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Information Investment Regulation and Portfolio Delegation

Abstract

We consider policies to achieve the social optimal level of investment in information acquisition by examining arbitrageur investment strategy and the likelihood of a market freeze in equilibrium. We show that if direct portfolio management is dominant, an investment subsidy may be better, whereas if delegated portfolio management is dominant, an investment tax is needed to prevent overinvestment, although this raises the possibility of a market freeze. We use this to evaluate the effect of the recent trend in hedge funds switching their operations to family offices and shed light on recent regulatory discussion of FinTech and Big Tech firms.

JEL Classification Codes: D86, G14, G33.

Keywords: adverse selection, delegated portfolio management, FinTech, information investment, market freeze.

1. Introduction

FinTech plays an increasingly important role in financial markets, particularly following the explosion in the use of artificial intelligence (AI) and big data by financial intermediaries (asset managers).[1,2] To exploit these technologies effectively, financial intermediaries need to invest in information acquisition, including developing greater expertise in processing information by managing data, tuning algorithms, and nurturing the entire process.

At the same time, financial regulators have begun discussing related policy measures because FinTech could also entail some risk. For instance, the Financial Stability Board (FSB), an international body that monitors the global financial system, reported on the varying regulations adopted by 20 jurisdictions (FSB, 2017). The report identifies supervisory and regulatory issues concerning FinTech, and finds that most regulations focus on technological aspects such as privacy security, investor protection, or operational resilience.[3] In contrast, the welfare effect of FinTech investment on the financial market has been discussed less frequently,[4] and there seems to be a lack of consensus about policy measures on information acquisition investment. This may be because it is not yet clear how such technologies affect the market, and there is limited availability of relevant data at this point. It is also evident in regulator statements that the risk of FinTech remains unclear, which leads to a "wait-and-see" attitude on the part of regulators (Didenko, 2018).[5]

[1] FinTech is a set of recently developed digital computing technologies applied to financial services.

[2] FinTech investment increased substantially in 2018, more than doubling from $50.8 billion in 2017 to $111.8 billion in 2018 (Pollari and Ruddenklau, 2019). Robo-advised assets under management will also grow from $0.3 trillion in 2016 to $2.2 trillion in 2020 (see O'Keefe, Warmund, and Lewis, 2016). In addition, Chen, Wu, and Yang (2019) argue that FinTech innovation brings a large amount of positive value for innovators.

[3] See Restoy (2019).

[4] Legal studies also focus on privacy or operational issues most frequently (see, e.g., Zhou, Arner, and Buckley, 2015; You, 2017; Xu and Xu, 2019).

[5] For example, the Bank of England announced in 2016 that one of its priorities was creating a regulatory approach to FinTech. However, a Bank of England official stated that "[i]t's very difficult to decide how to regulate something you don't quite know what it is" (https://uk.reuters.com/article/uk-boe-tech/boe-says-wont-stifleinnovation-as-wrestles-with-fintech-idUKKCN11E1O7). Meanwhile, the European Union's European Banking Authority has delayed a decision on whether FinTech actually requires new regulation. Its executive director stated that "[w]e should wait and see what uses the market is contemplating and whether that sort of use would imply the emergence of new risks" (https://www.reuters.com/article/us-fintech-bundesbank/fintech-sector-needs-more-regulatory-oversight-bundesbank-idUSKBN1591LV).

This paper explores information acquisition investment and desirable policy measures related to FinTech, from the perspective of information theory. Given the lack of evidence, it is important to examine any theoretical predictions before possible risks caused by information acquisition investment become known.[6] Furthermore, we also conduct welfare analysis to consider whether information acquisition investment tends to be too high or too low. This is essential to decide whether to promote or regulate such investments.

Information acquisition investment may affect at least two information problems in financial markets. First, it can create information asymmetry regarding asset types. Given there are informed and uninformed agents in the market, information asymmetry about assets can lead to adverse selection á la Akerlof (1970), and result in market freezes and fire sales, both of which were observed in the 2007–2008 global financial crisis. However, the investment in information acquisition by financial intermediaries could also make the prices of assets more informative, thereby alleviating the mispricing of assets and the extent of the limits of arbitrage (Shleifer and Vishny, 1997), and may also reduce the set of behavioral biases for all investors.[7]

Second, information investment can also invoke an additional information problem with portfolio delegation, which is widely adopted in practice. If the ability of the portfolio manager to acquire and process information is unknown by the public, and when his effort decision and his possessing of information are his own private information, informational asymmetry and moral hazard problems may arise, and these could impair the efficiency of financial markets. However, the portfolio manager has no incentive to internalize the various adverse selection and moral hazard costs when making his investment decisions. Hence, intensive investment in information acquisition may result in a substantial divergence between social and private values, even though price informativeness in the financial market increases.

To examine these issues, we develop two information-based theoretical models. The first

[6] Biais, Foucault, and Moinas (2015) is one of the few examples of such theoretical prediction, although their focus is on high-frequency trade.

[7] For empirical evidence concerning wealth-management robo-advisers in this respect, see D'Acunto, Prabhala, and Rossi (2019).

is a “direct portfolio management (DIR)” model, in which there are two types of informed agents (a capital-constrained risk-neutral seller and a risk-neutral talented arbitrageur) and two types of uninformed agents (risk-averse hedgers and a risk-neutral market maker). To meet liquidity needs, the seller can liquidate a nonmarketable asset or sell his holdings of a marketable asset with risky payoffs whose value is known only to the informed agents. The talented arbitrageur trades the marketable asset on his own account. He has skill in receiving private information with some probability by investing and exerting effort in information acquisition. Hedgers then trade the marketable asset to hedge optimally against an income shock and this hedging demand creates endogenous noise trading. Lastly, all the trade orders for the marketable asset are submitted to the market maker, who sets the price of the marketable asset at its expected value given all publicly available information.

The second is a “delegated portfolio management (DEL)” model. In this model, an arbitrageur is employed as a portfolio manager by an uninformed investor (the principal), who cannot observe asset types or the arbitrageur’s exerted effort. In the DEL model, arbitrageurs consist of a risk-neutral talented arbitrageur and a large number of incompetent arbitrageurs, and their type is private information. Hence, in this second model, the adverse selection regarding the arbitrageur’s type and the moral hazard regarding his effort are newly added to the adverse selection of assets. Furthermore, the principal does not know whether her employed arbitrageur has actually received an informative signal. Thus, as suggested by Dow and Gorton (1997), the optimal contract may also give the arbitrageur a distorted incentive to trade without information as a noise trader, that is, to churn.

Using these two models, our first result is that in the DIR model, underinvestment arises and the market freeze is more likely to occur in equilibrium than in the welfare-maximizing case, when liquidity in the marketable asset is sufficiently small relative to the seller’s endowment of the marketable asset. Next, in the DEL model, overinvestment always occurs. The likelihood of the market freeze is then smaller in equilibrium than in the welfare-maximizing case.

Intuitively, with the standard argument for an information externality, we typically think that underinvestment arises because the information-acquiring agent cannot receive enough benefit if asset prices partially reveal his acquired information. Otherwise, using the standard argument of adverse selection, we might likewise typically argue that overinvestment arises because information investment only serves a redistribution between uninformed and informed agents rather than as a resource expanding role. However, the mechanism for generating underinvestment (overinvestment) in our DIR (DEL) model differs from that in the standard argument.

In the DIR model, increasing investment in information acquisition involves the following trade-off. On the one hand, increasing investment makes the price of the marketable asset more informative and motivates the seller to meet liquidity needs by selling the marketable asset. This serves to improve the seller's allocative efficiency. On the other hand, as in the standard argument, increasing investment induces uninformed hedgers to hedge their income risk less because they would prefer not to lose to the more informed trader given the higher level of information investment. If the hedgers' income shock, that is, the liquidity in the marketable asset is sufficiently small relative to the seller's endowment of the marketable asset, the former effect dominates. Thus, the welfare-maximizing regulator favors the higher level of investment. However, the arbitrageur does not internalize these costs or benefits and thus underinvests in equilibrium.

By contrast, in the DEL model, the contract faces adverse selection regarding the arbitrageur's type so that it needs to exclude incompetent arbitrageurs. As such adverse selection is too severe, information investment is socially costly in the welfare-maximizing case. Hence, social need always limits information investment at a minimum level that can prevent incompetent arbitrageurs from participating in the contract. Hence, overinvestment always arises.[8]

Furthermore, in terms of the likelihood of the market freeze, increasing investment can make the price of the marketable asset more informative and mitigate adverse selection

[8] Our mechanism generating the inefficiency in information investment also differs from other studies on information acquisition in financial markets, such as Biais, Foucault, and Moinas (2015), Hauswald and Marquez (2006) and Glode, Green, and Lowery (2012), which stress an overinvestment problem caused by an arms race competition among investors.

regarding the asset type. Because the high-quality asset seller is then more willing to raise funds by selling the marketable asset, the likelihood of the market freeze is inversely related to the level of information investment, regardless of whether we employ the DIR or DEL models.

The second result is that portfolio delegation increases the equilibrium level of investment in information acquisition. However, while other things being equal the higher level of information investment decreases the likelihood of the market freeze, portfolio delegation conversely increases the equilibrium likelihood of the market freeze as long as liquidity in the marketable asset market is not sufficiently large. This result suggests that the recent trend in hedge funds switching their operations to family offices can not only decrease information acquisition investment but also decrease the likelihood of the market freeze as long as liquidity in the marketable asset market is not sufficiently large.

Intuitively, the main effect generated by portfolio delegation stems from the mechanism under which the portfolio delegation contract needs to exclude incompetent arbitrageurs, but still allow the competent arbitrageur to churn. Because the contract can induce the arbitrageur to churn, for a fixed investment level, the price of the marketable asset can be less informative under portfolio delegation. In this sense, portfolio delegation will raise the incentive for hedgers to increase their hedging demand, whereas it will reduce the incentive for the seller to supply the high-quality asset. Conversely, to deter incompetents from joining the contract, the principal needs to increase information investment.

In fact, as the effect on the incentive for hedgers is dominated by the latter two effects on the incentives for the seller and incompetent arbitrageurs, portfolio delegation induces the principal to increase the level of information investment so as to raise the incentive for the seller to supply the high-quality asset and prevent incompetents from joining the contract. Alternatively, for a fixed investment level, the likelihood of the market freeze in equilibrium is greater in the DEL model than in the DIR model if churning reduces the high-quality asset-seller's supply. Furthermore, using numerical calculations, we can show that churning is optimal when liquidity in the marketable asset is not sufficiently large. This tendency remains even though the investment level is endogenized. Accordingly, the likelihood of the market freeze in equilibrium is higher in the DEL model than in the DIR model as long as liquidity in the marketable asset is not sufficiently large.

Given these theoretical results, our analysis provides several implications for desirable regulation policies. In particular, to evaluate whether promoting or regulating information acquisition investment is welfare improving, we find it crucial to empirically distinguish an economy in which direct portfolio management prevails, from one in which delegated portfolio management is dominant. Generally, if direct portfolio management prevails, it is better to subsidize and promote investment in information acquisition when liquidity in the marketable asset market is sufficiently small relative to the seller's endowment of the marketable asset. In contrast, if delegated portfolio management is dominant, some investment tax is needed to prevent overinvestment, although this does raise the possibility of a market freeze.

As one possible application, suppose that a structured financial product, such as an asset-backed security, is originated to a sufficiently large extent, while liquidity in the market of the structured financial product is sufficiently small. Our model then suggests that the regulator can provide a subsidy for information investment if direct portfolio management prevails, but uses tax for information investment if delegated portfolio management is dominant. As another application, suppose that new financial products are created and supplied when the cost of information acquisition investment is relatively low as a result of technological innovation. Then, our analysis indicates that if delegated portfolio management is dominant, an investment tax should be used, but this raises the likelihood of a market freeze. Alternatively, the recent trend for hedge funds to transform their companies into family offices suggests that the investment subsidy may be used to improve the efficiency of information investment and reduce the likelihood of the market freeze. Finally, our analysis sheds some light on recent regulatory discussions of FinTech and Big Tech firms in both developed and emerging countries.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the basic setup of the model. Sections 4 and 5 characterize the equilibrium and its welfare properties in the DIR and DEL models, respectively. Section 6 provides a discussion of desirable regulation policies and Section 7 concludes. All proofs are presented in the Appendix.

2. Related Literature

This paper relates to three strands of research: market microstructure, delegated asset management, and information acquisition in the financial market. The market microstructure literature following Kyle (1985, 1989) examines how an informed trader's private information is revealed through his strategic trading in the asset trading process. Based on this line of inquiry, Dow and Han (2018) discuss the possibility of fire sales and a market freeze by considering the trading strategies of the capital-constrained informed seller, capital-constrained informed arbitrageurs, and well-capitalized uninformed hedgers. Extending the model in Dow and Han (2018) by incorporating observable investment in information acquisition as well as portfolio management delegation and by ruling out margin requirement constraints imposed on the arbitrageurs, we focus on the equilibrium and welfare properties of information investment and the likelihood of the market freeze. We also characterize the equilibrium allocation and welfare consequences under delegated portfolio management.[9]

Dow and Gorton (1997) and Kyle, Ou-Yang, and Wei (2011) extend the Kyle (1985) model by developing an integrated model of strategic informed trading and portfolio delegation. Dow and Gorton (1997) indicate that under the optimal contract, the delegated portfolio manager will trade like a noise trader, even though he has no information, and that such noise trade may be Pareto-improving.[10] Kyle, Ou-Yang, and Wei (2011) show that a higher-

[9] The observable investment decision of an informed trader in the market microstructure framework is investigated in Mendelson and Tunca (2004) by distinguishing between tractable and intractable information, although they do not consider a market freeze or delegated portfolio management. They argue that the informed trader acquires more information than is optimal for liquidity traders.

[10] Dasgupta and Prat (2006, 2008) and Guerrieri and Kondor (2012) introduce the reputational (career) concerns of portfolio managers into a strategically informed trading model, and suggest that reputational concerns lead to churning by portfolio managers. Taking a portfolio management contract as fixed, Allen and Gorton (1993) consider a model in which prices can diverge from fundamentals because of churning by portfolio managers. In contrast, our model discusses whether churning by portfolio managers actually arises in the absence of any reputational concerns when a portfolio management contract is endogenized.

powered linear contract induces the manager to exert more effort in information acquisition. However, none of these studies consider the welfare properties of the level of observable investment in information acquisition or the possibility of a market freeze. In addition, and in contrast to Dow and Gorton (1997), we show that the optimal contract does not necessarily involve the arbitrageur's random trading strategy when he has no informative signal.

A voluminous literature on information acquisition in financial markets discusses the importance of financial intermediaries in the production of information. Hauswald and Marquez (2006) suggest that banks use information asymmetries concerning borrower quality to soften price competition and to carve out and extend captive markets, and that the strategic role of information acquisition induces banks to overinvest in information acquisition. Glode, Green, and Lowery (2012) develop a bilateral trading model in which the acquisition of expertise by financial firms becomes an "arms race". They show that financial firms have an incentive to overinvest in financial expertise. Elsewhere, Philippon (2019) investigates the impact of the use of robo-advisors and big data on inequality. Importantly, unlike any of these studies, we consider the welfare properties of the level of observable investment in information acquisition and the possibility of a market freeze under strategic informed trading in the market, and examine the condition under what circumstances underinvestment in place of overinvestment arises. We also characterize the equilibrium allocation under delegated portfolio management.

3. Basic Setup

We consider two economies: the first is a benchmark economy in which an arbitrageur trades on his own account, and the second is an economy in which an arbitrageur is employed by a representative uninformed investor (principal) as a portfolio manager. We refer to the former as the "direct portfolio management (DIR)" model and the latter as the "delegated portfolio management (DEL)" model.[11] In the DIR model, there are five kinds of traders: a

[11] The justification for these two models is in the introduction. Unlike Dow and Han (2018), in both models we do not take account of margin requirement constraints.

seller of a risky asset, talented and incompetent arbitrageurs, hedgers, and a market maker. In the DEL model, we add a principal to the other five kinds of traders. There are three dates, 0, 1, and 2. For simplicity, we normalize the discount rate of all agents to be zero.

3.1. Assets and the seller.—

We consider a risky asset tradable by all market participants, denoted as a "marketable asset". The marketable asset is traded for cash at date 1, and pays a liquidating dividend v of either $v = 1$ or $v = 0$ with equal probability at date 2. If $v = 1$ ($v = 0$), the marketable asset is called a "high(low)-quality" asset. The type of asset is observable at date 2 to all traders, but is only observable at date 1 to traders with expertise.[12]

The seller of the marketable asset is risk-neutral and liquidity-constrained. He is endowed with $\overline{x}$ units of the marketable asset and knows the quality at date 1. He is also endowed with a "nonmarketable asset" (profit-generating operations of the firm), which cannot be transferred to other agents.[13] The nonmarketable asset yields a return y per unit of investment at date 2, which has probability density function $f(\cdot)$ and cumulative distribution function $F(\cdot)$ with support $[0, \overline{y}]$. The seller supplies his endowment of the marketable asset or liquidates the nonmarketable asset (equivalently, reduces investment in the nonmarketable asset) to meet his liquidity needs at date 1, where his liquidity shortage at date 1 is given by ℓ.[14] For simplicity, the seller has only a liquidity motive and decides to sell an amount of $x_a \in [0, \overline{x}]$ of the marketable asset, as assumed in Dow and Han (2018). He also reinvests in the nonmarketable asset any trading revenues from selling the marketable asset in excess of ℓ. If he reinvests (liquidates) a unit of the nonmarketable asset at date 1, he receives (loses) y at date 2. We assume that the realized value of y is uncertain at date 0 but is perfectly expected by all traders at date 1.[15]

[12]Examples of the marketable asset include equities, corporate bonds, and structured financial products.

[13]This implicitly implies that the cash flows from profit-generating operations cannot be pledged to their full extent because of moral hazard or nonverifiability, as suggested by Dow and Han (2018).

[14]The seller can be viewed as a firm that issues new equity or bonds because of its financial needs, or as a bank that makes loans and sells securities backed by loans because of capital requirement constraints.

[15]This is an innocuous assumption because the seller trades his assets only for liquidity motives and because we focus on the effect of information investment at date 0 on the investment and market freeze.

3.2. Arbitrageur.—

The arbitrageur chooses his trading strategy for speculative motives, even though he has no endowment of the marketable asset. Note that the arbitrageur trades on his own account in the DIR model, while he is employed as a portfolio manager in the DEL model.[16]

There are two types of risk-neutral arbitrageur. One is a talented arbitrageur, who may receive a private signal θ about v at date 1, where $\theta \in \{1, 0, \phi\}$. If $\theta = 1$ ($\theta = 0$), the arbitrageur has perfectly precise information that $v = 1$ ($v = 0$) at date 2. Alternatively, if $\theta = \phi$, the arbitrageur cannot obtain any informative signal about v. The other type applies to a large number of incompetent arbitrageurs, who have no chance of receiving an informative private signal. The arbitrageur's type is private information.[17]

To acquire information about v, the arbitrageur can decide on costly information investment at date 0. The investment determines the extent to which he is informed through an information acquisition process. In particular, upon investment of a level of i, the talented arbitrageur obtains a perfectly informative (uninformed) private signal $\theta \in \{1, 0\}$ ($\theta = \phi$) with probability $\alpha(i)$ $(1 - \alpha(i))$ at date 1, where $\alpha'(i) > 0$, $\alpha''(i) < 0$, and $\lim_{i \to \infty} \alpha(i) < 1$. Incompetent arbitrageurs cannot receive any informative signals either, even though they invest. The information investment imposes a cost on the arbitrageur, ci, where $c > 0$. We also assume that the arbitrageur must also exert effort in information acquisition by incurring a cost ei at date 0 if he wishes to acquire information about v. However, note that it is possible that the arbitrageur—in particular, an incompetent arbitrageur—expends investment cost but does not exert information acquisition effort.

We assume that the arbitrageur's investment in information acquisition is observable by all traders at date 1, whereas his information acquisition effort is unobservable. The former investment includes not only investment in AI technologies and big data but also investment in financial expertise associated with improvements in the academic education of employees and their compensation. These investments serve to collect and process information about

[16] The concept of an arbitrageur covers investment banks, hedge funds, and asset management firms.

[17] In fact, the assumption that the population of arbitrageurs consists almost entirely of incompetents is a simplifying assumption and is required for the refinement of equilibrium only under the DEL model. However, even if this assumption is relaxed, the main conclusions in the paper are qualitatively unaffected.

the values of assets traded. The latter is the disutility cost incurred when the arbitrageur actually engages in information acquisition and processing activity.

3.3. Hedgers.—

There are uninformed risk-averse hedgers, who participate in the market to hedge their income shock at date 1.[18] The income shock arises from a common risk factor that affects both hedgers' future income and the liquidating value v of the marketable asset. We assume that the income shock is positively (negatively) correlated with v with a 50 percent probability. For simplicity, we assume that all the hedgers are simultaneously hit by the same type of income shock at date 1. If the income shock is negatively correlated, hedgers will observe their income shock $z^h = -\overline{z}$ ($z^h = \overline{z}$) at date 2 if $v = 1$ ($v = 0$), and will be hedged by buying the marketable asset. If the income shock is positively correlated, hedgers will observe $z^h = \overline{z}$ ($z^h = -\overline{z}$) at date 2 if $v = 1$ ($v = 0$), and will be hedged by selling the marketable asset. Whether the hedgers' hedging need is positively or negatively correlated is their private information.

