KIER DISCUSSION PAPER SERIES

KYOTO INSTITUTE
OF
ECONOMIC RESEARCH

Discussion Paper No.1010

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Heterogeneous Elderly”

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July 2019

KYOTO UNIVERSITY

KYOTO, JAPAN

Pension, Retirement, and Growth in the Presence Heterogeneous Elderly*

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July 11, 2019

Abstract

This study explores the linkage between the labor force participation of the elderly and the long-run performance of the economy in the context of a two-period-lived overlapping generations model. We assume that the old agents are heterogeneous in their labor efficiency and they continue working if their income exceeds the pension that can be received in the case of full retirement. We inspect the long-run effects of changes in key factors that determine the labor force participation of the elderly. While the main part of the study treats a neoclassical growth model, we also discuss a model with endogenous growth.

Keywords: retirement decision, labor force participation, population aging, pension system, capital accumulation

JEL classification: E10, E62

*We thank Koichi Futagami, Tatsuro Iwaisako, Naoto Jinji, Kazutoshi Miyazawa, Shinichi Nishiyama, Ryosuke Shimizu, Koichi Yotsuya, and seminar participants at Osaka and Kyoto universities for their helpful comments on earlier versions of this paper. Mino's research is financially supported by JSPS KAKENHI project No.16KT0089 and No 17H02524.

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1 Introduction

In many developed countries, the labor force participation of the elderly substantially declined during the mid-1960s and the late 1980s. It was pointed out that the enhancement of social security programs in those countries promoted earlier retirement, which was the main source of the decrease in the labor force participation of the elderly. However, since the early 1990s, the rates of the labor force participation of the elderly in several industrialized countries have been increasing. For example, according to the Aging Society White Paper 2017 issued by the Japanese Cabinet Office, the labor force's share of the Japanese elderly aged 65 years and over increased from 5.9% in 1980 to 11.8% in 2016. Currently, more than 50% of the Japanese elderly aged 65–69 years engage in full-time- or part-time jobs. Such a trend stems from rises in longevity and health status of the elderly as well as from changes in the social environment that increase the activeness of the elderly. Additionally, researchers claim that a change in the elderly's labor supply is closely related to pension reforms. As mentioned below, several empirical studies, which use data of various countries, confirm that the recent rise in the labor force participation of the elderly had a statistically significant link to pension reforms implemented in those sampled industrialized countries after the 1990s.

In this study, we examine a model that may capture the basic facts mentioned above. First, we elucidate the key factors that determine the labor force participation of the elderly. Subsequently, we explore the long-run impacts of a rise in the labor force participation of the elderly on the aggregate economy. For this purpose, we construct a two-period-lived overlapping generations model in which the labor force participation of the old agents is endogenously determined. Based on this analytical framework, we investigate how changes in various factors that determine the labor force participation of the elderly affect the behaviors of key macroeconomic variables in the long-run.

Specifically, in our model economy, the young agents are assumed to be homogeneous and they fully devote their available time to working. However, the old agents are heterogeneous in the sense that their labor efficiency differs from each other. We assume that the old agents draw their labor efficiency from a given distribution function at the beginning of their old age. We also assume that there is a compulsory pay-as-you-go pension plan financed by taxation on the young generation's income. Given this setting, each old agent compares the expected pension revenue in the case of full retirement with the income that can be earned from the labor force participation; this determines the agent's decision to either fully retire or participate in the labor force . Such a decision determines a cutoff level of labor efficiency—the agents who have a lower efficiency level than the cutoff select full retirement, and the agents whose efficiency exceeds the cutoff continue working in their old age. We show that the pension scheme set by the government as well as the rate of population aging directly affect the threshold level of labor efficiency, thereby determining the labor force participation of the elderly.

The main part of this study assumes that the production side of the economy follows the standard neoclassical growth model. In this case, capital and income grow at an exogenously specified population growth rate, and per capita levels of capital and income stay constant in the steady state. Given this setting, we analytically show that an increase in the labor force participation of the old agents, generally, increases the steady state level of capital stock per population of the young generation. In addition to the theoretical discussion, we examine numerical examples to evaluate quantitative impacts of changes in the degree of population aging and the policy parameters on the labor force participation of the elderly and the steady state level of capital. Our numerical discussion reveals that magnitudes of those impacts are sensitive to the shape of the distribution function of the labor effi ciency of the old agents.

This study also examines the case of endogenous growth in which the presence of production externalities allow continuous growth of per capital income under constant population. Here, we examine the effect of a change in labor force participation of the elderly on the balanced growth rate of the aggregate economy. In this case, the relationship between labor supply of the elderly and long-run growth is more complex than the model with neoclassical technology. Thus, we mainly rely on numerical investigation in the case of endogenous growth.

