to (27) when  $n = 1$ . Figure 1 illustrates the  $(p + 3)$ -cycle in the case of a dynamically inefficient economy.

The following proposition summarizes the main findings of this paper.

**Proposition 3** *When  $\max \{0, (1 - 2\alpha)/(1 - \alpha)\} < \pi < \pi_n$ , there exists an equilibrium where for  $p$  consecutive periods, the economy is in the Schreft and Smith regime, and for  $n$  consecutive periods, in the Tirole regime. Along this equilibrium, the dynamics of  $k_t$  converge to a  $(p + n)$ -cycle.*

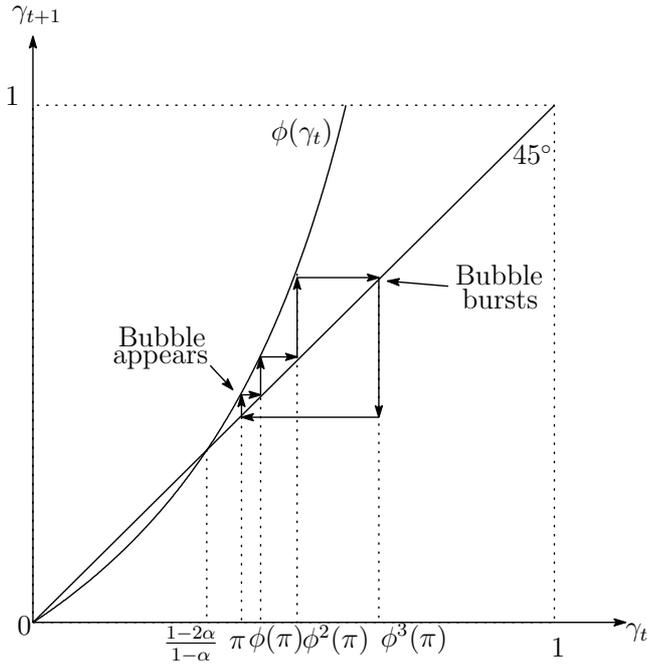


Figure 1: The  $(p + 3)$ -cycle in the case of  $\alpha < \frac{1}{2}$

Proposition 3 shows an indeterminacy of equilibrium because  $p$  and  $n$  are determined by the agents' expectations. Their coordinated expectations have real effects on the economy through the change in the banks' portfolio. As Michel and Wigniolle (2003) point out, such expectations can be coordinated by sunspots. I can then conclude that cyclical equilibria and sunspot equilibria can exist in an overlapping generations model with spatial separation and random relocation.

### 4.3 The Welfare Effects of Temporary Bubbles

Let us next consider the welfare analysis of bubbles. Figure 2 illustrates the expected utility of the representative agent of the  $(p + n)$ -cycle with  $n = 0, 1, 2, 3$  when the  $(p + 3)$ -cycle is possible. I refer to the expected utility as "welfare." Each cyclical bubble occurs at period  $t$  only once after the economy

is in the steady state of the Schreft and Smith regime for  $p$  consecutive periods. If the economy experiences no cyclical bubbles, the steady state welfare is  $-1.1258$ .<sup>7</sup> The solid line represents the welfare of generations  $t - 1$  through  $t + 4$  when the  $(p + 3)$ -cycle occurs. We see that under the  $(p + 3)$ -cycle, welfare rises sharply and then falls. As compared to the welfare of the economy that does not experience any bubbles, the  $(p + 3)$ -cycle provides benefit for generations  $t$ ,  $t + 1$ , and  $t + 2$ , but causes a significant loss for future generations after  $t + 3$ . Agents of generation  $t$  can coordinate their expectation and create bubbles. They can increase asset returns by reducing their investments in capital and demanding money. As a result, the welfare of generation  $t$  increases since they can increase asset returns and the degree of risk sharing while keeping the wage income at  $w(k^{SS})$ . Generation  $t + 1$  can also increase their welfare by reducing investments and demanding money. Note that increases in asset returns make up for the wage loss caused by the investment reduction of generation  $t$ . Generation  $t + 2$  is in the same situation as generations  $t$  and  $t + 1$ , and can increase their welfare by reducing investments significantly. However, bubbles must burst at  $t + 3$  because generation  $t + 3$  can not demand bubbles more than their wage income. Since the bubbles crowd out the capital stock, the negative effect of bursting bubbles on welfare for future generations depends on the scale of the bubbles. As compared to the welfare of the  $(p + 1)$ -cycle and the  $(p + 2)$ -cycle, the  $(p + 3)$ -cycle has a substantial negative impact on welfare for future generations after  $t + 3$ . Figure 2 indicates that prolonged bubbles develop a large welfare gap between the top and bottom and cause significant welfare losses for future generations after the bubbles burst.