Each hedger is financially unconstrained and has a quadratic utility function $U(w) = aw - \frac{1}{2}bw^2$, where w is the income of hedgers, $a > 0$, $b > 0$, and $a > b\overline{z}$. The final assumption ensures that the marginal utility of income is positive for $w \in [0, \overline{z}]$.[19]

Lastly, although the hedgers could be viewed as a continuum of small traders, they are instead interpreted as a representative investor who chooses his trading strategy for hedging motives. Thus, for convenience, they are referred to as "the hedger" in subsequent analysis, as in Dow and Gorton (1997).

3.4. Market maker, the marketable asset market, and the market freeze.—

The market maker is risk-neutral and financially unconstrained. She sets the price of the marketable asset at which she trades a quantity necessary to clear the market, as in Kyle (1985). More specifically, the seller, arbitrageurs, and the hedger submit their orders to the

[18] Such traders are typically pension funds, insurance companies, or sovereign wealth funds. Note that the income shock is only used to derive the hedgers' motive for hedging.

[19] This kind of quadratic function is used in the static market microstructure models in Vives (2011) and Rostek and Weretka (2012) and in the dynamic market microstructure model in Du and Zhu (2017).

market maker at date 1. The market maker observes the total order flow and sets the price equal to the expected liquidating value of the marketable asset, conditional on the market maker's information set at date 1. In fact, at date 1, the market maker is uniformed about the realization of v and cannot know the identity of any of the traders submitting orders, even though she observes the aggregate order flow. Hence, the market-clearing price is set equal to the expected value of v, conditional on the total order flow. In addition, no order flow of any trader can be observed by the other traders.

As in Dow and Han (2018), we assume, for simplicity, that the market does not open if there is no supply of the marketable asset by the seller, and that hedgers cannot open the market by themselves because they are of infinitesimal size.[20] We also define a market freeze to be an event where the high-quality asset fails to fully circulate in the market.

4. DIR Model

4.1. Definition of equilibrium.—

In the DIR model, incompetent arbitrageurs never trade in the marketable asset market, because they cannot receive any positive revenues from trading, even though they undertake information acquisition investment and exert information acquisition effort.[21] Hence, we only need to consider the decision chosen by the talented arbitrageur without loss of generality.

The timeline of the model is as follows.

1. At date 0, the arbitrageur decides whether to make information acquisition investment i and whether to exert information acquisition effort.

2. At date 1, the following events occur.

(i) The seller with liquidation value v submits his selling order x^s to the market maker and reinvests in (liquidates) the nonmarketable asset on the basis of v, the nonmarketable asset

[20] This assumption is particularly reasonable in the markets for initial public offerings, the primary markets for bonds, and the primary markets for structured financial products. In fact, if the market freeze is redefined as a situation in which only hedgers and the market maker trade in the market, the main results of this paper are not modified.

[21] For simplicity, we assume that arbitrageurs do not trade the marketable asset for a speculative motive if they cannot obtain any positive expected revenues from trading. This assumption can be justified in the DIR model if there is an infinitesimal trading cost.

return y, and i, by anticipating a price p.

(ii) The arbitrageur receives a private signal θ if he exerts information acquisition effort at date 0. Then, if the market opens, the arbitrageur submits his order x^a to the market maker on the basis of θ, y, and i.

(iii) The hedger knows whether the hedging need z^h is positively or negatively correlated with v. Let $\rho = +\ (-)$ if z^h is positively (negatively) correlated with v. If the market opens, the hedger submits the order x^h to the market maker on the basis of ρ, y, and i.

(iv) The market maker determines a price p based on x, y, and i, where $x = x^a + x^h - x^s$.

3. At date 2, v, y, and z^h are realized.

Note that y is perfectly predicted by all traders at date 1. An equilibrium is defined as (p, x^s, x^a, x^h, i) such that (i) (x^s, x^a, x^h) and i solve (1)–(3) defined below, respectively, (ii) $x = x^a + x^h - x^s$, and (iii) the price satisfies $p = E_1(v \mid x, y, i)$, where E_1 is the expectation operator at the beginning of date 1. Define $p(x; y, i) \equiv E_1(v \mid x, y, i)$.

The seller's problem is to maximize his expected profit by choosing $x^s \in [0, \overline{x}]$ at date 1:

$$\max_{x^s \in [0,\overline{x}]} E_1 \left\{ v(\overline{x} - x^s) + (1 + y)[p(x; y, i)x^s - \ell] \mid v, y, i \right\}, \tag{1}$$

where the first term is the revenue from the sale of the marketable asset and the second term is the revenue from the nonmarketable asset. As the seller trades only for the liquidity motive, he has no incentive to deviate from x^a determined by (1).

The arbitrageur maximizes his expected trading profit by choosing $i \geq 0$ at date 0 and x^a at date 1, and by deciding whether to exert information acquisition effort at date 0:

$$\max_{i \geq 0,\, \chi \in \{1,0\}} E_0 \left\{ \max_{x^a} E_1 \left[(v - p(x; y, i))x^a \mid \theta, y, \chi, i \right] \right\} - ci - ei\chi, \tag{2}$$

where E_0 is the expectation operator at date 0 and χ is the indicator function that satisfies $\chi = 1$ $(\chi = 0)$ if the arbitrageur exerts (does not exert) information acquisition effort.

The hedger maximizes expected utility by choosing x^h at date 1:

$$\max_{x^h} E_1 \left\{ U\left[(v - p(x; y, i))x^h + z^h \right] \mid \rho, y, i \right\}. \tag{3}$$

As the hedger consists of a continuum of infinitesimal agents who cannot affect the aggregate trading volume, the hedger has no incentive to deviate from x^h determined by (3).

4.2. Characterization of equilibrium.—

The seller's supply of the marketable asset is determined by the following lemma.

Lemma 1: *Given v, y,* and *i, the seller's supply is determined by*

$$x^s = \begin{cases} 0 & \text{if } E_1\left[p(x; y, i) \mid v, y, i\right] < \frac{v}{1+y}, \\ \overline{x} & \text{if } E_1\left[p(x; y, i) \mid v, y, i\right] \geq \frac{v}{1+y}. \end{cases}$$

Lemma 1 shows that the low-(high-)quality asset seller always supplies (does not always supply) the marketable asset because $v = 0$ ($v = 1$).[22] Lemma 1 also implies that the supply order of the high-quality asset seller is exactly equal to that of the low-quality asset seller if they supply their holdings of the marketable asset. Hence, the seller's order x^s does not depend on the asset type when the market freeze does not occur.

We next discuss the total order flow x of the marketable asset in equilibrium when the high-quality asset seller prefers to supply the marketable asset (that is, $x^s = \overline{x}$). For the present, we suppose that the hedger will trade $x^h = n_b \geq 0$ ($x^h = -n_s \leq 0$) if the hedger buys (sells) the marketable asset, that is, if the income shock is negatively (positively) correlated with v. n_b and n_s are derived later in solving the hedger's optimization problem.

To specify x, we need to investigate the arbitrageur's trading strategy x^a at date 1 when the market freeze does not arise. As mentioned at the beginning of Section 4.1, we need only consider the talented arbitrageur. Then, we impose the following out-of-equilibrium belief of the market maker when the market freeze does not occur: anticipating $x^s = \overline{x}$, she infers

[22] The seller is assumed to sell if $E_1\left[p(x; y, i) \mid v, y, i\right] = \frac{v}{1+y}$. If there are infinitesimal costs in trading the marketable asset and in liquidating the nonmarketable asset, this tie-breaking assumption can be justified if the former costs are smaller than or equal to the costs of the latter.

that any deviation of x from $n_b - \overline{x}$, $-\overline{x}$, or $-n_s - \overline{x}$ must come from the arbitrageur rather than the hedger, without changing the market maker's belief in the relative likelihood that the arbitrageur has an informative or uninformative signal. We also assume that the market maker believes that if the arbitrageur trades, any positive (negative) quantity other than n_s $(-n_b)$ is ordered by an arbitrageur who would otherwise have traded n_s $(-n_b)$.[23] Under these beliefs, the arbitrageur needs to camouflage his information-based trade by mimicking the trading of the hedger to obtain profits, because any other quantity would always reveal his information to the market maker and bring no profits. Given $x^s = \overline{x}$ and $x^h = n_b$ or $-n_s$, the arbitrageur will either buy n_s $(x^a = n_s)$ or sell n_b $(x^a = -n_b)$ to pool with the hedger.

As the arbitrageur buys n_s if $\theta = 1$, or sells n_b if $\theta = 0$, or does not trade if $\theta = \phi$, the market maker can observe five possible total order flows:

(i) $x = n_s + n_b - \overline{x}$ if $\theta = 1$, $x^a = n_s$, and $x^h = n_b$;

(ii) $x = n_b - \overline{x}$ if $\theta = \phi$, $x^a = 0$, and $x^h = n_b$;

(iii) $x = -\overline{x}$ if $\theta = 1$, $x^a = n_s$, and $x^h = -n_s$, or if $\theta = 0$, $x^a = -n_b$, and $x^h = n_b$;

(iv) $x = -n_s - \overline{x}$ if $\theta = \phi$, $x^a = 0$, and $x^h = -n_s$; and

(v) $x = -n_b - n_s - \overline{x}$ if $\theta = 0$, $x^a = -n_b$, and $x^h = -n_s$.

Thus, the trading strategy for the arbitrageur implies that he trades when $x \in \{n_s + n_b - \overline{x}, -\overline{x}, -n_b - n_s - \overline{x}\}$. If $x \in \{n_b - \overline{x}, -n_s - \overline{x}\}$, the market maker infers $\theta = \phi$ because she knows that the arbitrageur is not trading. The market maker also infers that $\theta = 1$ if $x = n_s + n_b - \overline{x}$, $\theta = 0$ if $x = -n_b - n_s - \overline{x}$, but cannot infer θ if $x^s + x^n = -\overline{x}$.

Now, the market maker and the other market participants infer the probability for each event of x at date 1 as follows if the market freeze does not arise.[24]

Lemma 2: *Suppose that the high-quality asset seller prefers to supply the marketable asset. The market maker and the other market participants infer the probability for each event of the total order flow at date 1 as: (i)* $\frac{\alpha(i)}{4}$ *if* $x = n_s + n_b - \overline{x}$*; (ii)* $\frac{1-\alpha(i)}{2}$ *if* $x = n_b - \overline{x}$*; (iii)*

[23] Because the market maker anticipates that the seller's supply is $x^s = \overline{x}$ while the hedger orders $x^a = n_b$ or $-n_s$, she can exactly infer the quantity of the arbitrageur's order unless $x = -\overline{x}$.

[24] Note that the other market participants can infer the total order flow and the probability associated with the total order flow by rationally anticipating each trader's strategy.

$\frac{\alpha(i)}{2}$ *if* $x = -\overline{x}$; *(iv)* $\frac{1-\alpha(i)}{2}$ *if* $x = -n_s - \overline{x}$; *and (v)* $\frac{\alpha(i)}{4}$ *if* $x = -n_b - n_s - \overline{x}$.

After observing x, the market maker sets a price p at which she trades the quantity necessary to clear the market. Using Lemma 2, the equilibrium price $p^*(x)$ is characterized by the following lemma if the market freeze does not arise.

Lemma 3: *Suppose that the high-quality asset seller prefers to supply the marketable asset. The equilibrium price* $p^*(x)$ *is then*

$$p^*(x) = \begin{cases} 1 & \text{if } x = n_s + n_b - \overline{x}, \\ \frac{1}{2} & \text{if } x \in \{n_b - \overline{x},\ -\overline{x},\ -n_s - \overline{x}\}, \\ 0 & \text{if } x = -n_b - n_s - \overline{x}. \end{cases}$$

Because prices are informative only if $x \in \{n_s + n_b - \overline{x},\ -n_b - n_s - \overline{x}\}$, it follows from Lemmas 2 and 3 with $\alpha'(i) > 0$ that an increase in i makes prices more informative.

If the high-quality asset seller does not prefer to supply the marketable asset, we obtain:

Lemma 4: *Suppose that the high-quality asset seller does not prefer to supply the marketable asset, then the equilibrium price is* $p^* = 0$.

Using Lemmas 1–4, we derive a threshold point of y below which the market freeze occurs.[25] Note that $\widehat{y}(i) < \overline{y}$ can be ensured by assuming that $\overline{y}$ is sufficiently large.

Lemma 5: *The market freeze occurs if and only if the return of the nonmarketable asset* y *is below* $\widehat{y}(i) \equiv \frac{2-\alpha(i)}{2+\alpha(i)}$, *that is,* $y < \widehat{y}(i)$. *Then, there is no trade in the market.*

When the quality of the marketable asset is private information, assets of different quality are traded at the same price. Then, if the nonmarketable asset has a relatively lower return, the high-quality asset seller never supplies the high-quality asset because he prefers to meet his liquidity needs by reducing investment in (liquidating) the nonmarketable asset. The seller is, thus, willing to supply only the low-quality asset to the market maker. An-

[25] We have already assumed that arbitrageurs do not trade the marketable asset for any speculative motive if they cannot obtain any positive revenues from trading. Furthermore, we also assume that the hedger does not trade the marketable asset for hedging motives if he cannot satisfy his hedging needs. Again, the latter assumption can be justified if there are infinitesimal trading costs.

ticipating this, neither the arbitrageur nor the hedger participates in the market when the nonmarketable asset has a relatively low return.

Now, using Lemmas 2, 3, and 5, we derive the hedger's decision:

Lemma 6:

$$n_b(i) = n_s(i) = \begin{cases} \max\left(2\left[\overline{z} - \frac{a}{b}\frac{\alpha(i)}{2-\alpha(i)}\right], 0\right) & \text{if } y \geq \widehat{y}(i), \\ 0 & \text{if } y < \widehat{y}(i). \end{cases} \tag{4}$$

For later use, define

$$\overline{n}(i) = 2\left[\overline{z} - \frac{a}{b}\frac{\alpha(i)}{2-\alpha(i)}\right]. \tag{5}$$

For simplicity, we focus on the case of $\overline{z} > \frac{a}{b}\frac{\alpha(i)}{2-\alpha(i)}$ for any i so that $\overline{n}(i) > 0$.[26]

To conclude the characterization of the equilibrium, we need to consider the arbitrageur's investment and effort decisions about information acquisition at date 0. The arbitrageur cannot earn any profits or invest when he does not incur the effort cost ei, because he cannot acquire any information about v. Thus, without loss of generality, we can focus on the case in which the arbitrageur always exerts effort in information acquisition.

It then follows from Lemmas 2–5 that the arbitrageur's problem is represented by

$$\max_{i \geq 0} \left[\frac{\alpha(i)}{4}\frac{1}{2}\overline{n}(i) + \frac{\alpha(i)}{4}\frac{1}{2}\overline{n}(i)\right]\left[1 - F(\widehat{y}(i))\right] - ci - ei, \tag{6}$$

where $\widehat{y}(i)$ is given by Lemma 5. Note that when $y \geq \widehat{y}(i)$, the arbitrageur's expected payoff can be positive only if he buys $\overline{n}(i)$ for $x = -\overline{x}$, $\theta = 1$, and $x^h = -\overline{n}(i)$, or he sells $\overline{n}(i)$ for $x = -\overline{x}$, $\theta = 0$, and $x^h = \overline{n}(i)$.

The first-order condition with respect to i is given by[27]

$$\frac{\alpha'(i)}{4}\left\{\left[\overline{n}(i) - \frac{a}{b}\frac{4\alpha(i)}{(2-\alpha(i))^2}\right]\left[1 - F(\widehat{y}(i))\right] + \frac{4\alpha(i)\overline{n}(i)f(\widehat{y}(i))}{(2+\alpha(i))^2}\right\} = c + e. \tag{7}$$

The first term in the largest bracket represents the direct effects of i on the expected

[26] A sufficient condition for $\overline{z} - \frac{a}{b}\frac{\alpha(i)}{2-\alpha(i)} > 0$ for any i is that $\overline{z}$ is large enough ($\overline{z}$ is sufficiently close to $\frac{a}{b}$) and/or $\lim_{i\to\infty} \alpha(i)$ is not sufficiently large.

[27] In the subsequent analysis, we assume that problem (6) has an interior solution.

trading profit of the arbitrageur when the market freeze does not occur. Specifically, the arbitrageur faces a trade-off between the two effects if he increases the likelihood of receiving an informative signal, $\alpha(i)$, by raising i when the market freeze does not occur. On the one hand, the higher $\alpha(i)$ creates greater profit opportunities; on the other hand, it causes greater informational asymmetry and reduces the volume of the hedger's demand, thus resulting in a thinner market and decreasing the arbitrageur's expected profits.

The second term in the largest bracket expresses the effect of i on the expected trading profit of the arbitrageur through the trading behavior of the high-quality asset seller. The higher $\alpha(i)$ makes the market maker's posterior belief become more accurate (see Lemma 2) and enhances the informativeness of prices (see the discussion under Lemma 3). As adverse selection regarding the asset is mitigated, the high-quality asset seller has more incentive to supply the marketable asset, thereby reducing the likelihood of the market freeze.[28] Then, an increase in i expands the profit opportunities of the arbitrageur.

Define i^* as the equilibrium investment level of the arbitrageur that satisfies (7). Then, for this i^*, the threshold of the market freeze, $\widehat{y}(i)$, is determined by Lemma 5 and the hedger's trading volume, $\overline{n}(i)$, is given by (5).

We discuss how i^* and $\widehat{y}(i^*)$ are affected by $\overline{z}$, $\overline{x}$, and c. We obtain:

Proposition 1: *i^* is increasing in $\overline{z}$, is independent of $\overline{x}$, and is decreasing in c.*

Proposition 2: *The market freeze is less likely to arise in equilibrium if $\overline{z}$ is larger and c is smaller, but is independent of $\overline{x}$.*

Thus, information investment (the likelihood of the market freeze) is larger (smaller) when the hedger's income shock is larger and the cost of investment is smaller, while these values are unaffected by the seller's endowment of the marketable asset.

To illustrate these results, we provide some numerical examples. We parametrize the information acquisition technology function, $\alpha(i) = \frac{e^i - e^{-i}}{e^i + e^{-i}}$ and choose the following set of basic parameters: $a = 5$, $b = 2.5$, $\overline{y} = 3$, $c = 0.001$, $e = 0.0001$, $\ell = 1.5$, $\overline{z} = 1$, and $\overline{x} = 11$. We also assume that y follows a uniform distribution on $[0, \overline{y}]$, where $\overline{y} = 3$.

[28] Note that the threshold of the market freeze, $\widehat{y}(i)$, defined by Lemma 5, is decreasing in $\alpha(i)$.

The solid line in Panel A of Figure 1 depicts the impact of a change in $\overline{z}$ on i^*, and shows that an increase in $\overline{z}$ raises i^*. The solid line in Panel C of Figure 1 plots the effect of a change in $\overline{z}$ on $\widehat{y}(i^*)$ in equilibrium, and shows that an increase in $\overline{z}$ reduces $\widehat{y}(i^*)$. Similarly, the solid lines in Panels A and C of Figure 2 indicate the effect of a change in $\overline{x}$ on i^* and $\widehat{y}(i^*)$, which means that i^* and $\widehat{y}(i^*)$ are independent of $\overline{x}$. The solid lines in Panels A and C of Figure 3 report that an increase in c reduces i^* and slightly raises $\widehat{y}(i^*)$.

The intuition for these results is as follows. For the investment, note that a larger $\overline{z}$ causes the hedger's income to become more volatile. This effect generates more motives to hedge, and induces the hedger to submit a larger order flow to the market maker. Then, by investing more to increase the likelihood of receiving an informative signal, the arbitrageur can potentially profit more from trading against the hedger. In addition, the arbitrageur can obtain positive expected profits only if the market freeze does not occur. This fact further strengthens the motive for the arbitrageur to increase i^* and enhances the informativeness of prices because the high-quality asset seller is more likely to supply the marketable asset as prices are less noisy (see the discussion in the next paragraph). Conversely, neither the hedger's demand nor the threshold of the high-quality asset seller supplying the marketable asset is affected by $\overline{x}$ (see Lemmas 1, 3, and 6). Thus, i^* is independent of $\overline{x}$. With regard to c, the result is evident because the information acquisition cost is then larger.

The result of the market freeze depends on an adverse selection mechanism regarding the asset. As the quality of the marketable asset is private information, assets of different quality trade at the same price. However, if prices are more informative, adverse selection regarding the asset is mitigated. Then, the high-quality asset seller supplies the marketable asset, even though divesting from the nonmarketable asset is less costly. Thus, the high-quality asset seller is more willing to supply the marketable asset for the higher i that improves the informativeness of the price. Indeed, Proposition 1 shows that i^* is larger if $\overline{z}$ is larger and c is smaller, but is independent of $\overline{x}$. Consequently, the likelihood of the market freeze is smaller if $\overline{z}$ is larger and c is smaller, but is independent of $\overline{x}$.

The result for $\overline{z}$ has interesting implications. An increase in $\overline{z}$ can be viewed as an increase in liquidity in the marketable asset market. Thus, Propositions 1 and 2 with Panels A and C from Figure 1 imply that a more liquid market is more likely to promote the arbitrageur's information investment and reduce the likelihood of the market freeze.