Related Studies

(i) Empirics

A number of authors conduct empirical studies on the relationship between social security and labor force participation of the elderly based on the data in various counties[1]. Concerning

[1] A well-cited earlier empirical study on this topic is Krueger and Pischke (1992).

the decline in the labor force participation of the elderly during the period 1960–1990, Gruber and Wise (1999, 2004, and 2007) provide meticulous research outcomes. Among others, Gruber and Wise (1999) present detailed empirical studies on 11 industrialized countries and reveal that an increase in the generosity of the social security plan contributed towards the international trend of the decline in the labor force participation of old persons[2]. In contrast, the recent studies focus on the persistent rise in the labor force participation of the elderly since the early 1990s. For example, Oshio et al. (2011) examine the impact of social security reforms on the labor force participation of the Japanese elderly over a period of 40 years (1968-2007). They find that the labor force participation of the elderly started increasing after the 1985 reform that reduced the generosity of social security benefits, including pension plans[3]. In a similar vein, Berkel et al. (2004) study the impact of pension reforms on the retirement decision of the German elderly population, while Bottazzi et al. (2008) and Bovini (2018) discuss the effect of pension reforms on the elderly's retirement decision and wealth accumulation in Italy. Additionally, Coil (2015) presents a useful survey on empirical studies on this topic[4].

Several authors point out that health condition is also important for the labor force participation of the elderly. Kalwji and Vermeulen (2008) investigate the impact of health condition on the labor force participation of the elderly in 11 European countries. They provide careful estimation results and clarify the multidimensional nature of the health condition of the elderly. Overall, their results reveal that health status is one of the major factors that affects the retirement decision of the elderly. Similar studies are conducted by Mete and Schlutz (2002) on Taiwan and by Cai (2007) on Australia[5].

(ii) Theory

Many authors modify Diamond's (1965) overlapping generations (OLG) model to introduce endogenous labor supply of the old agents. The most popular idea in the field is to assume that young agents fully devote their available time to working, whereas old agents make the labor-leisure choice. In this setting, an increase in the labor force participation of

[2] Gruber and Wise (1998) summarize their main findings.

[3] Higuchi et al. (2006), Shimizutani (2013), and Shimizutani and Oshio (2013) present further evidence.

[4] See also Coil and Levine (2018).

[5] Gacía-Pèrez et al. (2013) present an empirical study on the relationship between retirement incentive, pension, and employment status based on the Spanish data.

the elderly means that old agents select a lower level of leisure time. A sample of this type of formulation includes Zhang and Zhang (2009), Gon and Liu (2012), Mizuno and Yakita (2013), and Hirazawa and Yakita (2017). Since this modelling assumes that agents are homogeneous, all the old agents select the same level of partial retirement that corresponds to the length of leisure time they choose. A deficiency of this formulation is that it fails to capture the fact that, in reality, a substantial number of the elderly fully retire.

On the other hand, a few authors have examined models in which the full retirement decision of the elderly is endogenously determined. Among others, Matsuyama (2008) constructs a two-period-lived OLG model in which old agents make a discrete choice between working and retirement by comparing utilities obtained in alternative situations. In his model, agents in each cohort are homogeneous; this ensures that the decision regarding the choice between working and retirement is uniform among the elderly[6]. Aisa et al. (2015) introduce agent heterogeneity. In their model, each agent is endowed with a given labor efficiency, at the outset of the agent's life, and agents whose labor efficiency exceeds an endogenously determined threshold level work in their old age. Although the basic idea of Aisa et al. (2015) is similar to our model, the authors treat a two-period model, and hence the long-run effect of the old agents' labor supply on capital accumulation is not updated in their study[7]. When compared to the foregoing formulations mentioned above, our model, which emphasizes the heterogeneity of old agents, has an advantage— it can simultaneously determine full retirement of some old agents as well as the aggregate level of labor force participation of the elderly in a dynamic environment.

Concerning the link between the social security and labor force participation of the elderly, Diamond and Mirrlees (1978) present an early theoretical study. These authors explore optimal social insurance in the presence of endogenous retirement and asymmetric information. In macroeconomics research, Hu (1979) conducts one of the earliest studies on the impact of social security on labor supply of the elderly in the context of Diamond's (1965) OLG

[6] Matsuyama (2008) shows that the model involves multiple steady states so that a poverty trap arises. Gon and Liu (2012) indicate that if the model allows partial retirement, then it would imply that the steady state equilibrium is uniquely determined. Since the model discussed by Gon and Liu (2012) is a variant of the labor-leisure choice models mentioned above, it does not depict full retirement of some of the old agents.