The following interesting questions now arise: which cyclical bubbles are likely to emerge? Can the  $(p + 1)$ -or  $(p + 2)$ -cycle occur when the  $(p + 3)$ -cycle is

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<sup>7</sup>I set  $\alpha = 2/5$ ,  $\pi = 5/13$  and  $A = 1$ . Of course, these parameters satisfy the condition (28) with  $n = 3$ .

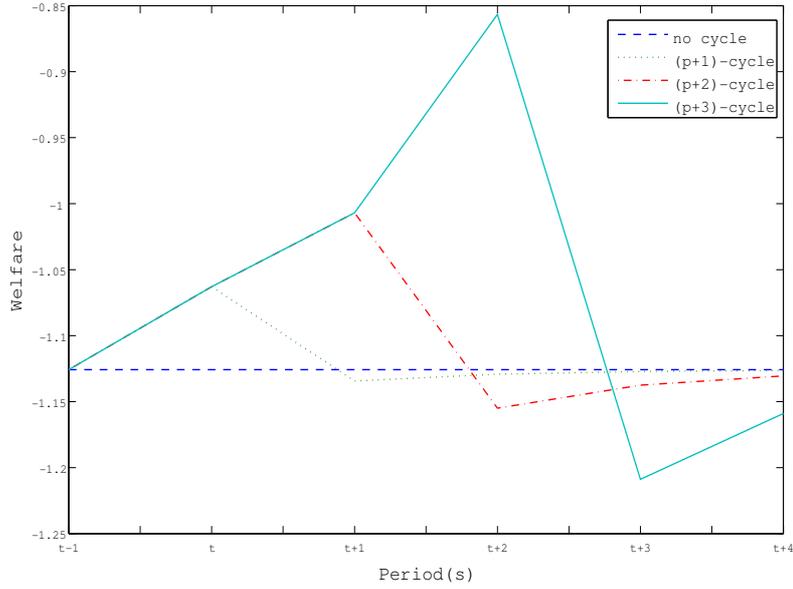


Figure 2: The welfare effects of cyclical bubbles

possible? To answer these questions, I adopt the principle that each generation decides whether to support bubbles to maximize their welfare. Then, it is easy to show that generations  $t + 1$  and  $t + 2$  always support bubbles. Figure 2 indicates that generations  $t+1$  and  $t+2$  can increase their welfare by supporting bubbles rather than bursting them. In addition, generation  $t$  always has an incentive to create bubbles because the welfare it maintains in the steady state is lower than the welfare brought about by bubble creation. As a result, the bubbles are created by generation  $t$  and bought by generations  $t + 1$  through  $t + 3$ , then burst. The  $(p + 3)$ -cycle is the most likely scenario when condition (28) with  $n = 3$  holds. This result will be extended in a more general form. That is, when condition (28) holds, the economy experiences the  $(p + n)$ -cycle, which features the most prolonged bubbles. These results provide a stabilizing role for central banks.