4.3. Welfare analysis.—

We focus on a constrained welfare maximization problem with respect to i, when the information structure of each trader is the same as that in equilibrium and there are no trade restrictions or regulations on any traders. This is equivalent to analyzing the case in which the regulator maximizes total welfare by choosing i without restricting any other actions of any traders when the regulator takes the information acquisition restrictions of any traders as given. If the regulator does not have any superior information, this analysis is reasonable. Then, we define total welfare as the sum of expected utilities at date 0 over all traders including the market maker:

$$W = E_0 \{ E_1 [v(\overline{x} - x^s) + (1+y)(p(x;y,i)x^s - \ell) \mid v, y, i] + E_1 [(v - p(x;y,i))x^a \mid \theta, y, i] \\ - (c+e)i + E_1 \left[U\left((v - p(x;y,i))x^h + z^h \right) \mid \rho, y, i \right] + E_1 [(p(x;y,i) - v)\, x \mid x, y, i] \} . \quad (8)$$

Note that the expected utilities of incompetent arbitrageurs are not included in (8) because they equal zero. We continue to focus on the case in which the talented arbitrageur always exerts effort in information acquisition.

Now, maximizing W with respect to i yields (see the Appendix for the derivation):

$$\frac{\alpha'(i)}{4} \left\{ \left[\overline{n}(i) - \frac{a}{b} \frac{4\alpha(i)}{(2-\alpha(i))^2} \right] [1 - F(\widehat{y}(i))] + \frac{4\alpha(i)\overline{n}(i) f(\widehat{y}(i))}{(2+\alpha(i))^2} \right\} \\ + \alpha'(i) [1 + \widehat{y}(i)] \left(1 - \frac{\alpha(i)}{2} \right) \frac{\overline{x} f(\widehat{y}(i))}{(2+\alpha(i))^2} - \frac{\alpha'(i)}{2} \left\{ \frac{b}{4} \overline{n}(i) \left[\overline{z} + \frac{a}{b} \frac{\alpha(i)}{2-\alpha(i)} \right] + \frac{a}{2} \overline{n}(i) \right\} [1 - F(\widehat{y}(i))] \\ + \frac{\alpha'(i)}{4} \frac{b(2-\alpha(i)) f(\widehat{y}(i))}{(2+\alpha(i))^2} (\overline{n}(i))^2 = c + e. \quad (9)$$

The first term on the left-hand side of (9) is the marginal speculation revenue of the arbitrageur in response to a change in i, which has the same expression as that on the left-hand side of (7). The second term on the left-hand side of (9) is due to an improvement in the seller's investment or disinvestment allocative efficiency in the nonmarketable asset. This effect of the improvement in the seller's allocative efficiency is positive when i increases. The third (fourth) term on the left-hand side of (9) measures an aggravation (improvement) in the risk sharing of the hedger's income shock when i increases. The third term shows the negative effect of the aggravation in the hedger's risk sharing when i increases. This arises because the arbitrageur can potentially profit more from trading against the hedger's hedging demand. Alternatively, the fourth term indicates the offsetting positive effect of the improvement in the hedger's risk sharing when i increases. This is because an increase in i reduces the likelihood of the market freeze that prevents the hedger's risk sharing. Let i_w^* denote i that satisfies (9).

The above three components regarding the effect of i on W on the left-hand side of (9) highlight the important factors involved in determining i_w^*. The first component regarding the effect on the arbitrageur's speculation profit is the same as the effect observed in the market equilibrium. The latter two components regarding the seller's allocative efficiency and the hedger's risk sharing involve the following trade-off. To start, the higher i is more likely to make prices more informative and induce the seller to avoid meeting liquidity needs by divesting the nonmarketable asset inefficiently rather than by selling the marketable asset. This improves allocative efficiency in the seller's nonmarketable investment, favors a higher i, and reduces the likelihood of the market freeze in total welfare maximization. Nonetheless, the hedger is less likely to hedge the income risk because the hedger prefers not to lose to the more informed trade as a result of the higher i. This effect favors a lower i and raises the likelihood of the market freeze in total welfare maximization. In fact, the hedger cannot insure against the income risk when the market freeze arises. This effect conversely favors a higher i and reduces the likelihood of the market freeze in total welfare maximization. However, the arbitrageur does not internalize the three effects created by the latter two components.

In the standard view, one might argue that underinvestment arises because the arbitrageur cannot receive enough profits when asset prices partially reveal his acquired information. One might also discuss that overinvestment occurs because information investment only serves the redistribution of profits between the arbitrageur and the hedger. However, the discussion of our DIR model is different from the standard view in the sense that the information investment serves to improve the seller's allocative efficiency and mitigates the likelihood of the market freeze.

Inspecting (9), we derive the following proposition.

Proposition 3: *The arbitrageur underinvests in information acquisition relative to the welfare-maximizing level, when the hedger's income shock $\overline{z}$ is sufficiently small and/or when the seller's endowment of the marketable asset $\overline{x}$ is sufficiently large.*

Proposition 4: *The market freeze is more likely to occur in equilibrium than in the welfare-maximizing case, when the hedger's income shock $\overline{z}$ is smaller and/or when the seller's endowment of the marketable asset $\overline{x}$ is larger.*

Because the welfare effects inevitably involve multiple forces moving in opposite directions, it is useful to provide numerical calculation results in Panels B and D of Figures 1 and 2 using the same set of basic parameters given in Section 4.2. We also provide the numerical calculation results regarding the effect of c in Figure 3.

Panel B of Figure 1 illustrates the effect of an increase in $\overline{z}$ on overinvestment (defined by $i^* - i^*_w$). The panel shows that if $\overline{z} \leq 0.802$ ($\overline{z} > 0.802$), underinvestment (overinvestment) occurs and the extent of the underinvestment (overinvestment) decreases (increases) with $\overline{z}$. Panel D of Figure 1 reports the effect of an increase in $\overline{z}$ on the difference between the likelihood of the market freeze in the equilibrium and welfare-maximizing cases (defined by $\widehat{y}(i^*) - \widehat{y}(i^*_w)$). In this panel, if $\overline{z} \leq 0.802$ ($\overline{z} > 0.802$), we find that the likelihood of the market freeze is larger (smaller) in equilibrium than in the welfare-maximizing case, and that $|\widehat{y}(i^*) - \widehat{y}(i^*_w)|$ is decreasing (increasing) in $\overline{z}$.

Panels B and D in Figure 2 illustrate the effect of an increase in $\overline{x}$ on $i^* - i^*_w$ and $\widehat{y}(i^*)$ $- \widehat{y}(i^*_w)$. Note that i^* and $\widehat{y}(i^*)$ are independent of $\overline{x}$. Then, in our parameter range of $\overline{x}$, we show that overinvestment always arises, and that the extent of the overinvestment decreases with $\overline{x}$. Indeed, the latter finding is consistent with Proposition 3. Furthermore, the likelihood of the market freeze is always smaller in equilibrium than in the welfare-maximizing case, whereas $|\widehat{y}(i^*) - \widehat{y}(i_w^*)|$ is decreasing in $\overline{x}$.

Panels B and D in Figure 3 depict the effect of an increase in c on $i^* - i_w^*$ and $\widehat{y}(i^*) - \widehat{y}(i_w^*)$. The panels indicate that in this range of c, overinvestment always arises, but the extent of the overinvestment is slightly decreasing in c. The likelihood of the market freeze is always smaller in equilibrium than in the welfare-maximizing case, while $|\widehat{y}(i^*) - \widehat{y}(i_w^*)|$ is slightly decreasing in c.

Intuitively, it follows from (9) that when $\overline{z}$ is small, the positive effects of i on the improvements in the seller's allocative efficiency and the hedger's risk sharing dominate the negative effect of i on the aggravation in the hedger's hedging demand. Thus, a decrease in $\overline{z}$ increases the social need of i. Because a decrease in $\overline{z}$ reduces the arbitrageur's private need of i (see the solid line in Panel A of Figure 1), underinvestment (or overinvestment) arises for $\overline{z} \leq 0.802$ (or $\overline{z} > 0.802$), while the likelihood of the market freeze is larger (or smaller) in equilibrium than in the welfare-maximizing case for such $\overline{z}$. As a result, $|\widehat{y}(i^*) - \widehat{y}(i_w^*)|$ is decreasing (or increasing) in $\overline{z}$ when $\overline{z} \leq 0.802$ (or $\overline{z} > 0.802$). An increase in $\overline{x}$ raises the effect of i on the improvement in the seller's allocative efficiency, while it does not affect the hedger's hedging demand. Thus, an increase in $\overline{x}$ increases the social need of i. As $\overline{x}$ does not affect the arbitrageur's private need of i (see the solid line in Panel A of Figure 2), an increase in $\overline{x}$ reduces the extent of overinvestment. This effect also decreases $|\widehat{y}(i^*) - \widehat{y}(i_w^*)|$. Lastly, a decrease in c reduces the cost of i in both the equilibrium and welfare-maximizing cases. However, for the set of the basic parameters, a decrease in c increases the arbitrageur's private need of i more than the social need of i (see Panel A of Figure 3) and raises the extent of overinvestment. As a result, a decrease in c increases $|\widehat{y}(i^*) - \widehat{y}(i_w^*)|$.

Several remarks are in order. First, an increase in $\overline{z}$ can be interpreted as an increase in liquidity in the marketable asset market. Thus, Propositions 3 and 4 with Panels B and D of Figure 1 suggest that if the liquidity is small (large), underinvestment (overinvestment) occurs in equilibrium and the market freeze is more (less) likely to arise in equilibrium than in the welfare-maximizing case, while the extent of underinvestment (overinvestment) and $|\widehat{y}(i^*) - \widehat{y}(i_w^*)|$ are decreasing (increasing) in the liquidity.

Second, $\overline{x}$ measures the magnitude of the issuance of new equities or bonds or structured financial products. Hence, Propositions 3 and 4 with Panels B and D of Figure 2 imply that if the issuance of these assets is larger, the extent of overinvestment and $|\widehat{y}(i^*) - \widehat{y}(i_w^*)|$ are reduced.

Third, a decrease in c can be viewed as improvements in information technology. Our numerical calculations suggest that improvements in information technology decrease the extent of overinvestment and $|\widehat{y}(i^*) - \widehat{y}(i_w^*)|$.

Finally, Glode, Green, and Lowery (2012), based on a bargaining model, suggest that financial firms have incentives to overinvest in financial expertise. This result is consistent with our result only if the hedger's income shock is not small. In their model, the ability of expertise to acquire more accurate information protects a trader from opportunistic bargaining by his counterparties and results in more favorable terms of trade. Thus, informative signals cannot lead to efficiency in allocation in such an arms race environment. Furthermore, the possibility of a trader acquiring more accurate information causes only adverse selection and induces his counterparties to avoid trading with him because they know that they will end up buying only when the true value is low (selling only when it is high). Hence, financial firms have incentives to overinvest in financial expertise, such that the overinvestment increases the likelihood of the market breakdown if asset value volatility rises. In our model, the higher possibility of the arbitrageur acquiring more accurate information is more likely to induce the hedger to avoid hedging the income risk because it aggravates the adverse selection problem; however, it improves the informativeness of prices, enables the informed high-quality seller to sell the marketable asset at more reasonable prices, and reduces the likelihood of the market breakdown. If the hedger's income shock is small, the latter effect dominates the former. Then, the social need of investment is larger than the arbitrageur's private need of investment so that underinvestment can occur.

5. DEL Model

5.1. Definition of equilibrium.—

Because portfolio delegation is widely observed in practice, we now consider where a representative, uninformed, risk-neutral investor (principal) entrusts her money to an arbitrageur, who serves as a portfolio manager protected by limited liability. We implicitly assume that it is substantially costly not only for the principal to acquire and process information because of the lack of financial and technological expertise, but also for the arbitrageur to operate as a stand-alone entity. Then, the arbitrageur needs to be compensated according to a contract designed by the principal at date 0. However, as the contract cannot condition directly on the arbitrageur's private information or his shirking decision, his incentives may be distorted. First, incompetent arbitrageurs may have an incentive to be employed as portfolio managers. As incompetent arbitrageurs are assumed to be dominant in the population, any contract that attracts them will oblige the principal to almost surely hire them. Thus, this possibility needs to be excluded in order to avoid entailing a positive payment in return for nothing. Second, the talented arbitrageur may have a distorted incentive to trade ("churning" incentive), even though he has received no informative signal. Finally, because the principal cannot observe whether the talented arbitrageur exerts information acquisition effort, the talented arbitrageur may have an incentive not to exert any information acquisition effort.

The timeline of the model is the same as that of the DIR model, except that:

1. At date 0, the principal decides whether to hire an arbitrageur as a portfolio manager and how much the arbitrageur invests in information acquisition.[29] If the principal hires the arbitrageur, she designs a contract with the arbitrageur. When employed, the arbitrageur decides whether to exert informational acquisition effort.

3. At date 2, v, y, and z^h are realized. The contract compensation is received by the arbitrageur.

Note that the investment in information acquisition is public information at date 1. The

[29] If the principal does not hire any arbitrageurs, she does not trade the marketable asset, because she does not have any private information and, thus, she cannot obtain any positive revenues from trading. Such a tie-breaking assumption can be justified if there are infinitesimal trading costs.

other information structure is also the same as that of the DIR model, except that the trading position of the arbitrageur is not observable to any other agents at date 1 but is observable and verifiable to the principal at date 2, and that the contract between the principal and the arbitrageur is observable to all the agents at date 1.

The talented arbitrageurs' (incompetent arbitrageurs') reservation payoff is denoted by $\underline{\Pi}^{ta}$ ($\underline{\Pi}^{ia}$). We assume that $\underline{\Pi}^{ta} > \underline{\Pi}^{ia}$. For simplicity, we also assume that $\underline{\Pi}^{ia} = 0$.

As in the preceding section, we start by assuming that the hedger will trade $x^h = n_b^d \geq 0$ ($x^h = -n_s^d \leq 0$) if he buys (sells) the marketable asset, that is, if the income shock is negatively (positively) correlated with v. Again, the quantities n_b^d and n_s^d are derived later in the hedger's optimization problem.

In the information structure of this model, the contract cannot condition directly on whether the arbitrageur is talented or incompetent, on whether he receives informative private information, or on whether he exerts information acquisition effort. Nevertheless, the contract can condition on the realized value of the marketable asset and on the trading position the arbitrageur took. At the end of Section 5.2, by imposing an out-of-equilibrium belief on the market maker, we show that the principal does not offer the arbitrageur any contract that rewards him for trading any quantities other than n_s^d, $-n_b^d$, and 0. Thus, the arbitrageur will either buy n_s^d or sell n_b^d or does not trade under the optimal contract.

To give the arbitrageur proper incentives, the principal needs to design nonnegative payments $m = (m_1, m_2, m_3, m_4, m_5, m_6) \geq 0$ to the arbitrageur in each contingency as follows: (i) when the market freeze does not occur, (a) m_1: the payment if $v = 1$ and $x^a = n_s^d$; (b) m_2: the payment if $v = 1$ and $x^a = -n_b^d$; (c) m_3: the payment if $v = 0$ and $x^a = n_s^d$; (d) m_4: the payment if $v = 0$ and $x^a = -n_b^d$; (e) m_5: the payment when the arbitrageur does not trade despite the absence of the market freeze; and (ii) when the market freeze arises, (f) m_6: the payment.[30] Note that the arbitrageur cannot be penalized with a negative payment because of limited liability. Then, as shown by Dow and Gorton (1997), the arbitrageur may trade n_s^d or $-n_b^d$ at random, even though he has no informative signal. Let ζ (ζ^u) denote the

[30] In the last two cases, although $v = 1$ or 0, there is no need to distinguish between these two possibilities.

probability of the talented arbitrageur trading n_s^d or $-n_b^d$ at random when he exerts (does not exert) information acquisition effort and does not receive any informative signal. We also define η as the probability of incompetent arbitrageurs trading n_s^d or $-n_b^d$. We assume that the arbitrageurs do not commit to the choice of ζ, ζ^u, or η before the market opens.

An equilibrium now consists of a price p, the trading strategies of the seller and the hedger (x^s, x^h), the trading strategy of the arbitrageurs $(x^a, \zeta, \zeta^u, \eta)$, a contract payment $m = (m_1, m_2, m_3, m_4, m_5, m_6)$, and an investment i such that (i) (x^s, x^h) solves the seller's and hedger's problems of (10) and (11) defined below, respectively, (ii) $(x^a, \zeta, \zeta^u, \eta, m, i)$ solves the principal's contracting problem of (20) given in the next subsection, (iii) $x = x^a + x^h - x^s$, and (iv) the price satisfies $p = E_1(v \mid x, y, i, m)$. Let $p(x; y, i, m) \equiv E_1(v \mid x, y, i, m)$.

The seller's problem is

$$\max_{x^s \in [0, \overline{x}]} E_1 \left\{ v(\overline{x} - x^s) + (1 + y)[p(x; y, i, m)x^s - \ell] \mid v, y, i, m \right\}; \tag{10}$$

whereas the hedger's problem is

$$\max_{x^h} E_1 \left\{ U\left[(v - p(x; y, i, m))x^h + z^h \right] \mid \rho, y, i, m \right\}.^{31} \tag{11}$$

5.2. Principal's contracting problem.—

We first need to characterize the equilibrium price $p^{**}(x)$ and the hedger's demand. To derive $p^{**}(x)$, we begin with the seller's optimal trading strategy:

Lemma 7: *Given v, y, i, and m, the seller's supply is determined by*

$$x^s = \begin{cases} 0 & \text{if } E_1\left[p(x; y, i, m) \mid v, y, i, m\right] < \frac{v}{1+y}, \\ \overline{x} & \text{if } E_1\left[p(x; y, i, m) \mid v, y, i, m\right] \geq \frac{v}{1+y}. \end{cases}$$

Next, to specify the total order flow x in equilibrium, we determine the arbitrageur's

[31] As in the DIR model, the seller has no incentive to deviate x^a determined by (10) because the seller trades only for the liquidity motive. Similarly, the hedger has no incentive to deviate x^h determined by (11) because the hedger consists of a continuum of infinitesimal agents who cannot affect the aggregate trading volume.

trading strategy x^a at date 1 when the market freeze does not occur. As mentioned, we only need to consider the talented arbitrageur in equilibrium because incompetent arbitrageurs must be excluded in the optimal contract. We also focus on the case in which the talented arbitrageur exerts information acquisition effort in equilibrium.[32] The arbitrageur buys n_s^d ($x^a = n_s^d$) if $\theta = 1$, and sells n_b^d ($x^a = -n_s^d$) if $\theta = 0$. This will be justified by the assumption of the contract compensation imposed at the beginning of Section 5.3. In addition, as argued in the contract payment in Section 5.1, the arbitrageur buys n_s^d or sells n_b^d at random with probability ζ if $\theta = \phi$, and does not trade with probability $1 - \zeta$ if $\theta = \phi$.

Then, the probability for each event of the total order flow at date 1 is inferred as follows.

Lemma 8: *Suppose that the high-quality asset seller prefers to supply the marketable asset. Then, the probability that the market participants infer for each event of the total order flow at date 1 is given by: (i)* $\frac{1}{4}\{\alpha(i) + [1 - \alpha(i)]\zeta\}$ *if* $x = n_s^d + n_b^d - \overline{x}$*; (ii)* $\frac{1}{2}[1 - \alpha(i)](1 - \zeta)$ *if* $x = n_b^d - \overline{x}$*; (iii)* $\frac{1}{2}\{\alpha(i) + [1 - \alpha(i)]\zeta\}$ *if* $x = -\overline{x}$*; (iv)* $\frac{1}{2}[1 - \alpha(i)](1 - \zeta)$ *if* $x = -n_s^d - \overline{x}$*; and (v)* $\frac{1}{4}\{\alpha(i) + [1 - \alpha(i)]\zeta\}$ *if* $x = -n_b^d - n_s^d - \overline{x}$.

Using Lemma 8, $p^{**}(x)$ is characterized by the following lemma.

Lemma 9: *(i) Suppose that the high-quality asset seller prefers to supply the marketable asset. The equilibrium price is then*

$$p^{**}(x) = \begin{cases} \frac{1}{2}\psi_1 & \text{if } x = n_s^d + n_b^d - \overline{x}, \\ \frac{1}{2} & \text{if } x \in \{n_b^d - \overline{x}, -\overline{x}, -n_s^d - \overline{x}\}, \\ \frac{1}{2}\psi_2 & \text{if } x = -n_b^d - n_s^d - \overline{x}, \end{cases}$$

where $\psi_1 = \frac{2\alpha(i)+[1-\alpha(i)]\zeta}{\alpha(i)+[1-\alpha(i)]\zeta}$ and $\psi_2 = \frac{[1-\alpha(i)]\zeta}{\alpha(i)+[1-\alpha(i)]\zeta}$.

(ii) Suppose that the high-quality asset seller does not prefer to supply the marketable asset. Then, the equilibrium price is $p^{**}(x) = 0$.

As prices are more informative only if $x \in \{n_s + n_b - \overline{x}, -n_b - n_s - \overline{x}\}$, it follows from Lemmas 8 and 9 with $\alpha'(i) > 0$ and $\zeta \in [0, 1]$ that an increase in i makes prices

[32] Otherwise, the analysis is trivial because the principal cannot earn any profits.

more informative for a fixed ζ.[33] However, if an uninformed arbitrageur chooses the random trading strategy ($\zeta > 0$), prices are less informative because $\frac{1}{2}\psi_1 < 1$ and $0 < \frac{1}{2}\psi_2$ for $\zeta > 0$.

Using Lemmas 7–9, we derive the threshold point of y below which the market freeze occurs.[34] Note that the arbitrageur's choice of ζ does not depend on y, as will be shown later in (18).