[7] It must be pointed out that Matsuyama (2008) briefly examines a modified model in which disutility levels of labor is heterogeneous among agents; this ensures that some old agents fully retire in equilibrium. However, he does not consider this line of formulation in detail.

model. In his model, the old agents make the labor-leisure choice and they receive a pension for their leisure time. Given this setting, Hu (1979) explores the long-run effect of a pay-as-you-go pension system on capital accumulation. Recently, Hu's modelling was employed by Miyazaki (2017), who examines the optimal social insurance in the presence of endogenous retirement. Cipriani (2018) also uses a similar setting to study the macroeconomic impact of population aging[8]. Since the analytical frameworks used by Hu (1979), Miyazaki (2017), and Cipriani (2018) are essentially the same as the models of Zhang and Zhang (2009) and others mentioned above, their discussions do not depict the full retirement decision of the elderly. Again, it is worth emphasizing that our model with heterogenous elderly can highlight the relationship between pension schemes and the full retirement of the elderly in a coherent manner[9].

The rest of the paper is organized as follows. Section 2 constructs the analytical framework for our discussion. Section 3 inspects the existence and stability of the steady state equilibrium of the model economy. Section 4 characterizes the relationship between the steady state equilibrium and the key parameters involved in the model. Section 5 re-examines our main results in the case of endogenous growth. Section 6 concludes the study.

2 Model

2.1 Households

Consider an overlapping generation economy in which each agent lives for two periods, young and old. We assume that young agents live for one period with certainty, but they face a probability of surviving to the old age. We denote the probability of surviving as $\pi \in (0, 1]$. Each cohort consists of a continuum of agents, and the mass of cohort grows at a constant rate of n. We denote the population of cohort born at the beginning of the period t as N_t. Then, it holds that $N_{t+1} = (1 + n) N_t$. Since the population share of the old generation in period t is $\pi N_t/(N_{t+1} + \pi N_t) = \pi/(1 + \pi + n)$, population aging means that a decline in the population growth rate, n and/or a rise in the probability of surviving, π.

[8] See also Phillippe and Pestieau (2013).

[9] Here, we focus on theoretical studies based on the two-period lived OLG models. There are several quantitative studies on the relationship between social security programs and labor supply in the context of calibrated multi-period lived OLG models: see, for example Kitao (2014 and 2015).

When young agents are homogeneous and each agent inelastically supplies one unit of labor. When old agents become heterogeneous in the sense that their labor efficiencies differs from each other. Such a difference in labor efficiency stems from differences in each agent's health status and the motivation for labor force participation, among others. We assume that the labor efficiency, denoted by h, is in between $\eta\,(>0)$ and 1. Namely, there is a minimum level of efficiency of old person's labor, η, and the most able old agents have the same level of labor efficiency as that of young agents. We also assume that the cumulative distribution function of h is specified as a truncated Pareto distribution in such a way that

$$F(h) = \frac{1-\left(\frac{h}{\eta}\right)^{-\psi}}{1-\left(\frac{1}{\eta}\right)^{-\psi}}, \qquad \psi > 1, \quad 0 < \eta < 1 \qquad h \in [\eta, 1]. \tag{1}$$

The functional form $F(h)$ is set for analytical convenience. Additionally, in our model, the income of an old agent who works increases with the agent's labor efficiency, h. Since foregoing empirical studies reveal that an upper tail of income distribution of the elderly follows a Pareto distribution, our specification may be an empirically plausible one[10].

In our model, if the agents born in period t survive in period $t+1$, then they draw their labor efficiency, h, from $F(h)$ at the beginning of period $t+1$. It is assumed that h is independent and identically distributed (i.i.d) over time as well as across agents. In addition, the probability distribution of h is assumed to be stationary. Therefore, the share of old agents with a particular level of labor efficiency is constant and the same in each cohort.

The objective function of an agent born at the beginning of period t is a discounted sum of expected utilities of consumption in both periods:

$$U_t = E_t \log\left[c_t + \pi\beta \log x_{t+1}\right] \qquad 0 < \beta < 1, \quad h \in [\eta, 1],$$

where c_t and x_{t+1} ,respectively, denote an agent's consumption in the agent's young and old ages, and β is a given discount factor. When the agent in cohort t is young, the agent's flow budget constraint is

$$c + s_t = w_t - \tau_t,$$

[10] Concerning the recent study on income distribution among the Japanese elderly, see Shirahase (2015) and Seiyama (2016).

where s_t is saving, w_t is the real wage, and τ_t represents an amount of income tax. In old age, the agent's budget constraint is given by

$$x_{t+1} = (1 + r_{t+1})\, s_t + m_{t+1},$$

where r_{t+1} is the real interest rate in period $t+1$, and m_{t+1} denotes the agent's expected income in old age[11]. Combining the budget constraints given above, we obtain the intertemporal budget constraint:

$$c_t + \frac{x_{t+1}}{1 + r_{t+1}} = w_t - \tau_t + \frac{m_{t+1}}{1 + r_{t+1}}. \tag{2}$$