## 5 Discount Window Policy

As seen in the previous section, the equilibrium in the Schreft and Smith regime is in under-accumulation ( $k^{SS} \leq k^*$ ) and is prone to bubbly cycles. The bubbles reduce capital accumulation and accelerate the degree of under-accumulation. In addition, the bubbles cause significant welfare losses for future generations after they burst. I now consider the following question: Is it possible to rule out temporary bubbles with an appropriate monetary policy? In contrast to Michel and Wigniolle (2005), I consider the discount window policy as a counter-bubble policy, following Haslag and Martin (2007).<sup>8</sup> In practice, a bubble crash in a developed economy leads the central bank to inject liquidity into the banking system to offset a credit crunch. The practice is often chosen ex post once the credit crunch is underway. In contrast, this paper focuses on the ex ante roles of the discount window, which reduces cash reserves of banks and increases investments in capital. The timing of central bank loans is as follows. At the end of each period, banks can borrow money for movers from the central bank. Then, in the next period, banks sell goods to movers from the other island and obtain the money necessary to repay the central bank loans. For simplicity, discount window loans are made at a nominal interest rate of zero, but with a limit on borrowing.

### 5.1 The Dynamically Inefficient Economy: $\alpha < 1/2$

Consider the situation where the economy is initially in the Schreft and Smith regime ( $\pi \geq (1 - 2\alpha)/(1 - \alpha)$ ) and is dynamically inefficient ( $\alpha < 1/2$ ). Let  $b_t$  denote the loan received from the central bank at period  $t$ . It is assumed

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<sup>8</sup>Michel and Wigniolle (2005) consider the situation where the central bank can control the money growth rate to rule out bubbles. This policy will also work in my model. In this paper, however, I do not change the money growth rate to obtain different policy implications.

that there is a cap  $\bar{b}$  on the amount a bank can borrow, so  $b_t \leq \bar{b}$ . Equations (4) and (5) become, respectively,

$$\pi d_t^m w_t = \frac{p_t}{p_{t+1}} (m_t + b_t), \quad (29)$$

$$(1 - \pi) d_t w_t = R_{t+1} s_t - \frac{p_t}{p_{t+1}} b_t. \quad (30)$$

Equations (29) and (30) contain the terms of a bank's borrowing from the central bank and repayment to the central bank, respectively. Because money is a dominated asset in the Schreft and Smith regime, it is always optimal for banks to borrow as much money as possible such that  $b_t = \bar{b}$ . Then, the banks' problem is to maximize the expected utility of agents subject to (3), (29), (30) and  $b_t = \bar{b}$ .

The solution for  $\gamma_t$  is given by

$$\gamma_t = \max \left\{ \pi - \frac{\bar{b}}{w_t} \left[ 1 - \pi + \frac{\pi}{I_t} \right], 0 \right\}, \quad (31)$$

where  $I_t \equiv R_{t+1} p_{t+1} / p_t$  denotes the gross nominal interest rate between  $t$  and  $t + 1$ . Since discount window loans are perfect substitutes for cash reserves, the optimal reserve-deposit ratio is decreasing in  $\bar{b}$ . Thus, the central bank can control the cash reserves of banks by changing the loan limit  $\bar{b}$ .

In an equilibrium, the market clearing conditions for money and capital, (10) and (11), respectively, must hold. Therefore, the equilibrium can be characterized as sequences  $\{k_t, m_t, I_t\}_{t=0}^{\infty}$  that must satisfy

$$k_{t+1} = (1 - \pi) w(k_t) + \bar{b} \left[ 1 - \pi + \frac{\pi}{I_t} \right], \quad (32)$$

$$I_t \equiv R_{t+1} \frac{p_{t+1}}{p_t} = R(k_{t+1}) \frac{m_t}{m_{t+1}}, \quad (33)$$

$$m_t = \pi w(k_t) - \bar{b} \left[ 1 - \pi + \frac{\pi}{I_t} \right]. \quad (34)$$

In the steady state, the inflation rate is zero, i.e.  $I = R(k)$ , and the level of capital stock,  $k$ , is the solution to the following equation:

$$k = (1 - \pi) w(k) + \bar{b} \left[ 1 - \pi + \frac{\pi}{R(k)} \right]. \quad (35)$$

It is straightforward to show that (35) has a unique solution and

$$\frac{\partial k}{\partial \bar{b}} = \frac{k \left[ 1 - \pi + \frac{\pi}{R(k)} \right]}{(1 - \alpha)(1 - \pi)w(k) + \bar{b} \left[ 1 - \pi + \frac{\pi(2 - \alpha)}{R(k)} \right]} > 0.$$

This result states that the capital stock of the steady state,  $k$ , is increasing in the cap,  $\bar{b}$ . The intuition is simple. As  $\bar{b}$  increases, banks can decrease cash reserves and increase the amount of goods invested.