Lemma 10: *The market freeze occurs if and only if the return of the nonmarketable asset* y *is below* $\widehat{y}^d(i,\zeta) \equiv \frac{1}{\frac{1}{4}[\alpha(i)+\frac{1}{2}(1-\alpha(i))\zeta]\psi_1+\frac{1}{2}[1-\alpha(i)](1-\zeta)+\frac{1}{4}[\alpha(i)+(1-\alpha(i))\zeta]+\frac{1}{8}[1-\alpha(i)]\zeta\psi_2} - 1$, *that is,* $y < \widehat{y}^d(i,\zeta)$. *Then, there is no trade in the market.*

Now, using Lemmas 8–10, we determine the hedger's decision at date 1:

Lemma 11: *For* i *and* ζ, *let* $n_b^d(i,\zeta)$ $(n_s^d(i,\zeta))$ *be the order flow of the hedger whose income shock is negatively (positively) correlated with* v. *Then,* $n_b^d(i,\zeta) = n_s^d(i,\zeta) \geq 0$. *If* $\zeta = 1$,

$$n_b^d(i,1) = n_s^d(i,1) = \begin{cases} 2\left[\overline{z} - \frac{a}{b}\frac{\alpha(i)}{2-(\alpha(i))^2}\right] & \text{if } y \geq \widehat{y}^d(i,1), \\ 0 & \text{if } y < \widehat{y}^d(i,1), \end{cases} \tag{12a}$$

where $\widehat{y}^d(i,1) \equiv \frac{2-(\alpha(i))^2}{2+(\alpha(i))^2}$; *and if* $\zeta = 0$,

$$n_b^d(i,0) = n_s^d(i,0) = \begin{cases} \overline{n}(i) = 2\left[\overline{z} - \frac{a}{b}\frac{\alpha(i)}{2-\alpha(i)}\right] & \text{if } y \geq \widehat{y}^d(i,0), \\ 0 & \text{if } y < \widehat{y}^d(i,0), \end{cases} \tag{12b}$$

where $\overline{n}(i)$ *is given by (5) and* $\widehat{y}^d(i,0) \equiv \frac{2-\alpha(i)}{2+\alpha(i)} = \widehat{y}(i)$.

Because $\overline{z} > \frac{a}{b}\frac{\alpha(i)}{2-\alpha(i)}$ has been assumed for any i, we have $n_b^d(i,1) = n_s^d(i,1) > n_b^d(i,0) = n_s^d(i,0) > 0$ for any i. For later use, define

$$\overline{\overline{n}}(i) = 2\left[\overline{z} - \frac{a}{b}\frac{\alpha(i)}{2-(\alpha(i))^2}\right]. \tag{13}$$

[33] Note that $\frac{\partial\psi_1}{\partial i} \geq 0$ and $\frac{\partial\psi_2}{\partial i} \leq 0$.

[34] We have already assumed that the hedger does not trade the marketable asset for hedging motives if the hedging need cannot be satisfied. We also assume that the arbitrageur does not trade the marketable asset if he cannot obtain any positive compensation from trading. The latter assumption can be justified in the DEL model if the arbitrageur incurs an infinitesimal disutility cost when trading. As we verify $m_6 = 0$ later, the arbitrageur does not trade if $y < \widehat{y}^d(i,\zeta)$.

We can now formalize the optimal contract between the principal and the arbitrageur. In the Appendix, we specify the expected payoff of the talented (incompetent) arbitrageur(s) at date 0, $\Pi^{ta}(i,\zeta)$ ($\Pi^{in}(i,\zeta)$), and the expected payoff of the principal with the talented arbitrageur at date 0, $\Pi^{p}(i,\zeta)$, relative to i and ζ as (A8)–(A10).

The next step is to describe several constraints to be satisfied by the optimal contract. First, the principal needs to induce the talented arbitrageur to enter into the contract. It follows from (A8) that his participation condition is

$$\Pi^{ta}(i,\zeta') \geq \underline{\Pi}^{ta}, \tag{14}$$

where $\zeta' = \zeta(i,m) = \arg\max_{0\leq\zeta\leq1}\left\{\left[\frac{\alpha(i)}{2}+\frac{(1-\alpha(i))\zeta}{4}\right](m_1+m_4)+\frac{[1-\alpha(i)]\zeta}{4}(m_2+m_3)+[1-\alpha(i)](1-\zeta)m_5\right\}$. As the arbitrageur cannot commit to any level of ζ before the market opens, ζ' is chosen at date 1 to maximize his expected payoff at date 1 after the market opens. Note that ζ' does not depend on y because (m_1,m_2,m_3,m_4,m_5) does not depend on y.

Second, as mentioned at the beginning of Section 5.1, the optimal contract must exclude the possibility of hiring any incompetent arbitrageurs. It follows from (A9) that the self-selection condition is represented by

$$\Pi^{in}(i,\zeta') \leq 0, \tag{15}$$

where ζ' is defined at (14). We assume that incompetent arbitrageurs cannot participate in the contract relation if they cannot obtain any positive revenues from trading.[35]

Third, as the talented arbitrageur needs to be induced to exert effort for information acquisition, using (A8), the following incentive-compatibility condition needs to be satisfied:

$$\Pi^{ta}(i,\zeta') \geq \int_{\widehat{y}^d(i,\zeta')}^{\overline{y}}\left\{\max_{0\leq\zeta^u\leq1}\left[\frac{\zeta^u}{4}(m_1+m_2+m_3+m_4)+(1-\zeta^u)m_5\right]\right\}dy+\int_0^{\widehat{y}^d(i,\zeta')}m_6dy-ci, \tag{16}$$

where ζ' is still defined at (14). Note that if the talented arbitrageur does not exert effort,

[35] This assumption can be justified if incompetent arbitrageurs incur infinitesimal disutility or monetary costs when they disguise themselves as talented arbitrageurs.

he only receives $\theta = \phi$ and uses the random trading strategy ζ^u after the market opens.

The remaining problem is to verify that the principal does not provide the arbitrageur any contract rewarding him for trading any quantities other than n_s^d, $-n_b^d$, or 0. We impose the following out-of-equilibrium belief on the market maker when the market freeze does not occur: anticipating the seller's optimal strategy $x^s = \overline{x}$, the market maker believes that any deviation of the total order flow from $-\overline{x} + n_s^d$ or $-\overline{x}$ or $-\overline{x} - n_b^d$ must come from the arbitrageur rather than the hedger, without changing the market maker's belief in the relative likelihood that the arbitrageur has an informative or uninformative signal. In addition, we assume that the market maker believes that any positive (negative) quantity other than n_s^d ($-n_b^d$) is ordered by the arbitrageur who would otherwise have traded n_s^d ($-n_b^d$).[36] As a result, the principal reduces her expected payoff if she gives the arbitrageur a contract rewarding him for trading any quantities other than n_s^d, $-n_b^d$, or 0.

The optimal contracting problem is now formalized by

$$\max_{(i,m)\geq 0} \Pi^p(i, \zeta'), \tag{17}$$

subject to (14)–(16) and ζ' is defined at (14).

5.3. Characterization of equilibrium.—

We consider a class of symmetric contract in which the payment is the same not only for both correct trading decisions, but also for both incorrect trading decisions: $m_1 = m_4 = \overline{m} > 0$ and $m_2 = m_3 = 0$. This justifies the assumption that the arbitrageur buys n_s^d if $\theta = 1$, and sells n_b^d if $\theta = 0$. Although there can be other contracts that are equivalent in terms of incentives and expected costs, we focus on this class of contract for simplicity, because the distinction is not a concern in this paper. In addition, increasing m_6 increases adverse incentives for incompetent arbitrageurs to disguise themselves as the talented one, and reduces the principal's expected payoff. Thus, we can set $m_6 = 0$.

We begin with deriving the optimal choices of ζ' (or ζ), η, and ζ^u.[37] Given the linearity

[36] As in footnote 23, the market maker can exactly infer the size of the arbitrageur's order unless $x = \overline{x}$.

[37] We assume that the arbitrageur chooses not to trade n_s^d or n_b^d if indifference between choices. This

of the expected payoffs of the talented and incompetent arbitrageurs at date 1 with respect to ζ', η, and ζ^u, we have

Lemma 12:

$$\zeta' = \eta = \zeta^u = \begin{cases} 1 & \text{if } \frac{1}{2}\overline{m} > m_5, \\ 0 & \text{if } \frac{1}{2}\overline{m} \leq m_5. \end{cases} \tag{18}$$

Next, we characterize the optimal choice of the investment level. Suppose that $\frac{1}{2}\overline{m} > m_5$, that is, $\zeta' = \eta = \zeta^u = 1$. Using Lemma 11 with (13) and (18), $\psi_1 = 1 + \alpha(i)$ and $\psi_2 = 1 - \alpha(i)$ from Lemma 9, and $m_1 = m_4 = \overline{m}$ and $m_2 = m_3 = m_6 = 0$, problem (17) is reduced to

$$\Pi_1^p \equiv \max_{(i,\overline{m},m_5)\geq 0} \left\{ \frac{\alpha(i)}{4}\overline{\overline{n}}(i) - \frac{1}{2}\left[1+\alpha(i)\right]\overline{m} \right\} \left[1 - F(\widehat{y}^d(i,1))\right], \tag{19a}$$

subject to

$$\frac{1}{2}\left[1+\alpha(i)\right]\overline{m}\left[1 - F(\widehat{y}^d(i,1))\right] \geq (c+e)i + \underline{\Pi}^{ta}, \tag{19b}$$

$$\frac{1}{2}\overline{m}\left[1 - F(\widehat{y}^d(i,1))\right] \leq ci, \tag{19c}$$

$$\frac{1}{2}\alpha(i)\overline{m}\left[1 - F(\widehat{y}^d(i,1))\right] \geq ei, \tag{19d}$$

$$\frac{1}{2}\overline{m} > m_5. \tag{19e}$$

Here, (19b)–(19d) correspond to (14)–(16), respectively; (19e) is the incentive-compatibility constraint for the arbitrageurs to trade n_s^d or $-n_b^d$ at random when they have no informative signal; and the hedger's trading amount is equal to $\overline{\overline{n}}(i)$ from (13).

Conversely, suppose that $\frac{1}{2}\overline{m} \leq m_5$, that is, $\zeta' = \eta = \zeta^u = 0$. It follows from Lemma 11 with (5) and (18), $\psi_1 = 2$ and $\psi_2 = 0$ from Lemma 9, and $m_1 = m_4 = \overline{m}$ and $m_2 = m_3 = m_6 = 0$ that problem (17) is reduced to

$$\Pi_0^p \equiv \max_{(i,\overline{m},m_5)\geq 0} \left\{ \frac{\alpha(i)}{4}\overline{n}(i) - \alpha(i)\overline{m} - \left[1-\alpha(i)\right]m_5 \right\} \left[1 - F(\widehat{y}^d(i,0))\right], \tag{20a}$$

assumption can be justified if the arbitrageur incurs infinitesimal disutility costs when trading.

subject to

$$\{\alpha(i)\overline{m} + [1-\alpha(i)]\, m_5\} \left[1 - F(\widehat{y}^d(i,0))\right] \geq (c+e)i + \underline{\Pi}^{ta}, \tag{20b}$$

$$m_5 \left[1 - F(\widehat{y}^d(i,0))\right] \leq ci, \tag{20c}$$

$$\alpha(i)(\overline{m} - m_5) \left[1 - F(\widehat{y}^d(i,0))\right] \geq ei, \tag{20d}$$

$$\frac{1}{2}\overline{m} \leq m_5. \tag{20e}$$

Note that (20b)–(20d) correspond to (19b)–(19d), respectively; (20e) is the incentive-compatibility constraint for the arbitrageurs not to trade n_s^d or $-n_b^d$ at random when they have no informative signal; and the hedger's trading amount is equal to $\overline{n}(i)$ from (5).

Solving problems (19) and (20), we derive the following lemmas.[38]

Lemma 13: *(i) If self-selection condition (19c) is not binding with equality, the optimal investment level in (19) is determined by*

$$\frac{\alpha'(i)}{4}\left\{\left[\overline{\overline{n}}(i) - \frac{a}{b}\frac{2\alpha(i)[2+(\alpha(i))^2]}{[2-(\alpha(i))^2]^2}\right]\left[1 - F(\widehat{y}^d(i,1))\right] + \frac{8(\alpha(i))^2\overline{\overline{n}}(i) f(\widehat{y}^d(i,1))}{[2+(\alpha(i))^2]^2}\right\} = c + e. \tag{21}$$

(ii) If (19c) is binding with equality while the talented arbitrageur's participation condition (19b) is not binding with equality, the optimal investment level in (19) is determined by

$$\frac{\alpha'(i)}{4}\left\{\left[\overline{\overline{n}}(i) - \frac{a}{b}\frac{2\alpha(i)[2+(\alpha(i))^2]}{[2-(\alpha(i))^2]^2}\right]\left[1 - F(\widehat{y}^d(i,1))\right] + \frac{8(\alpha(i))^2\overline{\overline{n}}(i) f(\widehat{y}^d(i,1))}{[2+(\alpha(i))^2]^2} - 4ci\right\}$$

$$= [1+\alpha(i)]c. \tag{22}$$

(iii) If both (19b) and (19c) are binding with equalities, the optimal investment level in (19) is determined by

$$\alpha(i) = \frac{e}{c} + \frac{\underline{\Pi}^{ta}}{ci}. \tag{23}$$

Lemma 14: *(i) If self-selection condition (20c) is not binding with equality, the optimal*

[38] We assume that the optimal investment level is positive in both of these problems (19) and (20).

investment level in (20) is determined by

$$\frac{\alpha'(i)}{4}\left\{\left[\overline{n}(i)-\frac{a}{b}\frac{4\alpha(i)}{(2-\alpha(i))^2}\right]\left[1-F(\widehat{y}^d(i,0))\right]+\frac{4\alpha(i)\overline{n}(i)f(\widehat{y}^d(i,0))}{(2+\alpha(i))^2}\right\}=c+e. \tag{24}$$

(ii) If (20c) is binding with equality while the talented arbitrageur's participation condition (20b) is not binding with equality, the optimal investment level in (20) is determined by

$$\frac{\alpha'(i)}{4}\left\{\left[\overline{n}(i)-\frac{a}{b}\frac{4\alpha(i)}{(2-\alpha(i))^2}\right]\left[1-F(\widehat{y}^d(i,0))\right]+\frac{4\alpha(i)\overline{n}(i)f(\widehat{y}^d(i,0))}{(2+\alpha(i))^2}-4ci\right\}$$

$$=[1+\alpha(i)]c. \tag{25}$$

(iii) If both (20b) and (20c) are binding with equalities, the optimal investment level in (20) is determined by (23).

If (19c) ((20c)) is binding with equality, adverse selection regarding the arbitrageur's type is severe. In addition, if both (19b) and (19c) ((20b) and (20c)) are binding with equalities, the adverse selection problem is too serious. Then, the principal finds it more difficult to mitigate the adverse selection problem by adjusting only compensation. Hence, the optimal investment level is determined by setting $\alpha(i)$ equal to $\frac{ei+\Pi^{ta}}{ci}$.

We now discuss whether the principal actually allows the arbitrageur to trade n_s^d or $-n_b^d$ at random under the optimal contract, when he has no informative signal. Comparing the solution in the case of $\frac{1}{2}\overline{m} \geq m_5$ with that in the case of $\frac{1}{2}\overline{m} < m_5$, we obtain:

Proposition 5: *(i) Under the optimal contract, the principal will not always allow the arbitrageur to trade n_s^d or $-n_b^d$ at random when he has no informative signal.*
(ii) Suppose that the return of the nonmarketable asset is uniformly distributed on $[0, \overline{y}]$. Then, if neither the ratio of liquidity in the marketable asset market to the upper bound of the return of the nonmarketable asset, $\frac{\overline{z}}{\overline{y}}$, nor the cost of information acquisition investment is large, the optimal contract involves the arbitrageur's random trading strategy when he has no informative signal.

Proposition 5 shows that churning is not necessarily optimal, even though Dow and Gorton

(1997) argue that it always is. However, if neither $\frac{\bar{z}}{\bar{y}}$ nor c is large, the optimal contract involves churning.

The intuition behind Proposition 5 is as follows. Suppose that the market freeze does not occur, which corresponds to the situation in Dow and Gorton (1997). Then, the random trading strategy of an uninformative arbitrageur enables the hedger to effectively insure the income risk at lower cost. This is because the price is less informative as ζ increases (see the discussion below Lemma 9). Thus, the hedger will respond by trading more. Because the principal can actually earn higher trading profits, she has an incentive to allow the arbitrageur to take the churning even when he has no informative signal. However, the less informative price from the churning discourages the seller from selling the high-quality asset as a result of the more severe adverse selection, thereby increasing the possibility that the market freeze arises. The churning then restricts the profit opportunities for the principal.

If the latter expected cost of the lost trading opportunities arising from the market freeze dominates the former expected trading profit resulting from the more hedging demand, the principal prefers to deter the arbitrageur from taking the churning by raising the payment, m_5, when he does not trade despite the absence of the market freeze (that is, when he is "actively" doing nothing), relative to the "success" payment to him, $\overline{m}$. This strategy is possible because the investment cost as well as m_5 can also preclude incompetent arbitrageurs from entering into the contact if the investment level is not sufficiently small. However, if liquidity in the marketable asset market relative to the upper bound of the return of the nonmarketable asset is not large, the expected cost of the lost trading opportunities arising from the market freeze is small. Then, the negative effect of the churning is relatively insignificant. In addition, if the investment cost is not large, the principal finds it difficult to exclude incompetent arbitrageurs. Hence, under these situations, the benefit brought about by the churning dominates its cost.

We now clarify the implications of varying $\overline{z}$, c, and $\overline{x}$ by providing numerical calculation results in Figures 4–6. In equilibrium under the DEL model, let i^{**} and ζ^{**} denote the investment in information acquisition and the likelihood of the churning, respectively.

The solid line in Panel A of Figure 4 shows that $\zeta^{**} = 1$ for $\overline{z} < 1.092$, while $\zeta^{**} = 0$ for $\overline{z} \geq 1.092$. This implies that for the higher range of $\overline{z}$, the optimal contract does not involve the arbitrageur's random trading strategy when he has no informative signal. Note that for fixed $\overline{y}$, this result is consistent with Proposition 5, which suggests that the optimal contract involves $\zeta^{**} = 1$ for a small $\frac{\overline{z}}{\overline{y}}$, whereas it does not always do so for any other $\frac{\overline{z}}{\overline{y}}$. The solid line in Panel B indicates that i^{**} is determined by either (22) for $\overline{z} < 1.092$, and (25) for $\overline{z} \geq 1.092$. The solid line in Panel C illustrates that i^{**} is increasing in $\overline{z}$. Because of Lemmas 10 and 11, the threshold of the occurrence of the market freeze depends on ζ^{**}, and $\widehat{y}^d(i^{**}, 1) > \widehat{y}^d(i^{**}, 0)$. Thus, the solid line in Panel D reveals that the market freeze is more likely to occur when $\overline{z}$ is smaller.

The solid lines in Panels A–D of Figure 5 illustrate the effects of an increase in $\overline{x}$. In this change, the optimal contract always involves $\zeta^{**} = 1$. Furthermore, i^{**} is determined by (22) and is independent of $\overline{x}$. Because ζ^{**} and i^{**} are independent of $\overline{x}$, the likelihood of the market freeze is also independent of $\overline{x}$, as suggested in Lemmas 10 and 11.

Lastly, Panels A–D of Figure 6 depict the effect of an increase in c. In this change of c, the optimal contract always involves $\zeta^{**} = 1$. i^{**} is determined by (22) and is slightly decreasing in c. Consequently, the threshold of the market freeze is slightly increasing in c, which implies that the market freeze is more likely to occur when c is larger.

Several remarks are in order. First, regardless of whether $\zeta^{**} = 1$ or $\zeta^{**} = 0$, only the self-selection condition is binding with equality in equilibrium.

Second, our results suggest that i^{**} is larger when $\overline{z}$ is larger and c is smaller, whereas it is independent of $\overline{x}$, like the DIR model, although the adverse selection problem regarding the arbitrageur's type and the churning of the arbitrageur are newly added under the DEL model. The reason is that under the DEL model, only the self-selection condition is binding with equality in equilibrium. Then, i^{**} is determined by (22) or (25), which is not a "corner" solution of (23). Hence, the effects of $\overline{z}$, $\overline{x}$, and c on i^{**} are similar to those on i^*.

Third, the likelihood of the market freeze is decreasing in $\overline{z}$ and increasing in c, while it is independent of $\overline{x}$. If $\overline{z}$ is large, the optimal contract involves $\zeta^{**} = 0$. Given that $\widehat{y}^d(i, 1)$

$> \widehat{y}^d(i,0)$ (see Lemma 11) and that i^{**} is increasing in $\overline{z}$, this explains the effect of changes in $\overline{z}$ on the likelihood of the market freeze. Conversely, for the variations in c and $\overline{x}$, $\zeta^{**} = 1$ is always optimal in these changes. As i^{**} is decreasing in c while it is independent of $\overline{x}$, the likelihood of the market freeze is increasing in c and is independent of $\overline{x}$.

Finally, $\zeta^{**} = 0$ is optimal only when $\overline{z}$ is large. The intuition why $\zeta^{**} = 1$ is optimal when $\overline{z}$ is not large is that the negative effect of the churning through an increase in the likelihood of the market freeze is relatively insignificant, and that c is relatively high in our parametric case, as explained in the intuitive discussion following Proposition 5.

5.4. Welfare analysis.—

We define total welfare W^d as the sum of expected utilities at date 0 over all traders in the DEL model. Again, we focus on the constrained welfare maximization problem with respect to information investment, when the information structure of each trader is the same as that specified in Section 5.1 and there is no trade restriction or regulation on any traders. This is equivalent to investigating the case in which the regulator maximizes total welfare by choosing the level of information investment, without restricting any delegated portfolio management contracts or any actions of any traders. Thus, the marketable asset price $p(x; m, i, y)$, the trading strategies of the seller and the hedger (x^s, x^h), the trading strategies of the arbitrageurs $(x^a, \zeta, \zeta^u, \eta)$, and the optimal contract $m = (m_1, m_2, m_3, m_4, m_5, m_6)$ are determined relative to i by the equilibrium derived in Section 5.3.