Maximizing U_t subject to (2), we find that the optimal levels of consumption and saving are, respectively, given by

$$c_t = \frac{1}{1 + \beta\pi}\left(w_t - \tau_t + \frac{m_{t+1}}{1 + r_{t+1}}\right), \tag{3}$$

$$x_{t+1} = \frac{\beta\pi\,(1 + r_{t+1})}{1 + \beta\pi}\left(w_t - \tau_t + \frac{m_{t+1}}{1 + r_{t+1}}\right), \tag{4}$$

$$s_t = w_t - \tau_t - c_t = \frac{\beta\pi}{1 + \beta\pi}(w_t - \tau_t) - \frac{m_{t+1}}{(1 + \beta\pi)\,(1 + r_{t+1})}. \tag{5}$$

The expected income in old age, m_{t+1}, is determined in the following manner. As mentioned earlier, in the beginning of $t+1$, an agent born in period t draws own labor efficiency from the given distribution of h. We assume that an old agent with ability h can offer $h \times 100\%$ a young worker's labor service. Given the specification of the distribution of labor efficiency, if an old agent born in period t retires, then the agent would receive a pension, p_{t+1}. However, if an old agent continues to work, then the agent would receive a part of the pension for the fully retired old, $(1-\mu)\, p_{t+1}$ $(0 < \mu < 1)$, as a basic pension. Since we have assumed that the old agent with labor efficiency h is $h \times 100\%$ that of an young worker, the competitive wage offered for an old agent with h is hw_{t+1}. The old agents hope to obtain the maximum

[11] In this paper, we assume that the wealth of unsurvived agents is consumed by the government. Alternatively, we may assume that there is a competitive annuity market with free entry. In this case, the budget constraint of an old agent becomes $x_{t+1} = \frac{(1+r_{t+1})}{\pi} s_t + m_{t+1}$. Our analytical outcomes are essentially the same under such an alternative setting.

amount of income in their old age, and hence the ex-post income in the old age is given by

$$\max \{p_{t+1}, hw_{t+1} + (1-\mu)p_{t+1}\}, \quad 0 < \mu < 1.$$

As a result, the cutoff level of labor efficiency of the old agents is

$$h_{t+1}^{*} = \mu \frac{p_{t+1}}{w_{t+1}}. \tag{6}$$

An old agent who draws $h < h_t^*$ fully retires, while an agent with $h \geq h_t^*$ continues working in old age[12]. Since the old agent retires with the probability $F\left(h_{t+1}^*\right)$ or continues working with the probability $1 - F\left(h_{t+1}^*\right)$, the income in the old age expected in period t will be determined by

$$m_{t+1} = F\left(h_{t+1}^*\right) p_{t+1} + \left[1 - F\left(h_{t+1}^*\right)\right] \left[(1-\mu)\, p_{t+1} + \int_{h_{t+1}^*}^{1} hw_{t+1} dF\left(h\right)\right], \tag{7}$$

where $\int_{h_{t+1}^*}^{1} hw_{t+1} dF\left(h\right)$ is the expected wage income for an old agent who stays in the labor force.

2.2 Firms

The production side of our model is the standard one. The final good and factor markets are competitive. There is a representative firm that produces a homogeneous good according to the following production technology:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}, \quad 0 < \alpha < 1, \tag{8}$$

where Y_t is the output, K_t is the aggregate capital stock, and L_t is the input of labor service. The firm employs both young- and old-workers' labor service, L_t^y, and L_t^o, respectively so that

$$L_t = L_t^y + L_t^o. \tag{9}$$

[12] We assume that if $h = h^*$, then the old agents would continue to select work.

We assume that one unit of hours of work of a young agent yields one unit of labor service, meaning that L_t^y also denotes the total hours of work of the young agents. Similarly, it is assumed that one unit of hours of work of an old agent whose labor efficiency is h gives h amount of labor service. Thus, if we denote the hours of work of type h old agents by $l_{h,t}$, then the aggregate input of old workers' labor service will be

$$L_t^o = \int_{h_t^*}^1 h l_{h,t} dF(h). \tag{10}$$

The profit of the firm is

$$\Pi_t = AK_t^\alpha \left(L_t^y + \int_{h_t^*}^1 h l_{h,t} dF(h) \right)^{1-\alpha} - r_t K_t - w_t L_t^y - \int_{h_t^*}^1 (w_t h) l_{h,t} dF(h).$$

The firm maximizes profit by selecting K_t, L_t^y and $l_{h,t}$, and the first-order conditions for an optimum are given by

$$\alpha A K_t^{\alpha-1} L_t^{1-\alpha} - r_t = 0, \tag{11}$$

$$(1-\alpha) A K_t^\alpha L_t^{-\alpha} - w_t = 0, \tag{12}$$

$$h(1-\alpha) A K_t^\alpha L_t^{-\alpha} - h w_t = 0 \quad \text{for } h \in [h_t^*, 1]. \tag{13}$$