Let us now consider the stabilization policy that restores efficiency and eliminates cyclical bubbles. Specifically, consider the policy that the central bank sets  $\bar{b}$  to satisfy  $\gamma^{SS} = (1 - 2\alpha)/(1 - \alpha)$  in Figure 3. In this case, the values  $k$  and  $\bar{b}$  satisfy (35) and

$$\pi - \frac{\bar{b}}{w(k)} \left[ 1 - \pi + \frac{\pi}{R(k)} \right] = \frac{1 - 2\alpha}{1 - \alpha}. \quad (36)$$

Solving (35) and (36) for  $k$  and  $\bar{b}$  yields

$$k^* = (\alpha A)^{\frac{1}{1 - \alpha}}, \quad (37)$$

$$\bar{b}^* = \frac{1 - \alpha}{\alpha} (\alpha A)^{\frac{1}{1 - \alpha}} \left[ \pi - \frac{1 - 2\alpha}{1 - \alpha} \right]. \quad (38)$$

If the central bank sets and announces  $\bar{b} = \bar{b}^*$ , the banks borrow as much money as possible from the central bank and are able to reduce their cash reserves and increase investments until the level of capital becomes the golden rule. Under this policy, the steady state in the Schreft and Smith regime is equivalent to that in the Tirole regime. This implies that the economy never experience cyclical bubbles.

In addition, the consumption of movers and non-movers is equalized as follows:

$$c^m = c^n = \frac{1 - \alpha}{\alpha} (\alpha A)^{\frac{1}{1 - \alpha}}. \quad (39)$$

Clearly, the allocation defined by (37) and (39) is equivalent to the first-best allocation. In the steady state, the appropriate loan limit allows banks to

increase investments in capital while keeping perfect risk sharing. I summarize this result in the following proposition.

**Proposition 4** *Suppose that the economy is initially in the Schreft and Smith regime and dynamically inefficient. The equilibrium in which the central bank can make loans up to  $\bar{b}^*$  achieves the social optimum and never experiences cyclical bubbles.*

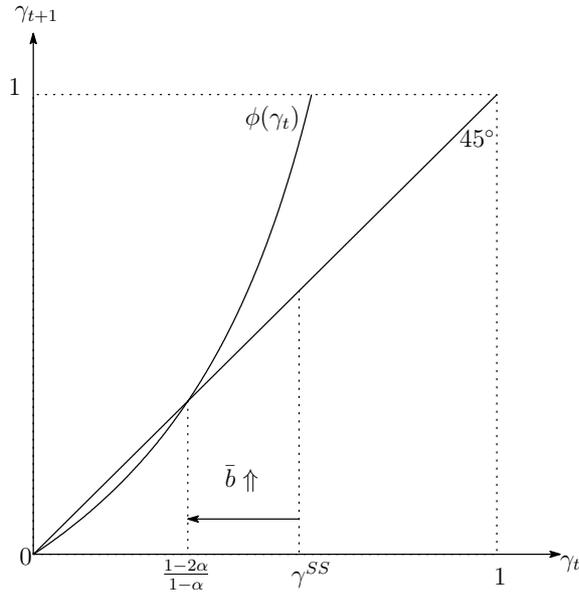


Figure 3: The discount window policy:  $\alpha < 1/2$

## 5.2 The Dynamically Efficient Economy: $\alpha \geq 1/2$

Next, consider the situation where the economy is initially in the Schreft and Smith regime and dynamically efficient ( $\alpha \geq 1/2$ ). In this case, there is no steady state in the Tirole regime that is equivalent to the social optimum, and the central bank does not have a target allocation to which its policy leads the economy. Since the economy is in under-accumulation, it is optimal for

the central bank to increase the loan limit sufficiently such that banks' money demand is zero, so  $\gamma^{SS} = 0$ . Under this policy, money does not circulate between generations. The central bank lends enough money to banks at period  $t$  for all movers so that banks do not need to sell goods to old agents to obtain money. The money is retired at period  $t + 1$  when banks repay their debts to the central bank. Since banks can invest all deposits in capital, the capital stock of the steady state,  $k^{SS}$ , satisfies  $k = w(k)$ , which is reduced to

$$k^{SS} = [(1 - \alpha)A]^{\frac{1}{1-\alpha}}.$$

This is equivalent to the steady-state capital stock of the Diamond nonmonetary economy. Note that the condition  $\alpha \geq 1/2$  is equivalent to  $k^{SS} \leq k^*$ . The economy is still in under-accumulation and never achieves the social optimum. However, the cyclical bubbles are eliminated since the steady state is also in the Tirole regime.