Now, we compare the welfare-maximizing level of investment, i^{**}_w, and the welfare-maximizing likelihood of market freeze, $\widehat{y}^d(i^{**}_w, \zeta^{**}_w)$, with the equilibrium ones, i^{**} and $\widehat{y}^d(i^{**}, \zeta^{**})$, by numerical calculations, where ζ^{**}_w denotes the welfare-maximizing churning strategy of the arbitrageur. Figures 4–6 provide the numerical calculation results by varying $\overline{z}$, c, and $\overline{x}$.

Panels E and F of Figure 4 illustrate the effects of an increase in $\overline{z}$ on $i^{**} - i^{**}_w$ and $\widehat{y}^d(i^{**}, \zeta^{**}) - \widehat{y}^d(i^{**}_w, \zeta^{**}_w)$. In the range of $\overline{z}$, overinvestment always arises, and the extent increases with $\overline{z}$. The likelihood of the market freeze is smaller in equilibrium than in the welfare-maximizing case, and $\left|\widehat{y}^d(i^{**}, \zeta^{**}) - \widehat{y}^d(i^{**}_w, \zeta^{**}_w)\right|$ is increasing in $\overline{z}$.

Panels E and F of Figure 5 indicate the effects of an increase in $\overline{x}$ on $i^{**} - i^{**}_w$ and

$\widehat{y}^d(i^{**}, \zeta^{**}) - \widehat{y}^d(i_w^{**}, \zeta_w^{**})$. In this range of $\overline{x}$, overinvestment always occurs, but the extent is independent of $\overline{x}$. The likelihood of the market freeze is smaller in equilibrium than in the welfare-maximizing case, while $\left|\widehat{y}^d(i^{**}, \zeta^{**}) - \widehat{y}^d(i_w^{**}, \zeta_w^{**})\right|$ is independent of $\overline{x}$.

Panels E and F of Figure 6 report the effects of an increase in c on $i^{**} - i_w^{**}$ and $\widehat{y}^d(i^{**}, \zeta^{**}) - \widehat{y}^d(i_w^{**}, \zeta_w^{**})$. In this range of c, overinvestment always occurs, and the extent is increasing in c. The likelihood of the market freeze is smaller in equilibrium than in the welfare-maximizing case, whereas $\left|\widehat{y}^d(i^{**}, \zeta^{**}) - \widehat{y}^d(i_w^{**}, \zeta_w^{**})\right|$ is increasing in c.

The intuition is as follows. The results in our parametric case show $\zeta_w^{**} = 0$ (see the dotted lines in Panel A of Figures 4–6). This does not alter the basic tendency observed in the DIR model, because the price of the marketable asset in the welfare-maximizing case in the DEL model is the same as that in the DIR model when $\zeta = 0$ (see Lemmas 3 and 9). However, the adverse selection regarding the arbitrageur's type is newly added in the DEL model because the optimal contract needs to rule out incompetent arbitrageurs by requiring a sufficient amount of information investment from the employed arbitrageur. If such adverse selection is too severe, information investment is socially costly in the welfare-maximizing case. Then, the social need limits information investment to a minimum level (given by (23)) that can not only deter incompetent arbitrageurs from participating in the contract but also induce the talented arbitrageur to enter into the contract. Hence, overinvestment occurs and the market freeze is more likely to arise in the welfare-maximizing case than in equilibrium. In addition, the minimum level discussed above is decreasing in c and independent of $\overline{z}$ and $\overline{x}$ (see (23)). As i^{**} is increasing in $\overline{z}$, slightly decreasing in c, and is independent of $\overline{x}$, the extent of overinvestment is increasing in $\overline{z}$ and c and is independent of $\overline{x}$. Then, $\left|\widehat{y}^d(i^{**}, \zeta^{**}) - \widehat{y}^d(i_w^{**}, \zeta_w^{**})\right|$ has a similar movement because the likelihood of the market freeze is inversely related to i.

In contrast to the DIR model, the adverse selection regarding the arbitrageur's type in the DEL model causes information investment to be socially costly in this model. The discussion of the welfare effect in the DEL model is unique in the sense that overinvestment arises because the optimal contract needs to exclude incompetent arbitrageurs by increasing information investment. Hence, the mechanism of arising overinvestment in the DEL model is different from that of Glode, Green, and Lowery (2012) because an arms race in financial expertise among financial intermediaries causes overinvestment in their model.

5.5. Comparison between the results of the DIR and DEL models.—

Our comparative static results in Figures 1–6 suggest the following:

(1) The investment in information acquisition in equilibrium is greater in the DEL model than in the DIR model.

(2) The likelihood of the market freeze in equilibrium is greater in the DEL model than in the DIR model as long as $\overline{z}$ is not sufficiently large, even though the likelihood of the market freeze is inversely related to investment in information acquisition in each model.

(3) Underinvestment (overinvestment) occurs in the DIR model as long as $\overline{z}$ is sufficiently (not sufficiently) small, whereas overinvestment always arises in the DEL model in our parametric range.

(4) The extent of overinvestment is increasing in $\overline{z}$ in both the DIR and DEL models.[39] The extent of overinvestment is decreasing in (independent of) $\overline{x}$ in the DIR (DEL) model, while it is decreasing (increasing) in c in the DIR (DEL) model.

The main reason why there are differences between the results of the DIR and DEL models depends on the fact that the portfolio delegation contracts needs to exclude incompetent arbitrageurs while it may allow the competent arbitrageur to churn.

We begin by discussing result (1). To rule out the incompetent arbitrageur while controlling the churning strategy of the talented arbitrageur, the principal needs to increase information investment (see self-selection conditions (19c) and (20c)). In fact, the price is less informative under portfolio delegation if $\overline{z}$ is not sufficiently large. This is because the contract then motivates the arbitrageur to trade n_s^d or $-n_b^d$ at random, even though he has no informative signals (see the discussion below Lemma 9). Thus, for a fixed i, the high-quality asset seller's supply will decrease, whereas the hedger's demand will increase. Indeed, the

[39]Note that the extent of underinvestment is the negative value of the extent of overinvestment.

final hedging effect is dominated by the two other effects. As a result, the portfolio delegation contract promotes investment in information acquisition.

Alternatively, for a fixed i, the likelihood of the market freeze in equilibrium is greater in the DEL model than in the DIR model when the talented arbitrageur undertakes churning behavior. This is because the churning aggravates the adverse selection problem regarding the marketable asset and decreases the high-quality asset seller's supply. This tendency remains even though the equilibrium level of information investment is greater in the DEL model than in the DIR model. However, if $\overline{z}$ is sufficiently large, the optimal contract does not involve the churning. Then, for a fixed investment level, the threshold of the market freeze in equilibrium in the DEL model is the same as that in the DIR model. Hence, if $\overline{z}$ is sufficiently large, the likelihood of the market freeze in equilibrium is smaller in the DEL model than in the DIR model because the equilibrium level of information investment is greater in the DEL model than in the DIR model.

For the efficiency of information investment in the DIR model, if $\overline{z}$ is sufficiently small, an increase in the level of information investment beyond equilibrium one is likely to improve the seller's allocative efficiency more than the hedger's hedging benefit. As the social need for information investment becomes relatively high, underinvestment arises as long as $\overline{z}$ is sufficiently small. In the DEL model, the adverse selection problem regarding the arbitrageur's type is newly added. As the adverse selection problem is too severe in the welfare-maximizing case, the social need for information investment is reduced so that overinvestment always arises in our parametric range.

The difference between the effects of $\overline{x}$ and c on the extent of overinvestment in the DIR and DEL models depends on the differences between the effects of $\overline{x}$ and c on the investment levels in information acquisition in the welfare-maximizing cases in the DIR and DEL models. In particular, the severe adverse selection problem regarding the arbitrageur's type in the DEL model makes i_w^{**} independent of $\overline{x}$ and causes the negative effect of c on i_w^{**} to be larger. However, as such an adverse selection problem does not exist, i_w^* is increasing in $\overline{x}$ and slightly decreasing in c in the DIR model.

Finally, given the results listed in this subsection, the recent trend of hedge funds converting their operations into family offices is more likely to decrease the investment in information acquisition and to reduce the likelihood of the market freeze if liquidity in the marketable asset is not sufficiently large. Furthermore, underinvestment is more likely to occur if liquidity in the marketable asset is sufficiently small.

6. Policy implications

We suppose that the regulator can levy a tax per unit of investment or give a subsidy per unit of investment. This tax (subsidy) raises (reduces) c. Because an increase in $\overline{z}$ can be interpreted as an increase in liquidity in the marketable asset market, we establish the following proposition by combining the results of Propositions 3 and 4 with the numerical calculation results regarding the effect of c in Section 4.3.

Proposition 6: *Suppose that the arbitrageur trades on his own account. Then, if liquidity in the marketable asset market is sufficiently small relative to the seller's endowment of the marketable asset, the investment subsidy is preferred, and can mitigate the underinvestment problem while reducing the likelihood of the market freeze.*

When the arbitrageur trades on his own account, the investment subsidy can reduce the likelihood of the market freeze without suffering any losses in the efficiency of information investment if liquidity in the marketable asset market is sufficiently small relative to the seller's endowment of the marketable asset. However, if these conditions are not met, it is possible that the investment tax is preferred and mitigates the overinvestment problem.

By contrast, the numerical calculation results in Section 5.4 lead to:

Proposition 7: *Suppose that the arbitrageur is employed as a portfolio manager. Then, the investment tax mitigates the overinvestment problem but raises the likelihood of the market freeze.*

Proposition 7 implies that if the arbitrageur is employed as a portfolio manager, the investment tax improves the social welfare although it also increases the likelihood of the market freeze.

Generally, these two propositions suggest the following. If direct portfolio management prevails, subsidizing and promoting investment in information acquisition is better when liquidity in the marketable asset market is sufficiently small relative to the seller's endowment of the marketable asset. In contrast, if delegated portfolio management is dominant, some investment tax is needed to prevent overinvestment although it raises the possibility of a market freeze.

For an application of these two propositions, suppose that a substantial amount of a structured financial product, such as an asset-backed security, is originated while liquidity in the market of the structured financial product is sufficiently small. Then, these propositions suggest that the regulator can use a subsidy for information investment if direct portfolio management prevails, but a tax for information investment if delegated portfolio management is dominant. As a result, in these situations, a subsidy (tax) for (of) information investment can improve the efficiency of information investment and reduce (raise) the likelihood of the market freeze when direct (delegated) portfolio management is dominant.

For another application, suppose that new financial products are created and supplied when the cost of investment in information acquisition is relatively low as a result of technological innovation. If delegated portfolio management is dominant, the investment tax should be used to restore the efficiency of information investment but raises the likelihood of the market freeze.

Furthermore, the recent trend in hedge funds transforming their companies into family offices suggests that the investment subsidy may be used to improve the efficiency of information investment and to reduce the likelihood of the market freeze.

De La Cruz, Medina, and Tang (2019) report that in many newly developed countries such as Chile, Mexico, Philippines, and Turkey, the sum of the holding shares of private corporations and strategic individuals is much larger than that of institutional investors. In particular, in these four countries, the sum of the holding shares of foreign investors is not large. Because asset markets other than the equity market are not well developed and liquidity in the equity market is sufficiently small, this finding roughly suggests that subsidizing and promoting investment in information is recommended in these countries.

Finally, our results suggest that in a market where professional investors such as investment banks, hedge funds, family offices, and FinTech firms manage only a small part of funds on their own account, the investment tax may be recommended even though it may increase the likelihood of the market freeze. Otherwise, the investment subsidy may be provided if the liquidity of the market is sufficiently small. However, the latter statement can be modified if political factors such as human rights and freedom or ethical factors such as privacy are important. Indeed, many FinTech firms have recently been active in various financial services. Big Tech firms may also have a financial subsidiary (e.g., Ant Financial for the Alibaba group) or have plans for entering into various financial services. However, there is ongoing regulatory discussion about these activities, particularly those of the Big Tech firms. In addition, the US government has strongly opposed the subsidies granted to IT industries by the Chinese government. Because the information acquisition investment in our model can also be interpreted as FinTech investment in these firms or industries, we can shed some light on these discussions. For example, if Big Tech firms are becoming more active and provide delegated portfolio management services in developed countries, the regulator may apply the investment tax on the investment of these firms to prevent overinvestment, although this policy may also increase the likelihood of the market freeze.

7. Concluding Remarks

This paper explores possible policies to achieve the social optimal level of investment in information acquisition by discussing the information acquisition investment strategy of arbitrageurs and the likelihood of a market freeze in financial market equilibrium and analyzing the welfare consequences.

We derive the following theoretical result. In the direct portfolio management model, underinvestment arises when liquidity in the marketable asset is sufficiently small relative to the seller's endowment of the marketable asset. A market freeze is then more likely to occur in equilibrium than in the welfare-maximizing case under the above condition. In the delegated portfolio management model, overinvestment occurs in our parametric case and the likelihood of the market freeze is then smaller in equilibrium than in the welfare-maximizing case. Indeed, the mechanisms causing underinvestment in the direct portfolio management model and overinvestment in the delegated portfolio management model are different from those in the standard view.

Furthermore, portfolio delegation increases information investment, while it also increases the likelihood of the market freeze as long as liquidity in the marketable asset market is not sufficiently large. The effects generated by the introduction of portfolio delegation result from the fact that the portfolio delegation contract needs to exclude incompetent arbitrageurs whereas it can allow the competent arbitrageur to use the random trading strategy when he has no informative signal. The result of the effect of portfolio delegation provides some implications for the recent trend in hedge funds converting their operations into family offices.

Given these theoretical results, we show that the effects of different policy measures depend on whether direct or delegated portfolio management is dominant. More specifically, if direct portfolio management prevails, subsidizing and promoting investment in information acquisition is better when liquidity in the marketable asset market is sufficiently small relative to the seller's endowment of the marketable asset. In contrast, if delegated portfolio management is dominant, some investment tax is needed to prevent overinvestment, although this does raise the possibility of a market freeze.

Finally, our theoretical result also indicates that in the delegated portfolio management model, the optimal contract does not necessarily involve the talented arbitrageur's random trading strategy when he has no informative signal, although Dow and Gorton (1997) argue that the optimal contract always induces an uninformed arbitrageur to trade randomly. However, if neither liquidity in the marketable asset market relative to the upper bound of the return of the nonmarketable asset nor the cost of information acquisition investment is large, we qualitatively show that churning is optimal. Intuitively, if the market freeze does not occur, the churning enables hedgers to effectively insure their income risk at lower cost because it causes the price of the marketable asset to be less informative. As hedgers will respond by trading more, the principal can earn higher trading profits. However, making the price of the marketable asset less informative discourages the seller from selling the high-quality asset. Hence, the churning increases the likelihood that the market freeze arises. As a result, the profit opportunities for the principal are likely to be restricted. If this cost is sufficiently large, the principal prevents the churning by raising the arbitrageur's compensation scheme when he does not trade despite the absence of the market freeze. This scheme is feasible in the present model because an increase in information investment in conjunction with the arbitrageur's compensation can also be used to deter incompetent arbitrageurs from entering into the contact because of their increasing investment costs. However, if neither liquidity in the marketable asset market relative to the upper bound of the return of the nonmarketable asset nor the investment cost is large, the churning is optimal because its benefit of higher trading profits dominates its cost of losing out on profit opportunities.

Appendix

Proof of Lemma 1: The result is evident from the seller's problem (1). ■

Proof of Lemmas 2 and 3: The market maker can observe the five possible total order flows given in the text above Lemma 2. We assume that $v = 1$ or $v = 0$ with equal probability, and that the talented arbitrageur receives a perfectly informative signal $\theta \in \{1, 0\}$ with probability $\alpha(i)$. Because of $v = 1$ for $\theta = 1$ or $v = 0$ for $\theta = 0$, the talented arbitrageur receives $\theta = 1$ or 0 (that is, he chooses $x^a = n_s$ or $-n_b$) with equal probability $\frac{1}{2}\alpha(i)$, but he receives only $\theta = \phi$ (that is, he chooses $x^a = 0$) with probability $1 - \alpha(i)$. Furthermore, because we assume that the hedger's income shock is positively or negatively correlated with v with equal probability, the hedger will trade $x^h = n_b \geq 0$ or $x^h = -n_s \leq 0$ with equal probability. Given these arguments, the statement of Lemma 2 is obtained. In addition, $E_1(v \mid x, y, i) = \frac{1}{2}$ if the market maker cannot infer v from x, whereas $E_1(v \mid x, y, i) = 1$ ($E_1(v \mid x, y, i) = 0$) if the market maker can infer $v = 1$ ($v = 0$) from x. Hence, it follows from $p^* = p(x; y, i) \equiv E_1(v \mid x, y, i)$ that the result of Lemma 3 is obtained. ■

Proof of Lemma 4: Lemma 1 shows that the high-quality asset seller does not supply the marketable asset if $E_1[p(x; y, i) \mid 1, y, i] < \frac{1}{1+y}$. As the market maker can anticipate this, she expects that the quality of the asset is low. Thus, the market maker sets $p^* = 0$. ■

Proof of Lemma 5: Repeating the argument of the proof of Lemmas 2 and 3, we can show that the probability for each event of the total order flow conditional on $v = 1$ at date 1 inferred by the high-quality asset seller is given by: (i) $\frac{\alpha(i)}{2}$ if $x = n_s + n_b - \overline{x}$; (ii) $\frac{1-\alpha(i)}{2}$ if $x = n_b - \overline{x}$; (iii) $\frac{\alpha(i)}{2}$ if $x = -\overline{x}$; (iv) $\frac{1-\alpha(i)}{2}$ if $x = -n_s - \overline{x}$; and (v) 0 if $x = -n_b - n_s - \overline{x}$.[40] Thus, it follows from Lemma 3 that $E_1[p(x; i, y) \mid 1, i, y] = \frac{1}{2} + \frac{\alpha(i)}{4}$. Given Lemma 1, the high-quality asset seller does not prefer to supply the marketable asset if $\frac{1}{2} + \frac{\alpha(i)}{4} < \frac{1}{1+y}$, that is, if $y < \widehat{y}(i)$. Then, it follows from Lemma 4 that neither the arbitrageur nor the hedger

[40] Note that the conditional probability of each event of the total order flow for the high-quality asset seller is the same as that for the low-quality asset seller if $x = n_b - \overline{x}$, $x = -\overline{x}$, or $x = -n_s - \overline{x}$. Accordingly, for these sizes of total order flows, the conditional probability of each event of the total order flow for the high-quality asset seller is the same as the probability of each event of the total order flow for the market maker.

trades the marketable asset because of the reasons discussed in footnote 25. Consequently, the result of this lemma is verified. ■

Proof of Lemma 6: We first investigate the case of a hedger whose income shock z^h is *negatively* correlated with v (that is, $z^h = -\overline{z}$ for $v = 1$ and $z^h = \overline{z}$ for $v = 0$). Hence, the hedger always needs to buy the marketable asset ($x^h = n_b \geq 0$).

Suppose that $y \geq \widehat{y}(i)$ and the high-quality asset is supplied. Then, from the viewpoint of the above hedger with $x^h = n_b$, there are three possible cases: (i) $x = n_s + n_b - \overline{x}$. From Lemma 2, this case occurs with probability $\frac{\alpha(i)}{2}$ for the hedger conditional on the event that z^h is *negatively* correlated with v. It also follows from Lemma 3 that $p^*(x) = 1$. Because this case happens when $v = \theta = 1$, $x^a = n_s$, and $z^h = -\overline{z}$, the income of the hedger at date 2 is then $-p^*(x)n_b + vn_b - \overline{z} = -\overline{z}$. (ii) $x = n_b - \overline{x}$. It follows from Lemmas 2 and 3 that the conditional probability of this case for the hedger is $1 - \alpha(i)$ and that $p^*(x) = \frac{1}{2}$. As this case occurs when the arbitrageur is uninformative (that is, $\theta = \phi$ and $x^a = 0$), either $v = 1$ and $z^h = -\overline{z}$ or $v = 0$ and $z^h = \overline{z}$ is realized with equal probability. Thus, the income of the hedger at date 2 becomes $-\frac{1}{2}n_b + n_b - \overline{z} = \frac{1}{2}n_b - \overline{z}$ or $-\frac{1}{2}n_b + \overline{z}$ with equal probability. (iii) $x = -\overline{x}$. Because the hedger needs to buy $x^h = n_b$, this case corresponds to only the case of $v = \theta = 0$, $x^a = -n_b$, and $z^h = \overline{z}$. It then follows from Lemmas 2 and 3 that the conditional probability of this case for the hedger is $\frac{\alpha(i)}{2}$ and that $p^*(x) = \frac{1}{2}$. The income of the hedger at date 2 is, thus, $-\frac{1}{2}n_b + \overline{z}$. The optimization problem for this hedger is then

$$\max_{n_b \geq 0} \left\{ \frac{\alpha(i)}{2} U(-\overline{z}) + \frac{1-\alpha(i)}{2} \left[U\left(\frac{1}{2}n_b - \overline{z}\right) + U\left(-\frac{1}{2}n_b + \overline{z}\right) \right] + \frac{\alpha(i)}{2} U\left(-\frac{1}{2}n_b + \overline{z}\right) \right\}. \tag{A1}$$

It follows from $U(w) = aw - \frac{1}{2}bw^2$ and the first-order condition to (A1) that $n_b(i) = \max\left(2\left[\overline{z} - \frac{a}{b}\frac{\alpha(i)}{2-\alpha(i)}\right], 0\right)$.