Since conditions (12) and (13) are symmetric, the total demand of labor service is

$$L_t = [(1-\alpha) A]^{\frac{1}{\alpha}} w_t^{-\frac{1}{\alpha}} K_t. \tag{14}$$

2.3 Government

We assume that there is a pay-as-you-go pension plan in which the tax for social security is levied on the young agents' wage income[13]. The tax revenue of the government is $\tau_t = \tau w_t$, where $\tau \in (0,1)$ is the flat rate of the wage tax. For simplicity, we assume that the government's tax revenue is spent on pensions alone and there is no government debt. Hence,

[13] In reality, income tax also applies to the old agents' wage income. In our model, we ignore any government spending, except for pension payments, we assume that old agents' wage income is free from taxation.

the government's budget constraint in period t is given by

$$\tau w_t N_t = \left[F\left(h_t^*\right) + (1-\mu)\left(1 - F\left(h_t^*\right)\right)\right] p_t \pi N_{t-1}. \tag{15}$$

The left-hand side of (15) is the aggregate tax revenue, and the right-hand side expresses the aggregate pension for the elderly. It must be noted that, by our assumption, the mass of the old agents in period t is πN_{t-1}.

2.4 Market Equilibrium Conditions

In period t, the young agents' total labor supply is N_t, while the old agent's total supply labor service is given by

$$\pi N_{t-1} \int_{h_t^*}^{1} h dF(h) = \frac{\psi}{1 - \left(\frac{1}{\eta}\right)^{-\psi}} \int_{h_t^*}^{1} \left(\frac{h}{\eta}\right)^{-\psi} = \frac{\psi \eta^{\psi-1}}{(1-\eta^{\psi})(\psi-1)} \left[(h^*)^{1-\psi} - 1\right] \pi N_{t-1}.$$

Therefore, the aggregate labor supply denoted by N_t^s is

$$N_t^s = N_t \left\{1 + \frac{\psi \eta^{\psi-1}}{(\eta^{\psi}-1)(\psi-1)} \left[1 - (h^*)^{1-\psi}\right] \frac{\pi}{1+n}\right\}, \tag{16}$$

and the labor market equilibrium condition is

$$N_t^s = L_t. \tag{17}$$

Since only young agents save, the market equilibrium condition of capital stock is

$$K_{t+1} = N_t s_t. \tag{18}$$

3 Equilibrium Dynamics

3.1 Determinants of Labor Supply of the Elderly

To derive a complete dynamic system, we first inspect the government budget constraint (15) . Using (1) , (15) , and $N_{t+1} = (1+n) N_t$, we obtain ,

$$\frac{(1+n)\tau}{\pi h_t} = \frac{1}{\mu} - 1 + \frac{1 - \left(\frac{h_t}{\eta}\right)^{-\psi}}{1 - \left(\frac{1}{\eta}\right)^{-\psi}}. \tag{19}$$

This condition determines a stationary cutoff level of labor efficiency, h^*. As depicted by Figure 1, the left-hand side (LHS) of (19) monotonically decreases with h, while the right-hand side (RHS) of (19) monotonically increases with h. It is easy to see that there is a unique and stationary cutoff that satisfies $h_t^* \in (\eta, 1)$ if the following conditions hold:

$$\frac{1}{\mu} > \frac{(1+n)\tau}{\pi\eta} > \frac{1}{\mu} - 1. \tag{20}$$

Given (20) , h^* is uniquely expressed as a function of parameters in such a way that

$$h^* = H(n, \pi, \tau, \mu, \psi, \eta). \tag{21}$$

[Figure 1]

Inspecting Figure 1, we see that a fall in n or a rise in π yields a downward shift of the graph in the left-hand side (LHS) of (19), so that h^* decreases. Since the labor force participation rate of the elderly is $\frac{1-h^*}{1-\eta}$, population aging promotes the labor force participation of old agents. Conversely, a rise in the rate of payroll tax, τ , leads to an upward shift in the LHS of (19) , which gives a higher level of h^*. Thus, a more generous pension plan lowers the labor force participation of the elderly. However, a fall in μ, which means that a more generous pension reform for the old agents who stay in the labor force yields an upward shift in the RHS of (19) , decreases h^* and increases the rate of labor force participation of the elderly. This is because a more favorable pension plan for the old agents who do not retire makes staying in the labor force more attractive.