### 5.3 Discussion

From what has been discussed above, discount window lending is unambiguously beneficial in the economy of the Schreft and Smith regime regardless of whether the economy is dynamically efficient. Such a policy increases investments in capital and stabilizes bubbly fluctuations. If there is another friction that is absent from my framework, such as asymmetric information, however, the presence of a lender of last resort can generate moral hazard, resulting in excessive risk taking by banks and therefore greater uncertainty about a bank's ability to repay the loan. In this case, there exists a trade-off between the benefits from productive efficiency and economic stability and inefficiency due to moral hazard.

Another policy that can eliminate bubbly fluctuations is the reserve requirement. Under this policy, there exists a trade-off between productive efficiency

and stability. Consider the situation where the government imposes a reserve requirement on banks so that  $\gamma^{SS} \geq 1 - \alpha$ . If  $\pi < 1 - \alpha$ , the optimal reserve-deposit ratio of banks binds  $1 - \alpha$ , and the cyclical bubbles are eliminated. Under the reserve requirement, however, the capital stock decreases, and the total goods for consumption also decrease. Using the previous parameter setting, the steady-state welfare with the reserve requirement calculated at -1.2419 is lower than the steady-state welfare without the reserve requirement, -1.1258. Is this reserve requirement a “good” policy? To answer this question, I must consider the cost of temporary bubbles. A bubble at some period is beneficial for the generation living during that period because it increases returns on savings. It is detrimental, however, for following generations because it reduces capital stock in an economy that is experiencing under-accumulation. As Michel and Wigniolle (2005) point out, if agents are under the veil of ignorance and use a Rawlsian criterion between the different periods, they will prefer to live in the bubbly periods. If agents are sufficiently risk averse, the existence of cyclical bubbles reduces “ex-ante welfare.” I conjecture that if the reserve requirement is not too restrictive and eliminates bubbles, this policy is justified by the circumstances. I leave detailed analysis to answer the above question for future research.

## 6 Conclusion

This paper studies temporary bubbly equilibria in the monetary growth model where spatial separation and limited communication create endogenous roles for money. The model makes three contributions. First, the level of relocation shock is quite important in determining the equilibrium regime. High liquidity demands tend to ensure that the economy is in a Schreft and Smith regime. Second, the existence of the equilibria with temporary bub-

bles first studied by Michel and Wigniolle (2003) is confirmed in a different model in which money holding is microfunded. The value of money increases for some periods and then falls. The bursting bubbles can cause significant welfare losses for future generations. The economy can experience this event periodically. Such indeterminacies have not been pointed out in the overlapping generations model with random relocations by Schreft and Smith (1997, 1998). In addition, the model shows that the equilibria with regime changes do not always exist even when there exists an intertemporal equilibrium in which money is a dominated asset. In other words, there exists an equilibrium in the Schreft and Smith regime in which the economy cannot experience any regime switching. This result is in contrast to that of the model with the cash-in-advance constraint, and implies that imposing the cash-in-advance constraint may exclude some sets of equilibria from the research objects.

Finally, the discount window policy can play two important roles: (i) it can increase productive efficiency and (ii) it stabilizes bubbly fluctuations. If the economy is dynamically inefficient and is in the Schreft and Smith regime, this policy plays these roles so that the economy achieves the social optimum and is free from bubbly cycles. In the case that the economy is dynamically efficient, the policy can stabilize the economy. The main point here is that the decentralized economy is prone to bubbly fluctuations and that the discount window is a powerful monetary policy to stabilize these fluctuations.

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