Conversely, if $y < \widehat{y}(i)$, we show that $n_b(i) = 0$, as indicated in Lemma 5.

Next, we examine the case of a hedger with z^h that is *positively* correlated with v. Then, the hedger needs to sell the marketable asset ($x^h = -n_s \leq 0$). Repeating a similar argument, we can derive $n_s(i) = \max\left(2\left[\overline{z} - \frac{a}{b}\frac{\alpha(i)}{2-\alpha(i)}\right], 0\right)$ if $y \geq \widehat{y}(i)$; and $n_s(i) = 0$ if $y < \widehat{y}(i)$.

Summarizing these arguments, we verify the result of this lemma. ■

Proof of Proposition 1: Given (5), totally differentiating both hands sides of (7) with respect to i, $\overline{z}$, $\overline{x}$, and c yields

$$\Gamma di = -\frac{1}{2}\left\{\alpha'(i)\left[1 - F(\widehat{y}(i))\right] + \frac{4\alpha(i)\alpha'(i)f(\widehat{y}(i))}{\left[2+\alpha(i)\right]^2}\right\} d\overline{z} + 0d\overline{x} + dc, \tag{A2}$$

where Γ is the derivative of the left-hand side of (7) with respect to i. Because $\Gamma < 0$ from the second-order condition, i^* is increasing in $\overline{z}$, independent of $\overline{x}$, and is decreasing in c. ■

Proof of Proposition 2: Given Lemma 5, we need only examine the effects of $\overline{z}$, $\overline{x}$, and c on $\widehat{y}(i^*) \equiv \frac{2-\alpha(i^*)}{2+\alpha(i^*)}$. Indeed, we show $\frac{\partial \widehat{y}(i^*)}{\partial j} = -\frac{4\alpha'(i^*)}{[2+\alpha(i^*)]^2}\frac{di^*}{dj}$, for $j = \overline{z}, \overline{x}, c$. Then, the statement of this proposition is evident from the result of Proposition 1. ■

The derivation procedure of (9): Using $p(x; y, i) \equiv E_1(v \mid x, y, i)$, total welfare (8) at date 0 can then be rewritten as:

$$\begin{aligned}
W = {} & \frac{\alpha(i)}{4}\overline{n}(i)\left[1 - F(\widehat{y}(i))\right] - ci - ei \\
& + \frac{1}{2}\left\{\int_{\widehat{y}(i)}^{\overline{y}} (1+y)\left[\frac{1}{2}\left(1 + \frac{\alpha(i)}{2}\right)\overline{x} - \ell\right] dy + \int_0^{\widehat{y}(i)} \left[\overline{x} - (1+y)\ell\right] dy\right\} \\
& + \frac{1}{2}\left\{\int_{\widehat{y}(i)}^{\overline{y}} (1+y)\left[\frac{1}{2}\left(1 - \frac{\alpha(i)}{2}\right)\overline{x} - \ell\right] dy - \int_0^{\widehat{y}(i)} (1+y)\ell dy\right\} \\
& + \left\{\frac{1}{2}\left[\frac{\alpha(i)}{2}U(-\overline{z}) + \frac{1-\alpha(i)}{2}\left(U\left(\frac{\overline{n}(i)}{2} - \overline{z}\right) + U\left(-\frac{\overline{n}(i)}{2} + \overline{z}\right)\right) + \frac{\alpha(i)}{2}U\left(-\frac{\overline{n}(i)}{2} + \overline{z}\right)\right]\right. \\
& \left. + \frac{1}{2}\left[\frac{\alpha(i)}{2}U\left(-\frac{\overline{n}(i)}{2} + \overline{z}\right) + \frac{1-\alpha(i)}{2}\left(U\left(-\frac{\overline{n}(i)}{2} + \overline{z}\right) + U\left(\frac{\overline{n}(i)}{2} - \overline{z}\right)\right) + \frac{\alpha(i)}{2}U(-\overline{z})\right]\right\} \\
& \times \left[1 - F(\widehat{y}(i))\right] + \left[\frac{1}{2}U(-\overline{z}) + \frac{1}{2}U(\overline{z})\right] F(\widehat{y}(i)).
\end{aligned} \tag{A3}$$

The derivation procedure of (A3) is as follows. Repeating the argument of the proof of Lemma 5, we have $E_1[p(x; i, y) \mid 0, i, y] = \frac{1}{2} - \frac{\alpha(i)}{4}$. Using (1) and Lemma 1 with $E_1[p(x; i, y) \mid 1, i, y] = \frac{1}{2} + \frac{\alpha(i)}{4}$ and $E_1[p(x; i, y) \mid 0, i, y] = \frac{1}{2} - \frac{\alpha(i)}{4}$, the expected payoff of the seller is represented as the second line for $v = 1$, and the third line for $v = 0$ on the right-hand side of (A3). Substituting (4) and (5) into (A1) with $\overline{n}(i) > 0$, we show that the expected utility of the hedger whose income shock is negatively (positively) correlated with v is expressed as the fourth (fifth) line on the right-hand side of (A3) multiplied by $[1 - F(\widehat{y}(i))]$ if the market freeze does not occur. If the market freeze arises, the expected utility of the hedger is equal to $\left[\frac{1}{2}U(-\overline{z}) + \frac{1}{2}U(\overline{z})\right] F(\widehat{y}(i))$ in the sixth line because the hedger is unable to insure against risk. Using (6), the expected payoff of the talented arbitrageur is represented as the first on the right-hand side of (A3). Total welfare (8) at date 0 is, therefore, rewritten as (A3).

Now, using the envelope theorem, maximizing the right-hand side of (A3) with respect to i yields (9). ■

Proof of Propositions 3 and 4: Taking the investment level as given, we see that if $\overline{z}$ is sufficiently small so that $\overline{n}(i)$ is sufficiently close to zero, the value of the sum of the terms in the second line on the left-hand side of (9) becomes positive. On the other hand, an increase in $\overline{x}$ increases the value of the first term in the second line on the left-hand side of (9) but does not affect the values of any other terms on the left-hand side of (9). Note that the welfare-maximizing investment level i_w^* is determined by (9), whereas the equilibrium one i^* is determined by (7) of which the left-hand side is the same as the first line on the left-hand side of (9). These findings imply that $W'(i^*) > 0$ occurs when $\overline{z}$ is sufficiently small and/or when $\overline{x}$ is sufficiently large. If we assume $W''(i) < 0$ for a range including i_w^* and i^*, the statement of Proposition 3 is immediate because of $W'(i_w^*) = 0$. Repeating the proof procedure of Proposition 2, we can also verify the statement of Proposition 4. ■

Proof of Lemma 7: The result is evident from the seller's problem (10). ■

Proof of Lemmas 8 and 9: Given the arbitrageur's trading strategy, the market maker can observe five possible total order flows: (i) $x = n_s^d + n_b^d - \overline{x}$ if $\theta = 1$, $x^a = n_s^d$, and $x^h = n_b^d$ or if $\theta = \phi$, $x^a = n_s^d$, and $x^h = n_b^d$; (ii) $x = n_b^d - \overline{x}$ if $\theta = \phi$, $x^a = 0$, and $x^h = n_b^d$; (iii) $x = -\overline{x}$ if $\theta = 1$, $x^a = n_s^d$, and $x^h = -n_s^d$, if $\theta = \phi$, $x^a = n_s^d$, and $x^h = -n_s^d$, if $\theta = 0$, $x^a = -n_b^d$, and $x^h = n_b^d$, or if $\theta = \phi$, $x^a = -n_b^d$, and $x^h = n_b^d$; (iv) $x = -n_s^d - \overline{x}$ if $\theta = \phi$, $x^a = 0$, and $x^h = -n_s^d$; and (v) $x = -n_b^d - n_s^d - \overline{x}$ if $\theta = 0$, $x^a = -n_b^d$, and $x^h = -n_s^d$ or if $\theta = \phi$, $x^a = -n_b^d$, and $x^h = -n_s^d$.

Under the contract, if the talented arbitrageur receives an uninformative signal $\theta = \phi$ with probability $1 - \alpha(i)$, he trades n_s^d with probability $\frac{\zeta}{2}$, $-n_b^d$ with probability $\frac{\zeta}{2}$, and 0 with probability $1 - \zeta$. Then, repeating the proof procedure presented in Lemma 2, the statement of Lemma 8 is obtained. In addition, if the market maker cannot infer the arbitrageur's position, $E_1(v \mid x, y, i, m) = \frac{1}{2}$. Thus, it follows from $p^{**} = p(x; y, i, m) \equiv E_1(v \mid x, y, i, m)$ that $p(n_s^d + n_b^d - \overline{x}; y, i, m) = \frac{1}{2}\psi_1$, $p(n_b^d - \overline{x}; y, i, m) = p(-\overline{x}; y, i, m) = p(-n_s^d - \overline{x}; y, i, m)$ $= \frac{1}{2}$, $p(-n_b^d - n_s^d - \overline{x}; y, i, m) = \frac{1}{2}\psi_2$. Hence, the result of Lemma 9(i) is obtained. Finally, repeating the proof procedure of Lemma 4, the statement of Lemma 9(ii) is evident. ■

Proof of Lemma 10: Repeating the argument of the proof of Lemmas 8 and 9, we can show that the probability for each event of the total order flow conditional on $v = 1$ at date 1 inferred by the high-quality asset seller is given by: (i) $\frac{1}{2}\{\alpha(i) + \frac{1}{2}[1 - \alpha(i)]\zeta\}$ if $x = n_s^d + n_b^d - \overline{x}$; (ii) $\frac{1}{2}[1 - \alpha(i)](1 - \zeta)$ if $x = n_b^d - \overline{x}$; (iii) $\frac{1}{2}\{\alpha(i) + [1 - \alpha(i)]\zeta\}$ if $x = -\overline{x}$; (iv) $\frac{1}{2}[1 - \alpha(i)](1 - \zeta)$ if $x = -n_s^d - \overline{x}$; and (v) $\frac{1}{4}[1 - \alpha(i)]\zeta$ if $x = -n_b^d - n_s^d - \overline{x}$.[41] Thus, it follows from Lemma 9(i) that $E_1[p(x; i, y) \mid 1, i, y] = \frac{1}{4}[\alpha(i) + \frac{1}{2}(1 - \alpha(i))\zeta]\psi_1 + \frac{1}{2}[1 - \alpha(i)](1 - \zeta)$ $+ \frac{1}{4}[\alpha(i) + (1 - \alpha(i))\zeta] + \frac{1}{8}[1 - \alpha(i)]\zeta\psi_2$ if the market freeze does not occur. Then, it is immediately from Lemma 7 that the result is derived. ■

Proof of Lemma 11: We begin with examining the case of a hedger whose income shock z^h is *negatively* correlated with v (that is, $z^h = -\overline{z}$ for $v = 1$ and $z^h = \overline{z}$ for $v = 0$). Then, the hedger always needs to buy the marketable asset, $x^h = n_b^d \geq 0$. Suppose that $y \geq \widehat{y}^d(i, \zeta)$. From the viewpoint of the above hedger, there are three possible cases:[42] (i) $x = n_s^d + n_b^d - \overline{x}$. For the above hedger, given that either $v = 1$ or $v = 0$ is realized with equal probability and that $x^a = n_s^d$ is chosen with probability 1 ($\frac{1}{2}\zeta$) if $\theta = 1$ ($\theta = \phi$), this case happens when $v = 1$, $\theta = 1$ or ϕ, $x^a = n_s^d$, and $z^h = -\overline{z}$ with probability $\frac{1}{2}\alpha(i) + \frac{1}{4}[1 - \alpha(i)]\zeta$; or when $v = 0$, $\theta = \phi$, $x^a = n_s^d$, and $z^h = \overline{z}$ with probability $\frac{1}{4}[1 - \alpha(i)]\zeta$. As Lemma 9 shows that $p^{**}(x) = \frac{1}{2}\psi_1$ in this case, the income of the hedger at date 2 is $(-\frac{1}{2}\psi_1 + 1)n_b^d - \overline{z}$ when v $= 1$, $\theta = 1$ or ϕ, $x^a = n_s^d$, and $z^h = -\overline{z}$; or is $-\frac{1}{2}\psi_1 n_b^d + \overline{z}$ when $v = 0$, $\theta = \phi$, $x^a = n_s^d$, and $z^h = \overline{z}$. (ii) $x = n_b^d - \overline{x}$. Because $x^a = 0$ is chosen with probability $1 - \zeta$ if $\theta = \phi$, this

[41] See the statement of footnote 40.

[42] Because of $x^h = n_b^d$, we need not consider any cases that involve $x^h = -n_s^d$.

case occurs when $v = 1$, $\theta = \phi$, $x^a = 0$, and $z^h = -\overline{z}$ with probability $\frac{1}{2}[1 - \alpha(i)](1 - \zeta)$ or when $v = 0$, $\theta = \phi$, $x^a = 0$, and $z^h = \overline{z}$ with probability $\frac{1}{2}[1 - \alpha(i)](1 - \zeta)$. As Lemma 9 implies that $p^{**} = \frac{1}{2}$ in this case, the income of the hedger at date 2 is $\frac{1}{2}n_b^d - \overline{z}$ or $-\frac{1}{2}n_b^d + \overline{z}$, respectively. (iii) $x = -\overline{x}$. This case occurs when $v = 1$, $\theta = \phi$, $x^a = -n_b^d$, and $z^h = -\overline{z}$ with probability $\frac{1}{4}[1 - \alpha(i)]\zeta$, or when $v = 0$, $\theta = 0$ or ϕ, $x^a = -n_b^d$, and $z^h = \overline{z}$ with probability $\frac{1}{2}\alpha(i) + \frac{1}{4}[1 - \alpha(i)]\zeta$.[43] As Lemma 9 again indicates that $p^{**} = \frac{1}{2}$, the income of the hedger at date 2 is $\frac{1}{2}n_b^d - \overline{z}$ or $-\frac{1}{2}n_b^d + \overline{z}$, respectively. The optimization problem for the above hedger is then represented by

$$\begin{aligned}\max_{n_b^d \geq 0} \Big\{ &\left[\tfrac{1}{2}\alpha(i) + \tfrac{1}{4}(1 - \alpha(i))\zeta\right] U\left((-\tfrac{1}{2}\psi_1 + 1)n_b^d - \overline{z}\right) + \tfrac{1}{4}[1 - \alpha(i)]\zeta U\left(-\tfrac{1}{2}\psi_1 n_b^d + \overline{z}\right) \\ &+ \tfrac{1}{2}[1 - \alpha(i)](1 - \zeta)\left[U(\tfrac{1}{2}n_b^d - \overline{z}) + U(-\tfrac{1}{2}n_b^d + \overline{z})\right] \\ &+ \tfrac{1}{4}[1 - \alpha(i)]\zeta U\left(\tfrac{1}{2}n_b^d - \overline{z}\right) + \left[\tfrac{1}{2}\alpha(i) + \tfrac{1}{4}(1 - \alpha(i))\zeta\right] U\left(-\tfrac{1}{2}n_b^d + \overline{z}\right)\Big\}.\end{aligned} \tag{A4}$$

Similarly, repeating the above argument indicates that if $y \geq \widehat{y}(i, \zeta)$, the optimization problem for the hedger whose z^h is positively correlated with v is expressed by

$$\begin{aligned}\max_{n_s^d \geq 0} \Big\{ &\left[\tfrac{1}{2}\alpha(i) + \tfrac{1}{4}(1 - \alpha(i))\zeta\right] U\left(\tfrac{1}{2}\psi_2 n_s^d - \overline{z}\right) + \tfrac{1}{4}[1 - \alpha(i)]\zeta U\left((\tfrac{1}{2}\psi_2 - 1)n_s^d + \overline{z}\right) \\ &+ \tfrac{1}{2}[1 - \alpha(i)](1 - \zeta)\left[U(\tfrac{1}{2}n_s^d - \overline{z}) + U(-\tfrac{1}{2}n_s^d + \overline{z})\right] \\ &+ \tfrac{1}{4}[1 - \alpha(i)]\zeta U\left(\tfrac{1}{2}n_s^d - \overline{z}\right) + \left[\tfrac{1}{2}\alpha(i) + \tfrac{1}{4}(1 - \alpha(i))\zeta\right] U\left(-\tfrac{1}{2}n_s^d + \overline{z}\right)\Big\}.\end{aligned} \tag{A5}$$

If $y < \widehat{y}(i, \zeta)$, the high-quality asset seller does not prefer to supply the marketable asset. Because the hedger cannot hedge the income shock by buying or selling the marketable asset, we must have $n_b^d = n_s^d = 0$.

We are now in a position to prove $n_b^d(i, \zeta) = n_s^d(i, \zeta)$ (≥ 0). If $y < \widehat{y}(i, \zeta)$, as argued above, $n_b^d(i, \zeta) = n_s^d(i, \zeta) = 0$ is trivial. If $y \geq \widehat{y}(i, \zeta)$, solving problems (A4) and (A5) yields the

[43] Note that this case does not arise if $v = 1$ and $\theta = 1$. This is because the hedger whose z^h is negatively correlated with v needs to buy $x^h = n_b^d$, while the arbitrageur must also buy x^a to earn positive returns if $v = 1$ and $\theta = 1$.

following first-order conditions:

$$\frac{1}{2}\left[\alpha(i)+\frac{1}{2}(1-\alpha(i))\zeta\right]\left(-\frac{1}{2}\psi_1+1\right)U'\left((-\frac{1}{2}\psi_1+1)n_b^d-\overline{z}\right)-\frac{1}{4}[1-\alpha(i)]\zeta\frac{1}{2}\psi_1U'\left(-\frac{1}{2}\psi_1n_b^d+\overline{z}\right)$$
$$+\frac{1}{4}[1-\alpha(i)](1-\zeta)\left[U'\left(\frac{1}{2}n_b^d-\overline{z}\right)-U'\left(-\frac{1}{2}n_b^d+\overline{z}\right)\right]-\frac{1}{4}\alpha(i)U'\left(-\frac{1}{2}n_b^d+\overline{z}\right)$$
$$+\frac{1}{8}[1-\alpha(i)]\zeta\left[U'\left(\frac{1}{2}n_b^d-\overline{z}\right)-U'\left(-\frac{1}{2}n_b^d+\overline{z}\right)\right]\leq 0, \quad \text{(A6)}$$

$$\frac{1}{2}\left[\alpha(i)+\frac{1}{2}(1-\alpha(i))\zeta\right]\frac{1}{2}\psi_2U'\left(\frac{1}{2}\psi_2n_s^d-\overline{z}\right)-\frac{1}{4}[1-\alpha(i)]\zeta(-\frac{1}{2}\psi_2+1)U\left((\frac{1}{2}\psi_2-1)n_s^d+\overline{z}\right)$$
$$+\frac{1}{4}[1-\alpha(i)](1-\zeta)\left[U'\left(\frac{1}{2}n_s^d-\overline{z}\right)-U'\left(-\frac{1}{2}n_s^d+\overline{z}\right)\right]-\frac{1}{4}\alpha(i)U'\left(-\frac{1}{2}n_s^d+\overline{z}\right)$$
$$+\frac{1}{8}[1-\alpha(i)]\zeta\left[U'\left(\frac{1}{2}n_s^d-\overline{z}\right)-U'\left(-\frac{1}{2}n_s^d+\overline{z}\right)\right]\leq 0, \quad \text{(A7)}$$

where the inequality of (A6) ((A7)) is binding if $n_b^d>0$ ($n_s^d>0$). The definitions of ψ_1 and ψ_2 in Lemma 9 imply that $1-\frac{1}{2}\psi_1=\frac{1}{2}\psi_2$ and $\frac{1}{2}\psi_1=1-\frac{1}{2}\psi_2$. Hence, it follows from (A6) and (A7) that $n_b^d(i,\zeta)=n_s^d(i,\zeta)$ (≥ 0).