We also find that the properties of the cumulative distribution function of h also affect the labor force participation rate of the elderly. First, consider the effect of an increase in the shape parameter, ψ. Letting the RHS of (19) be $\frac{1}{\mu} - 1 + F(h; \psi, \eta)$. Then, it is shown that $\frac{\partial F(h;\psi,\eta:)}{\partial \psi} > 0$, so that the graph of RHS shifts upward. Thus, h^* decreases. It must be noted that the magnitude of ψ represents the degree of heterogeneity of the labor efficiency among the elderly. Hence, if old agents are more homogeneous, that is, ψ take a higher value, then the labor force participation rate of the elderly will increase. However, we find that $\frac{\partial F(h;\psi,\eta:)}{\partial \eta} < 0$, which means that a rise in the minimum efficiency of labor, η yields a downward shift in the graph of RHS. As a result, a higher η yields a higher h^* and decreases the labor force participation of the elderly.

In order to present an economic implication of this comparative statics' result as to ψ and η, it is useful to remember that the expected value of labor efficiency of old agents is given by

$$\int_{\eta}^{1} h dF(h) = \frac{\psi \eta^{\psi}}{1 - \eta^{\psi}} \int_{n}^{1} h^{-\psi} dh = \frac{\psi \eta^{\psi - 1} \left(\eta^{1-\psi} - 1 \right)}{(1 - \eta^{\psi})(\psi - 1)}.$$

We can confirm that a fall in ψ or a rise in η increases the expected value of h. Namely, if the old agents become more heterogeneous or if the minimum level of labor efficiency increases, then the average productivity of an entire old generation would expand. A low level of ψ means that the distribution function of the density of h has a "long and fat tail," which gives rise to a high level of average productivity of the old agents. A higher η may imply that the health status of the elderly is, generally, in good condition. Alternatively, it may reflect the social and institutional environments that encourage the elderly to participate in the labor force. Since, from (6) the equilibrium value of the pension-wage ration is determined by $h^* = \mu \frac{p_{t+1}}{w_{t+1}}$, under a given μ, a lower average productivity of the old agents gives rise to a lower level of pension-wage ratio. In other words, a more homogeneous society realizes a relatively high level of labor force participation of the elderly. By contrast, if a wide heterogeneity among the elderly leads to a relative increase in their average productivity, then h^* will be high. In this case, the labor force of the elderly is concentrated in the agents with a relatively high level of efficiency, which can sustain a high level of pension-wage ratio.

In sum, the impacts of changes in the parameter values on the cutoff level of labor efficiency are as follows:

$$\frac{\partial h^*}{\partial n} < 0, \; \frac{\partial h^*}{\partial \pi} > 0, \; \frac{\partial h^*}{\partial \tau} > 0, \; \frac{\partial h^*}{\partial \mu} > 0, \; \frac{\partial h^*}{\partial \psi} < 0, \; \frac{\partial h^*}{\partial \eta} > 0.$$

Consequently, we obtain the following proposition:

Proposition 1 *If $\frac{1}{\mu} > \frac{(1+n)\tau}{\pi\eta} > \frac{1}{\mu} - 1$, then there is a unique level of cutoff in labor efficiency, $h^* \in (\eta, 1)$. The rate of labor force participation of the elderly, $\frac{1-h^*}{1-\eta}$,rises with a higher longevity, π, a lower population growth rate, n, a lower payroll tax rate, τ, a lower deduction of pension for the old agents who continue working, and with a lower level of heterogeneity among the elderly.*

3.2 Existence and Stability of the Steady State

In view of (5) and (18), we see that the aggregate capital stock per population of young agents, $k_t = K_t/N_t$, follows

$$k_{t+1} = \frac{1}{1+n}\left[\frac{\beta\pi}{1+\beta\pi}(1-\tau)\,w_t - \frac{1}{(1+\beta\pi)(1+r_{t+1})}m_{t+1}\right]. \tag{22}$$

Here, m_{t+1} is expressed as

$$\begin{aligned} m_{t+1} &= F(h^*)\,p_{t+1} + (1-\mu)\left[1-F(h^*)\right]\left[p_{t+1} + \int_{h^*}^{1} w_{t+1} h dF(h)\right] \\ &= \left[1-\mu+\mu F(h^*)\right]p_{t+1} + \frac{\psi\eta^{\psi-1}}{(1-\eta^{\psi})(\psi-1)}\left[1-F(h^*)\right]\left[(h^*)^{1-\psi}-1\right]w_{t+1}, \end{aligned}$$

Equation (19) means that the threshold level of h^* stays constant over time and the pension is proportional to the real wage rate :

$$p_t = \frac{1}{\mu}h^* w_t \text{ for all } t \geq 0. \tag{23}$$

Furthermore, using (14), (16) and (17), we express the labor market equilibrium condition in the following manner:

$$\left[(1-\alpha)A\right]^{\frac{1}{\alpha}} w_t^{-\frac{1}{\alpha}} K_t. = N_t\left\{1 + \frac{\psi\eta^{\psi-1}}{(1-\eta^{\psi})(\psi-1)}\left[(h^*)^{1-\psi}-1\right]\frac{\pi}{1+n}\right\}.$$