Next, given $\widehat{y}^d(i,\zeta)$ indicated in Lemma 10, we have $\widehat{y}^d(i,1)\equiv\frac{2-(\alpha(i))^2}{2+(\alpha(i))^2}$ and $\widehat{y}^d(i,0)\equiv\frac{2-\alpha(i)}{2+\alpha(i)}$. Using $U'(w)=a-bw$, it follows from (A6) and (A7) that $n_b^d(i,1)$ and $n_s^d(i,1)$ ($n_b^d(i,0)$ and $n_s^d(i,0)$) are obtained in the forms for $\zeta=1$ ($\zeta=0$) provided in this lemma. ■

Representation of the expected payoffs of the talented and incompetent arbitrageurs at date 0 **and the expected payoff of the principal with the talented arbitrageur at date** 0**:** We begin with examining the expected payoff of the talented arbitrageur at date 0. Suppose that $y\geq\widehat{y}^d(i,\zeta)$. Note that the contract compensation is given in Section 5.1, that $v=0$ or 1 occurs with equal probability, and that the uninformed talented arbitrageur buys n_s^d or sells n_b^d at random with probability ζ.[44] Then, the talented arbitrageur faces five possible cases: (i) $v=1$ and $x^a=n_s^d$. This case happens when he receives $\theta=1$ with probability $\frac{1}{2}\alpha(i)$, or when he receives $\theta=\phi$ and buys n_s^d in the event

[44] For brevity, we drop the dependence of (i,ζ) from $n_b^d(i,\zeta)$ and $n_s^d(i,\zeta)$ in the subsequent analysis.

of $v=1$ with probability $\frac{1}{4}[1-\alpha(i)]\zeta$. His payoff is m_1 in both events. (ii) $v=1$ and $x^a=-n_b^d$. This case arises only when he receives $\theta=\phi$ and sells n_b^d in the event of $v=1$ with probability $\frac{1}{4}[1-\alpha(i)]\zeta$. His payoff is m_2. (iii) $v=0$ and $x^a=n_s^d$. This case arises only when he receives $\theta=\phi$ and buys n_s^d in the event of $v=0$ with probability $\frac{1}{4}[1-\alpha(i)]\zeta$. His payoff is m_3. (iv) $v=0$ and $x^a=-n_b^d$. This case occurs when he receives $\theta=0$ with probability $\frac{1}{2}\alpha(i)$, or when he receives $\theta=\phi$ and sells n_b^d in the event of $v=0$ with probability $\frac{1}{4}[1-\alpha(i)]\zeta$. His payoff is m_4 in both events. (v) $x^a=0$. This case occurs when he receives $\theta=\phi$ and does not trade with probability $[1-\alpha(i)](1-\zeta)$. His payoff is m_5. Next, suppose that $y<\widehat{y}^d(i,\zeta)$. Then, as the market freeze arises (see Lemma 10), his payoff is m_6. For a fixed z, the expected payoff of the talented arbitrageur at date 0 is thus represented by

$$\Pi^{ta}(i,\zeta)=\int_{\widehat{y}^d(i,\zeta)}^{\overline{y}}\left\{\left[\frac{\alpha(i)}{2}+\frac{(1-\alpha(i))\zeta}{4}\right](m_1+m_4)\right.$$
$$\left.+\frac{[1-\alpha(i)]\,\zeta}{4}(m_2+m_3)+[1-\alpha(i)](1-\zeta)m_5\right\}dy+\int_0^{\widehat{y}^d(i,\zeta)}m_6dy-(c+e)i. \qquad \text{(A8)}$$

Note that the arbitrageur incurs the investment and effort costs.[45]

We next deal with the expected payoff of the incompetent arbitrageur at date 0. As he cannot receive any informative signals, he only has to buy n_s^d or sell n_b^d at random with probability η. Hence, if $y\geq\widehat{y}^d(i,\zeta)$, he has five possible cases: (i) $v=1$ and $x^a=n_s^d$. This case happens when he buys n_s^d in the event of $v=1$ with probability $\frac{1}{4}\eta$. His payoff is m_1. (ii) $v=1$ and $x^a=-n_b^d$. This case arises when he sells n_b^d in the event of $v=1$ with probability $\frac{1}{4}\eta$. His payoff is m_2. (iii) $v=0$ and $x^a=n_s^d$. This case arises when he buys n_s^d in the event of $v=0$ with probability $\frac{1}{4}\eta$. His payoff is m_3. (iv) $v=0$ and $x^a=-n_b^d$. This case occurs when he sells n_b^d in the event of $v=0$ with probability $\frac{1}{4}\eta$. His payoff is m_4. (v) $x^a=0$. This case occurs when he does not trade with probability $1-\eta$. If $y<\widehat{y}^d(i,\zeta)$, his payoff is m_6. As argued in the text, he does not commit to the choice of η before the market opens. Hence, he would also optimally choose η at date 1 after the market opens. Thus, his

[45] In reality, this assumption is reasonable because investment banks, hedge funds, and asset management firms need to invest ahead in FinTech innovation such as the use of AI and big data and in the acquisition of expertise.

expected payoff at date 0 is expressed by

$$\Pi^{in}(i,\zeta) = \int_{\widehat{y}^d(i,\zeta)}^{\overline{y}} \left\{ \max_{0\leq\eta\leq 1} \left[\frac{\eta}{4}(m_1 + m_2 + m_3 + m_4) + (1-\eta)m_5 \right] \right\} dy + \int_0^{\widehat{y}^d(i,\zeta)} m_6 dy - ci. \tag{A9}$$

Note that the principal can identify any incompetent arbitrageur if the incompetent arbitrageur does not make the same level of investment in information acquisition as the talented arbitrageur. Hence, any incompetent arbitrageur must incur the same investment cost as the talented arbitrageur if he desires to participate in the contract relation. However, as the information acquisition effort is not observable, no incompetent manager exerts any information acquisition efforts because he cannot have any informative signals.

To specify the expected payoff at date 0 of the principal who employs the talented arbitrageur, we need to use Lemmas 9 and 10 with the argument of contract compensation used to derive (A8). Suppose that $y \geq \widehat{y}^d(i,\zeta)$. Note that either $v = 1$ or $v = 0$ is realized with equal probability, that the arbitrageur receives an informative signal, $\theta = 1$ or 0, with equal probability $\frac{1}{2}\alpha(i)$, that $x^a = n_s^d$ is chosen for $\theta = 1$ ($\theta = \phi$) with probability 1 ($\frac{1}{2}\zeta$) while $x^a = -n_b^d$ is chosen for $\theta = 0$ ($\theta = \phi$) with probability 1 ($\frac{1}{2}\zeta$), and that $x^h = n_b^d$ or $-n_s^d$ occurs with equal probability. Then, the principal earns the following: (i)(a) $v = \theta = 1$ and $x^a = n_s^d$. Her expected payoff is $\frac{1}{4}\alpha(i)[(1 - \frac{1}{2}\psi_1)n_s^d - m_1]$ for $x = n_s^d + n_b^d - \overline{x}$, and $\frac{1}{4}\alpha(i)[(1 - \frac{1}{2})n_s^d - m_1]$ for $x = -\overline{x}$. (b) $v = 1$, $\theta = \phi$, and $x^a = n_s^d$. Her expected payoff is $\frac{1}{8}[1 - \alpha(i)]\zeta[(1 - \frac{1}{2}\psi_1)n_s^d - m_1]$ for $x = n_s^d + n_b^d - \overline{x}$, and $\frac{1}{8}[1 - \alpha(i)]\zeta[(1 - \frac{1}{2})n_s^d - m_1]$ for $x = -\overline{x}$. (ii) $v = 1$, $\theta = \phi$, and $x^a = -n_b^d$. Her expected payoff is $\frac{1}{8}[1 - \alpha(i)]\zeta[(-1 + \frac{1}{2})n_b^d - m_2]$ for $x = -\overline{x}$, and $\frac{1}{8}[1 - \alpha(i)]\zeta[(-1 + \frac{1}{2}\psi_2)n_b^d - m_2]$ for $x = -n_s^d - n_b^d - \overline{x}$. (iii) $v = 0$, $\theta = \phi$, and $x^a = n_s^d$. Her expected payoff is $\frac{1}{8}[1 - \alpha(i)]\zeta[(0 - \frac{1}{2}\psi_1)n_s^d - m_3]$ for $x = n_s^d + n_b^d - \overline{x}$, and $\frac{1}{8}[1 - \alpha(i)]\zeta[(0 - \frac{1}{2})n_s^d - m_3]$ for $x = -\overline{x}$. (iv)(a) $v = \theta = 0$ and $x^a = -n_b^d$. Her expected payoff is $\frac{1}{4}\alpha(i)[(0 + \frac{1}{2})n_b^d - m_4]$ for $x = -\overline{x}$, and $\frac{1}{4}\alpha(i)[(0 + \frac{1}{2}\psi_2)n_b^d - m_4]$ for $x = -n_s^d - n_b^d - \overline{x}$. (b) $v = 0$, $\theta = \phi$, and $x^a = -n_s^d$. Her expected payoff is $\frac{1}{8}[1 - \alpha(i)]\zeta[(0 + \frac{1}{2})n_b^d - m_4]$ for $x = -\overline{x}$, and $\frac{1}{8}[1 - \alpha(i)]\zeta[(0 + \frac{1}{2}\psi_2)n_s^d - m_4]$ for $x = -n_s^d - n_b^d - \overline{x}$. (v) $\theta = \phi$ and $x^a = 0$. As $x^a = 0$ is chosen for $\theta = \phi$ with probability $1 - \zeta$, her expected payoff is $-[1 - \alpha(i)](1$

$-\zeta)m_5$. On the other hand, if $y \geq \widehat{y}^d(i,\zeta)$, she earns $-m_6$. Now, her expected payoff at date 0 is

$$\Pi^p(i,\zeta) = \int_{\widehat{y}^d(i,\zeta)}^{\infty} \left\{ \frac{1}{4}\alpha(i)[(1-\frac{1}{2}\psi_1)n_s^d - m_1] + \frac{1}{4}\alpha(i)(\frac{1}{2}n_s^d - m_1) + \frac{1}{8}[1-\alpha(i)]\zeta[(1-\frac{1}{2}\psi_1)n_s^d - m_1] \right.$$

$$+\frac{1}{8}[1-\alpha(i)]\zeta(\frac{1}{2}n_s^d - m_1) + \frac{1}{8}[1-\alpha(i)]\zeta(-\frac{1}{2}n_b^d - m_2) + \frac{1}{8}[1-\alpha(i)]\zeta[(-1+\frac{1}{2}\psi_2)n_b^d - m_2]$$

$$+\frac{1}{8}[1-\alpha(i)]\zeta(-\frac{1}{2}\psi_1 n_s^d - m_3) + \frac{1}{8}[1-\alpha(i)]\zeta(-\frac{1}{2}n_s^d - m_3) + \frac{1}{4}\alpha(i)(\frac{1}{2}n_b^d - m_4)$$

$$+\frac{1}{4}\alpha(i)(\frac{1}{2}\psi_2 n_b^d - m_4) + \frac{1}{8}[1-\alpha(i)]\zeta(\frac{1}{2}n_b^d - m_4)$$

$$\left. +\frac{1}{8}[1-\alpha(i)]\zeta(\frac{1}{2}\psi_2 n_s^d - m_4) - [1-\alpha(i)](1-\zeta)m_5 \right\} dy - \int_0^{\widehat{y}^d(i,\zeta)} m_6 dy. \tag{A10}$$

■

Proof of Lemma 12: If the market freeze does not occur, the expected payoffs of the talented and incompetent arbitrageurs at date 1 are linear in ζ, η, and ζ^u respectively (see (16), (A8), and (A9)). Thus, it follows from $m_1 = m_4 = \overline{m}$ and $m_2 = m_3 = 0$ that these choices are given (18). ■

Proof of Lemmas 13 and 14: We first prove that (20e) can be set binding with equality in problem (20) without loss of generality. Suppose that (20e) is not binding with equality. Then, taking $\alpha(i)\overline{m} + [1-\alpha(i)]\, m_5$ as fixed, decreasing m_5, and increasing $\overline{m}$, we can set (20e) binding with equality while (20b)–(20d) hold and the value of (20a) remains fixed.

Next, substituting $m_5 = \frac{1}{2}\overline{m}$ into (20a)–(20d) yields

$$\Pi_0^p \equiv \max_{(i,\overline{m})\geq 0} \left\{ \frac{\alpha(i)}{4}\overline{n}(i) - \frac{1}{2}\left[1+\alpha(i)\right]\overline{m} \right\} \left[1 - F(\widehat{y}^d(i,0))\right], \tag{A11a}$$

subject to

$$\frac{1}{2}\left[1+\alpha(i)\right]\overline{m}\left[1 - F(\widehat{y}^d(i,0))\right] \geq (c+e)i + \underline{\Pi}^{ta}, \tag{A11b}$$

$$\frac{1}{2}\overline{m}\left[1 - F(\widehat{y}^d(i,0))\right] \leq ci, \tag{A11c}$$

$$\frac{1}{2}\alpha(i)\overline{m}\left[1 - F(\widehat{y}^d(i,0))\right] \geq ei. \tag{A11d}$$

Provided m_5 satisfies $\frac{1}{2}\overline{m} > m_5$ in problem (19), inspecting problems (19) and (A11) verifies that the functional forms of the objective function and the constraints of problem (19) are the same as those of problem (A11), except that $\overline{\overline{n}}(i)$ and $\widehat{y}^d(i,1)$ are used in problem (19) instead of $\overline{n}(i)$ and $\widehat{y}^d(i,0)$ in problem (A11).

In the subsequent proof, we focus on solving problem (19) and derive the results of Lemma 13. Repeating the same procedure, we can show the results of Lemma 14.

Suppose that (19d) is binding with equality. Then, combining (19c) and (19d) yields $\frac{1}{2}[1 + \alpha(i)]\overline{m}[1 - F(\widehat{y}^d(i,1))] \leq (c+e)i$, which contradicts (19b) because of $\underline{\Pi}^{ta} > 0$. Thus, (19d) must not be binding with equality if the optimal solution to (19) exists.

Now, suppose that (19c) is not binding with equality. Then, the above discussion indicates that neither (19c) nor (19d) is satisfied with equality. Because the profit maximization of the principal means that (19b) must be satisfied with equality, problem (19) is reduced to

$$\max_{i\geq 0} \frac{\alpha(i)}{4}\overline{\overline{n}}(i)\left[1 - F(\widehat{y}^d(i,1))\right] - (c+e)i - \underline{\Pi}^{ta}. \tag{A12}$$

As we assume that the optimal investment level is positive, the first-order condition to (A12) with respect to i is given by (21). Note that $\widehat{y}^d(i,1)$ is defined in Lemma 11.

Next, suppose that (19c) is binding with equality but (19b) is not binding with equality. As has been verified, (19d) is not binding with equality, either. Then, using (19c) with equality, problem (19) is reduced to

$$\max_{i\geq 0} \frac{\alpha(i)}{4}\overline{\overline{n}}(i)\left[1 - F(\widehat{y}^d(i,1))\right] - [1+\alpha(i)]\, ci. \tag{A13}$$

Thus, we obtain (22) as the first-order condition to (A13) with respect to i.

Lastly, suppose that both (19b) and (19c) are binding with equalities. Combining (19b) and (19c) yields (23), which gives the optimal level of investment in (19) in this case. ■

Proof of Proposition 5: As shown at the second paragraph of the proof of Lemmas 13 and 14, we need only compare the solutions to (19) and (A11) without loss of generality. For any $(i, \overline{m})$ satisfying the constraints of problem (A11), it follows from (A11a) and (A11b) that

$$\left[\frac{\alpha(i)}{4}\overline{n}(i) - \frac{1+\alpha(i)}{2}\overline{m}\right]\left[1 - F(\widehat{y}^d(i,0))\right] \leq \frac{\alpha(i)}{4}\overline{n}(i)\left[1 - F(\widehat{y}^d(i,0))\right] - (c+e)i - \underline{\Pi}^{ta}. \quad \text{(A14)}$$

As verified in the proof of Lemmas 13 and 14, (19b) must be binding with equality in (19) if (19c) is not binding with equality or if both (19b) and (19c) are binding with equalities. Then, it follows from (19a) and (19b) with equality that (19a) is reduced to (A12).

As verified in the proof of Lemmas 13 and 14, the remaining case is the situation under which only (19c) is binding with equality. Then, it follows from (19a) and (19c) with equality that (19a) is reduced to (A13).

Let i_0^{**} denote an optimal level of investment in problem (A11). We first compare the right-hand side of (A14) with (A12) at $i = i_0^{**}$. Subtracting the right-hand side of (A14) from (A12) and evaluating it at $i = i_0^{**}$ with $\overline{n}(i)$ from (5), $\overline{\overline{n}}(i)$ from (13), $\widehat{y}^d(i,1)$ and $\widehat{y}^d(i,0)$ from Lemma 11, and the uniform distribution of $F(\cdot)$ on $[0,\, \overline{y}]$, we obtain

$$\begin{aligned}\frac{\alpha(i_0^{**})}{2}&\left\{\frac{\overline{z}}{\overline{y}}\left[\frac{2-\alpha(i_0^{**})}{2+\alpha(i_0^{**})} - \frac{2-(\alpha(i_0^{**}))^2}{2+(\alpha(i_0^{**}))^2}\right] + \frac{a\alpha(i_0^{**})}{b}\left[\frac{1}{2-\alpha(i_0^{**})} - \frac{1}{2-(\alpha(i_0^{**}))^2}\right]\right.\\ &\left.+\frac{a\alpha(i_0^{**})}{b\overline{y}}\left[\frac{1}{2+(\alpha(i_0^{**}))^2} - \frac{1}{2+\alpha(i_0^{**})}\right]\right\}. \qquad \text{(A15)}\end{aligned}$$

Given $\alpha \in [0,\, 1)$, note that the first term in (A15) is negative, whereas the remaining two terms in (A15) are positive. If $\overline{z}$ is sufficiently small relative to $\overline{y}$, the remaining two terms dominate the former. Then, (A15) is positive.

Similarly, subtracting the right-hand side of (A14) from (A13) and evaluating it at $i = i_0^{**}$

with (5), (13), Lemma 11, and the uniform distribution of $F(\cdot)$ on $[0,\, \overline{y}]$, we obtain

$$\frac{\alpha(i_0^{**})}{2}\left\{\frac{\overline{z}}{\overline{y}}\left[\frac{2-\alpha(i_0^{**})}{2+\alpha(i_0^{**})}-\frac{2-(\alpha(i_0^{**}))^2}{2+(\alpha(i_0^{**}))^2}\right]+\frac{a\alpha(i_0^{**})}{b}\left[\frac{1}{2-\alpha(i_0^{**})}-\frac{1}{2-(\alpha(i_0^{**}))^2}\right]\right.$$
$$\left.+\frac{a\alpha(i_0^{**})}{b\overline{y}}\left[\frac{1}{[2+(\alpha(i_0^{**}))^2]}-\frac{1}{[2+\alpha(i_0^{**})]}\right]-\frac{2}{\alpha(i_0^{**})}\left[(\alpha(i_0^{**})c-e)i_0^{**}-\underline{\Pi}^{ta}\right]\right\}. \tag{A16}$$

Note that the first term in (A16) is negative, whereas the sum of the remaining three terms in (A16) is positive if c is sufficiently small. Hence, if $\overline{z}$ is sufficiently small relative to $\overline{y}$ and if c is sufficiently small, (A16) is positive.

Let $(i_0^{**}, \overline{m}_0^{**})$ denote a solution to problem (A11). Now, if there is a set of $(\overline{m}, m_5)$ that enables i_0^{**} to satisfy the constraints of problem (19), Π_1^p defined by (19a) is larger than or equal to the value of (A12) ((A13)) evaluated at i_0^{**} if (19b) (only (19c)) is binding with equality at the optimal solution to (19). Thus, if $\frac{\overline{z}}{\overline{y}}$ and c are sufficiently small, Π_1^p is larger than or equal to the left-hand side of (A14) at i_0^{**}, which is equal to Π_0^p. As a result, if these conditions hold, the principal prefers an optimal contract that allows an uninformed arbitrageur to use a random trading strategy.

Even if there are no sets of $(\overline{m}, m_5)$ that cause i_0^{**} to satisfy all the constraints of problem (19), we show that (19c) is satisfied at $(i_0^{**}, \overline{m}_0^{**})$ because $\widehat{y}^d(i,0) < \widehat{y}^d(i,1)$ for a fixed i. Furthermore, by adjusting $(\overline{m}, m_5)$, we can find a set of $(i_0^{**}, \overline{m})$ that makes (19c) bind with equality. If (19c) is binding with equality, we have $\frac{1}{2}\overline{m} = \frac{ci}{1-F(\widehat{y}^d(i,1))}$. Then, (19b) and (19d) are reduced to $\alpha(i)ci \geq ei + \underline{\Pi}^{ta}$ and $\alpha(i)ci \geq ei$, respectively. In fact, these constraints can be satisfied at $(i_0^{**}, \overline{m}_0^{**})$ because i_0^{**} satisfies (A11b) and (A11c) so that $\alpha(i_0^{**})ci_0^{**} \geq ei_0^{**} + \underline{\Pi}^{ta}$. However, this contradicts the initial assumption that there are no sets of $(\overline{m}, m_5)$ that cause i_0^{**} to satisfy all the constraints of problem (19).

Combining these arguments, we can establish the statement of this proposition. ■

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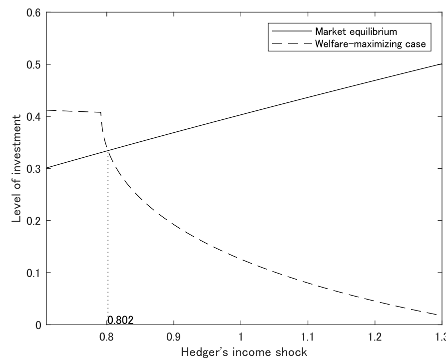
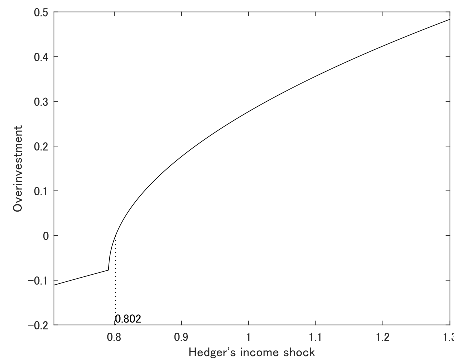
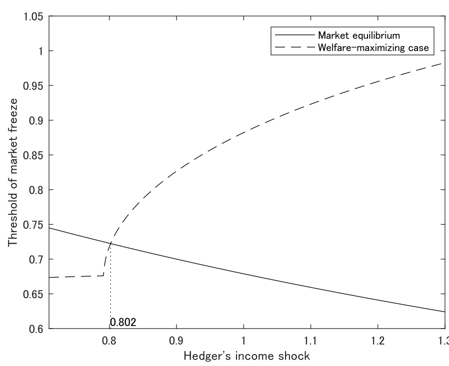


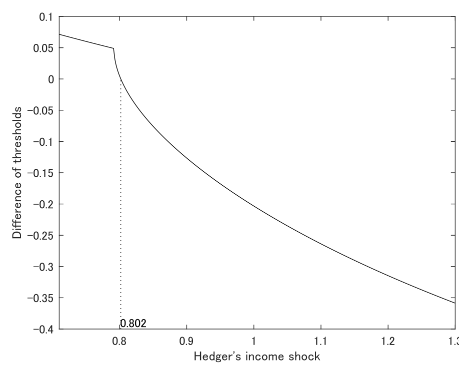
A. Investment levels (i^* and i_w^*)



B. Overinvestment ($i^* - i_w^*$)



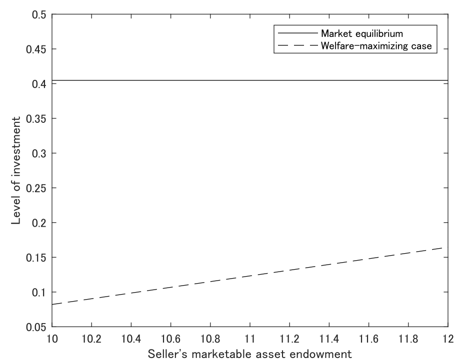
C. Likelihood of the market freeze ($\hat{y}(i^*)$ and $\hat{y}(i_w^*)$)



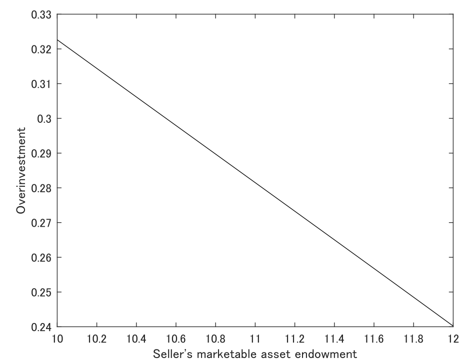
D. Difference in $\hat{y}$ ($\hat{y}(i^*) - \hat{y}(i_w^*)$)

Figure 1: Effects of the hedger's income shock ($\bar{z}$) in the DIR model

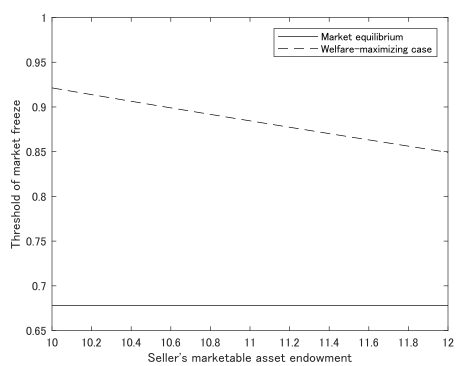
Notes: This figure depicts the impact of an increase in $\bar{z}$ on the investment levels (Panel A), the level of overinvestment (Panel B), the likelihood of the market freeze (Panel C), and the difference in the likelihood of the market freeze (Panel D) in the DIR model.



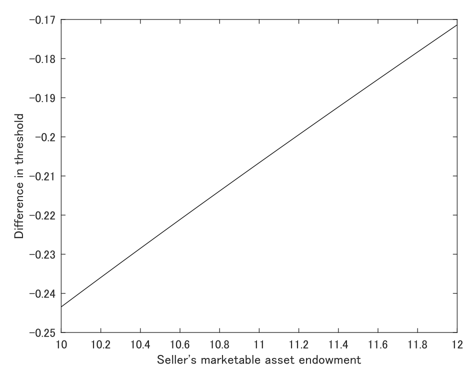
A. Investment levels (i^* and i^*_w)



B. Overinvestment ($i^* - i^*_w$)



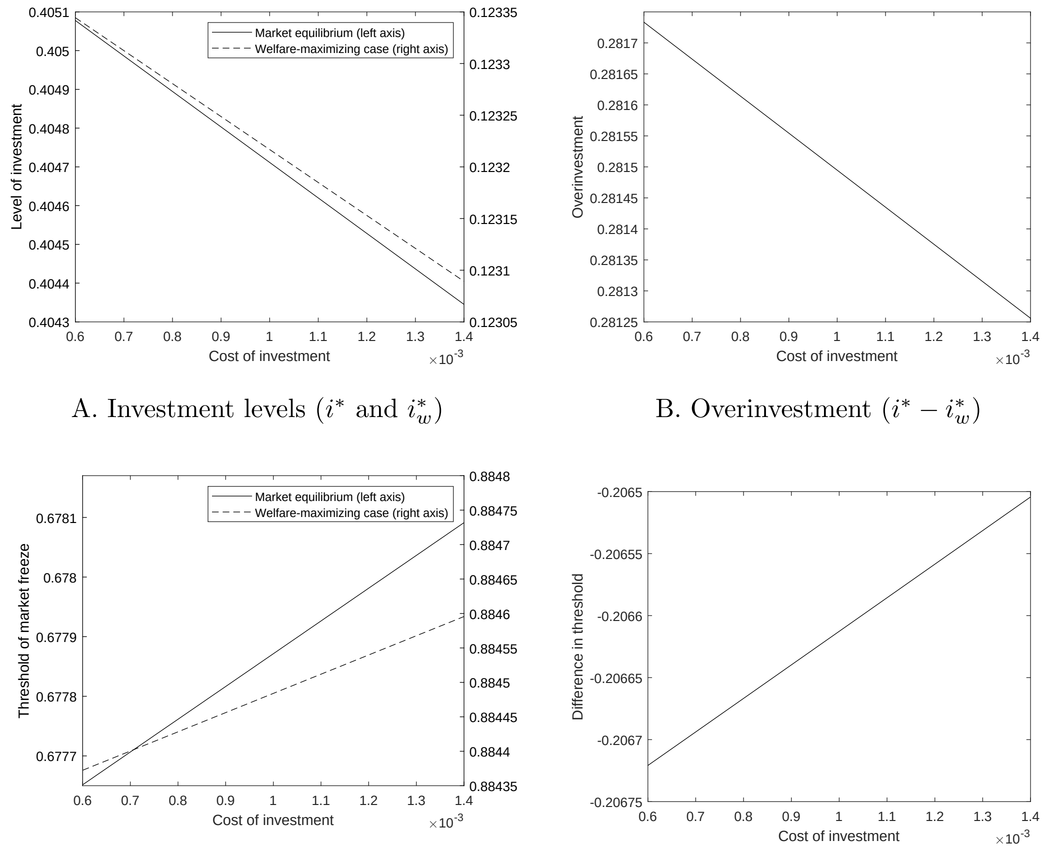
C. Likelihood of the market freeze ($\hat{y}(i^*)$ and $\hat{y}(i^*_w)$)



D. Difference in $\hat{y}$ ($\hat{y}(i^*) - \hat{y}(i^*_w)$)

Figure 2: Effects of the seller's endowment of the marketable asset ($\bar{x}$) in the DIR model

Notes: This figure depicts the impact of an increase in $\bar{x}$ on the investment levels (Panel A), the level of overinvestment (Panel B), the likelihood of the market freeze (Panel C), and the difference in the likelihood of the market freeze (Panel D) in the DIR model.



A. Investment levels (i^* and i_w^*)

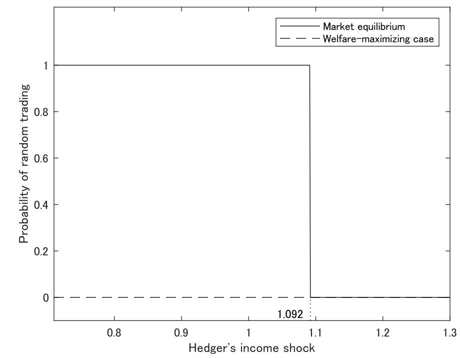
B. Overinvestment ($i^* - i_w^*$)

C. Likelihood of the market freeze ($\hat{y}(i^*)$ and $\hat{y}(i_w^*)$)

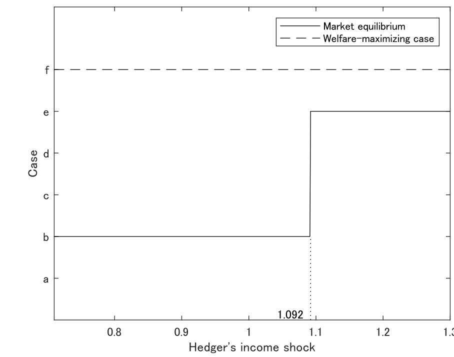
D. Difference in $\hat{y}$ ($\hat{y}(i^*) - \hat{y}(i_w^*)$)

Figure 3: Effects of the cost of investment (c) in the DIR model

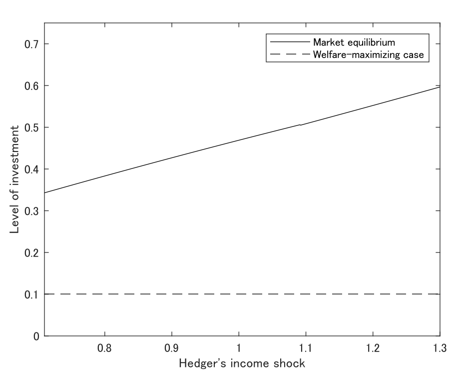
Notes: This figure depicts the impact of an increase in c on the investment levels (Panel A), the level of overinvestment (Panel B), the likelihood of the market freeze (Panel C), and the difference in the likelihood of the market freeze (Panel D) in the DIR model.



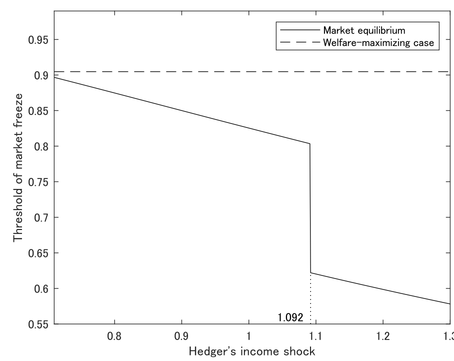
A. Probability of random trading (ζ^{**})



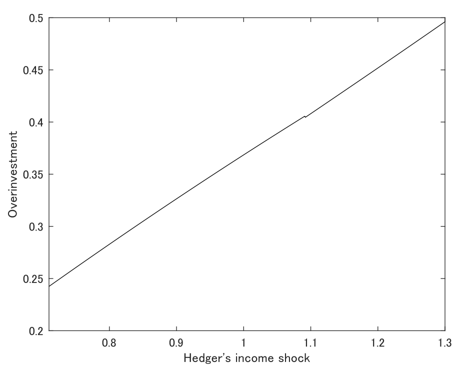
B. Chosen cases



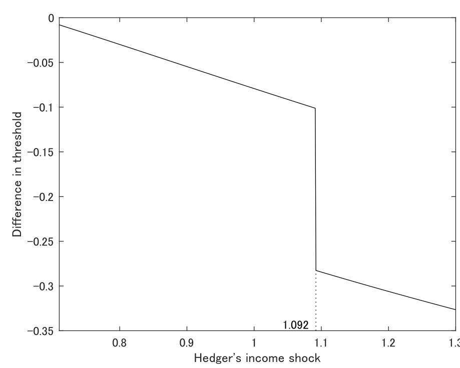
C. Investment levels (i^{**} and i_w^{**})



D. Likelihood of the market freeze ($\hat{y}(i^{**})$ and $\hat{y}(i_w^{**})$)



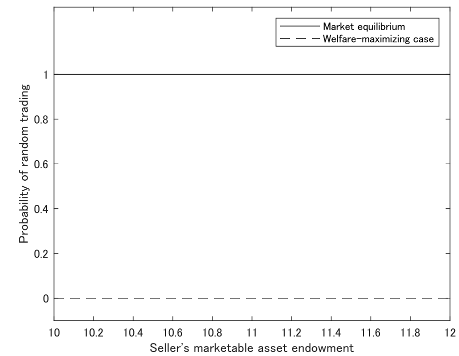
E. Overinvestment ($i^{**} - i_w^{**}$)



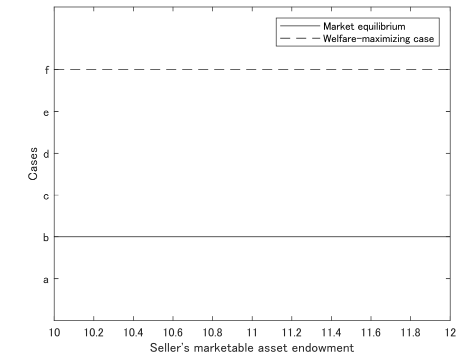
F. Difference in $\hat{y}$ ($\hat{y}(i^{**}) - \hat{y}(i_w^{**})$)

Figure 4: Effects of the hedger's income shock ($\bar{z}$) in the DEL model

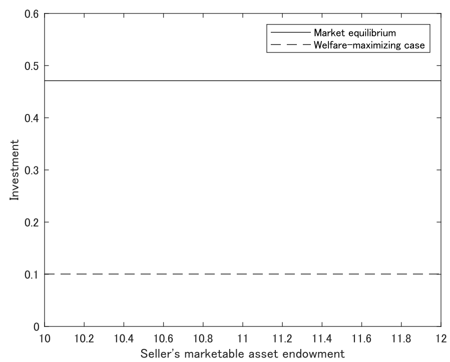
Notes: This figure depicts the impact of an increase in $\bar{z}$ on the value of ζ (Panel A), the chosen cases (Panel B), the investment levels (Panel C), the likelihood of the market freeze (Panel D), the level of overinvestment (Panel E), and the difference in the likelihood of the market freeze (Panel F) in the DEL model. In Panel B, cases (a), (b), and (c) correspond to the cases in which the investment levels (i^{**} and i_w^{**}) are determined by Lemmas 13(i), 13(ii), and 13(iii) when $\zeta^{**} = 1$, respectively, while cases (d), (e), and (f) correspond to the cases in which the investment levels (i^{**} and i_w^{**}) are determined by Lemmas 14(i), 14(ii), and 14(iii) when $\zeta^{**} = 0$, respectively.



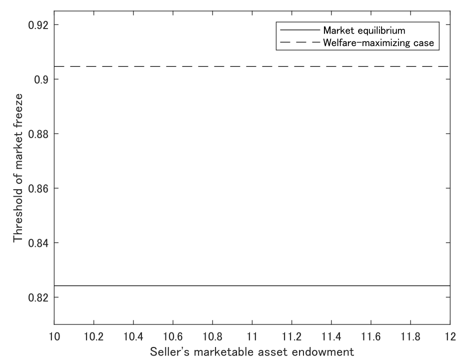
A. Probability of random trading (ζ^{**})



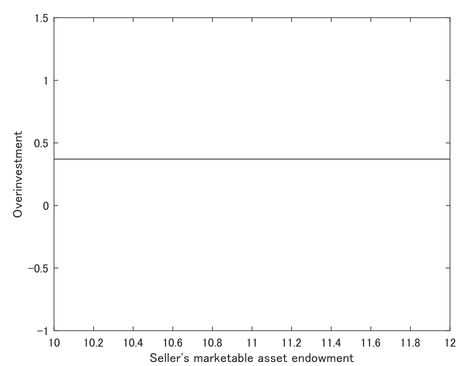
B. Chosen cases



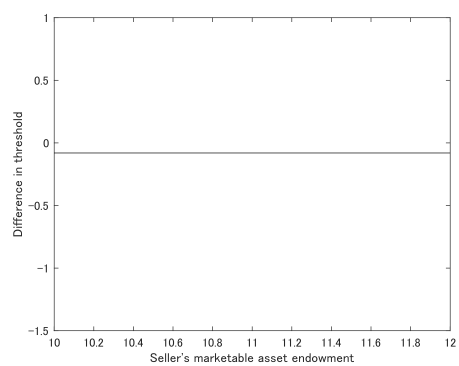
C. Investment levels (i^{**} and i_w^{**})



D. Likelihood of the market freeze ($\hat{y}(i^{**})$ and $\hat{y}(i_w^{**})$)



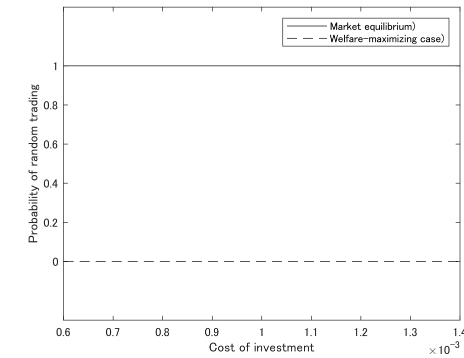
E. Overinvestment ($i^{**} - i_w^{**}$)



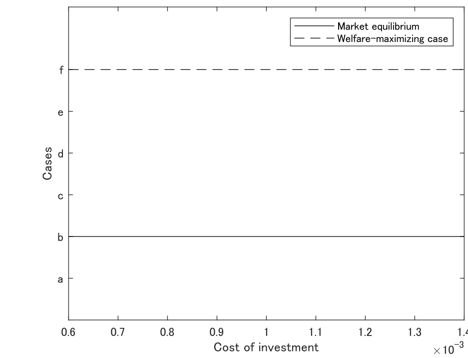
F. Difference in $\hat{y}$ ($\hat{y}(i^{**}) - \hat{y}(i_w^{**})$)

Figure 5: Effects of the seller's endowment of the marketable asset ($\bar{x}$) in the DEL model

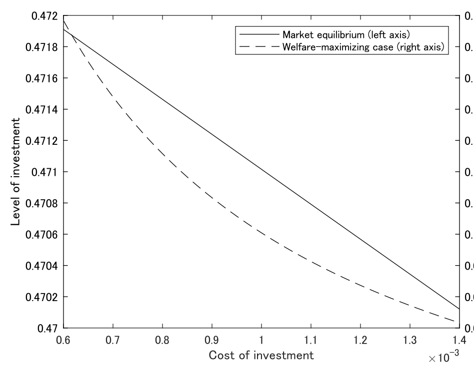
Notes: This figure depicts the impact of an increase in $\bar{x}$ on the value of ζ (Panel A), the chosen cases (Panel B), the investment levels (Panel C), the likelihood of the market freeze (Panel D), the level of overinvestment (Panel E), and the difference in the likelihood of the market freeze (Panel F) in the DEL model. In Panel B, cases (a), (b), and (c) correspond to the cases in which the investment levels (i^{**} and i_w^{**}) are determined by Lemmas 13(i), 13(ii), and 13(iii) when $\zeta^{**} = 1$, respectively, while cases (d), (e), and (f) correspond to the cases in which the investment levels (i^{**} and i_w^{**}) are determined by Lemmas 14(i), 14(ii), and 14(iii) when $\zeta^{**} = 0$, respectively.



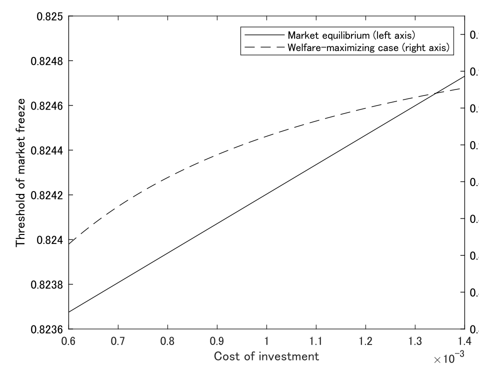
A. Probability of random trading (ζ^{**})



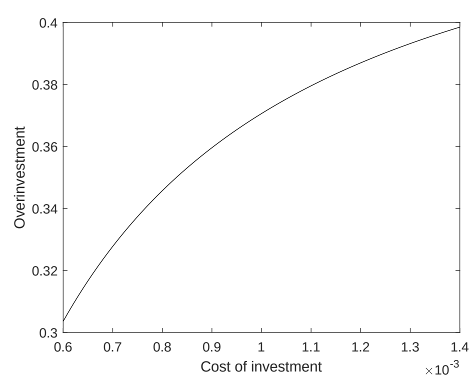
B. Chosen cases



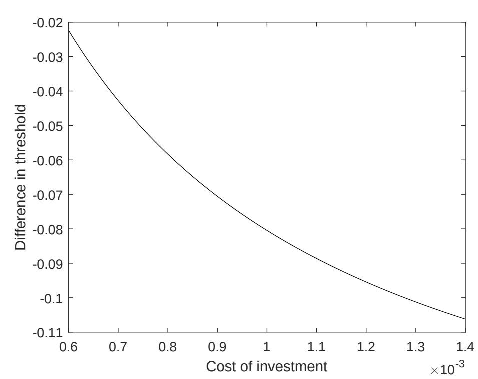
C. Investment levels (i^{**} and i_w^{**})



D. Likelihood of the market freeze ($\hat{y}(i^{**})$ and $\hat{y}(i_w^{**})$)



E. Overinvestment ($i^{**} - i_w^{**}$)



F. Difference in $\hat{y}$ ($\hat{y}(i^{**}) - \hat{y}(i_w^{**})$)

Figure 6: Effects of the cost of investment (c) in the DEL model

Notes: This figure depicts the impact of an increase in c on the value of ζ (Panel A), the chosen cases (Panel B), the investment levels (Panel C), the likelihood of the market freeze (Panel D), the level of overinvestment (Panel E), and the difference in the likelihood of the market freeze (Panel F) in the DEL model. In Panel B, cases (a), (b), and (c) correspond to the cases in which the investment levels (i^{**} and i_w^{**}) are determined by Lemmas 13(i), 13(ii), and 13(iii) when $\zeta^{**} = 1$, respectively, while cases (d), (e), and (f) correspond to the cases in which the investment levels (i^{**} and i_w^{**}) are determined by Lemmas 14(i), 14(ii), and 14(iii) when $\zeta^{**} = 0$, respectively.