This relationship gives the ratio between the real wage rate and the capital stock per young generation, $k_t = K_t/N_t$ as follows:

$$k_t = \Phi \left[(1-\alpha) A\right]^{-\frac{1}{\alpha}} w_t^{\frac{1}{\alpha}}, \tag{24}$$

where

$$\Phi = 1 + \frac{\psi \eta^{\psi-1}}{\left(\eta^{\psi}-1\right)(\psi-1)} \left[1-(h^*)^{1-\psi}\right] \frac{\pi}{1+n}. \tag{25}$$

It must be noted that, like in Diamond's (1965) standard setting, if the elderly are homogeneous (, i.e., $\psi = +\infty$) and they do not work, then $\Phi = 1$, so that (24) represents the relationship between the real wage rate and an optimal capita-labor ratio selected by the firms.

Finally, the relationship between r_t and w_t stems from (11) :

$$r_t = \alpha A \left[(1-\alpha) A\right]^{\frac{1}{\alpha}-1} w_t^{1-\frac{1}{\alpha}}. \tag{26}$$

As a result, from (22), (23) , (24) , and (26) , we derive a complete dynamic system of the real wage as follows:

$$\begin{aligned}
&(1+n)\Phi\left[(1-\alpha)A\right]^{-\frac{1}{\alpha}} w_{t+1}^{\frac{1}{\alpha}} \\
&= \frac{\beta\pi}{1+\beta\pi}(1-\tau) w_t - \frac{\left[\frac{1}{\mu}-1+\mu F(h^*)\right] h^* w_{t+1} + \frac{\psi\eta^{\psi-1}}{(1-\eta^{\psi})(\psi-1)}\left[1-F(h^*)\right]\left[(h^*)^{1-\psi}-1\right] w_{t+1}.}{(1+\beta\pi)\left(1+A\left[(1-\alpha)A\right]^{\frac{1}{\alpha}-1} w_{t+1}^{1-\frac{1}{\alpha}}\right)}.
\end{aligned}$$

We rewrite the above system as

$$w_t = \Gamma\left(w_{t+1}\right), \tag{27}$$

where

$$\begin{aligned}
\Gamma\left(w_{t+1}\right) = &\frac{(1+n)(1+\beta\pi)}{(1-\tau)\beta\pi}\Phi[(1-\alpha)A]^{-\frac{1}{\alpha}} w_{t+1}^{\frac{1}{\alpha}} \\
&+ \frac{1+\beta\pi}{(1-\tau)\beta\pi} \times \frac{\left[\frac{1}{\mu}-1+\mu F(h^*)\right] h^* w_{t+1} + \frac{\psi\eta^{\psi-1}}{(1-\eta^{\psi})(\psi-1)}\left[1-F(h^*)\right]\left[(h^*)^{1-\psi}-1\right] w_{t+1}}{(1+\beta\pi)\left(1+A\left[(1-\alpha)A\right]^{\frac{1}{\alpha}-1} w_{t+1}^{1-\frac{1}{\alpha}}\right)}.
\end{aligned}$$

It is easy to see that $\alpha \in (0,1)$ means that $\Gamma(0)=0,\ \Gamma(+\infty)=+\infty$, and that $\Gamma(w_{t+1})$ is monotonically increasing in w_{t+1}. Additionally, as confirmed in the Appendix to this study, we see that if $\alpha \in [0.5,1)$, then $\Gamma(w_{t+1})$ is strictly convex in w_{t+1}; additionally, it holds that $\lim_{w_{t+1}\to 0}\Gamma'(w_{t+1}) = 0$[14]. These results ensure that there is a unique and strictly positive level of real wage, w, that satisfies $w = \Gamma(w)$. Since function $\Gamma(.)$ is invertible, (27) is expressed as

$$w_{t+1} = \Gamma^{-1}(w_t),$$

where $\Gamma^{-}1(w_t)$ is monotonically increasing and a strictly concave function of w_t that satisfies $\lim_{w\ t} \to \infty\Gamma^{-}1(w_t) = +\infty$. Consequently, the dynamic system of the real wage has a unique and globally stable steady state in which $w = \Gamma(w)$ holds: see Figure 2.

[Figure 2]

As for the aggregate dynamics of our economy, we then find:

Proposition 2 *If* $0.5 < \alpha < 1$, *then the economy has a unique steady state equilibrium that satisfies global stability.*

4 Impacts of the Labor Supply of the Elderly

4.1 Steady-State Characterization

In the previous section, we focus on the single difference equation of w_t to inspect the existence and stability of the steady state equilibrium. To investigate the long-run effects of old agents' labor supply on the steady state values of key variables, it will be convenient to use the following two conditions:

$$k = \Phi\left[(1-\alpha)A\right]^{-\frac{1}{\alpha}} w^{\frac{1}{\alpha}}. \tag{28}$$

[14]Since the final good market is assumed to be competitive, α corresponds to the income share of the capital, which ensures that it is, generally, set at around $1/3$. One idea to set $\alpha > 0.5$ is to follow Mankiw, Romer and Weil (1995) and assume that K_t includes human capital that is perfectly substitutable with physical capital. It must be noted that, as shown in the Appendix, the strict convexity of $\Gamma(w_{+1})$ may hold even if $0 \in (0, 0.5)$.

$$k = \frac{1}{1+n}\left\{\frac{\beta\pi}{1+\beta\pi}(1-\tau)w - \frac{w}{(1+\beta\pi)\left(1+A\left[(1-\alpha)A\right]^{\frac{1}{\alpha}-1}w^{1-\frac{1}{\alpha}}\right)}\left[\frac{1}{\mu}-1+\mu F(h^*)\right]h^* \right.$$
$$\left. - + \frac{\psi\eta^{\psi-1}}{(1-\eta^{\psi})(\psi-1)}\left[1-F(h^*)\right]\left[(h^*)^{\psi-1}-1\right]\right\}. \tag{29}$$

In the above, k and w, respectively, denote the steady state values of k_t and w_t. As mentioned before, (28) represents the relationship between the capital stock per mass of each generation and the real wage rate. A change in the labor force participation of the elderly changes the level of Φ. Equation (29) expresses the per capita saving of the young generation, which is positively related the young agents' after-tax income, $(1-\tau)w$, and negatively related to the per-capita consumption in their old age.

Figure 3 depicts the graphs of (28) and (29). We assume that the strict convexity condition for $\Gamma(w_{t+1})$ given in Proposition 2 is fulfilled, which establishes an inverse U-shaped relation of (29). In this case, the steady state levels of k_t and w_t are uniquely determined.

[Figure 3]

Once k and w are fixed, the steady state values of the key variables are determined in the following manner. The per capita level of income the steady state is

$$\frac{AK_t^{\alpha}L_t^{1-\alpha}}{N_t+\pi N_{t-1}} = A\left(\frac{1+n}{1+n+\pi}\right)k^{\alpha}\Phi^{1-\alpha}. \tag{30}$$

Additionally, the rate of return to capital and per capita pension are, Respectively, given by

$$r = A\left[(1-\alpha)A\right]^{\frac{1}{\alpha}-1}w^{1-\frac{1}{\alpha}}, \tag{31}$$

$$p = \frac{h^*w}{\mu}. \tag{32}$$

4.2 Comparative Statics in the Long-run

Using the steady state conditions derived above, we conduct some comparative statics in the long-run equilibrium.

(i) Population Aging

As mentioned earlier, in our setting, population aging means a fall in the population expansion rate, n, and/or a rise in the survival rate, π. Other things being equal, a decline in the population growth rate, n, increases the discount rate $1/(1+n)$, which increases Φ defined by (25). In addition, a lower n directly reduces h^*. As a result, both graphs of (28) and (29) shift upward, and, hence, k increases (see Figure 5-a). On the other hand, the effect on the steady-state real wage is analytically ambiguous. This is because, while a decrease in population of the young agents increases the equilibrium level of real wage, an increase in the old agents' labor supply depresses the competitive real wage. The resulting change in the long-run rate of real wage hinges on the relative strength of those opposing effects.

Similarly, we see that a rise in the probability of surviving, π, also increases k. The shifts of graphs (28) and (29) are the same as Figure 4-a: a higher π means that the discount factor of the households, $\beta\pi$, increases, which promotes savings of the young agents. Additionally, a rise in π increases the labor participation of the elderly, accelerating production and capital accumulation. Again, the effect of a rise in π on w is not clarified without further restrictions on parameter values involved in the model.

Concerning the impact of population aging on per capita real income, (30) yields

$$\frac{Y_t}{N_t+\pi N_{t-1}} = \left(\frac{1+n}{1+n+\pi}\right)\left(\frac{\pi}{1+n}\right)^{1-\alpha} k^{\alpha}\left\{1+\frac{\psi\eta^{\psi-1}}{(1-\eta^{\psi})(\psi-1)}\left[(h^*)^{1-\psi}-1\right]\right\}^{1-\alpha}. \tag{33}$$

Hence, a decline in n or a rise in π leads to three effects—the two indirect effects that are represented by an increase in k and a decrease in h^*; these effects positively impact the per capita income in the steady state. The direct effect expressed by terms $\left(\frac{1+n}{1+n+\pi}\right)$ and $\left(\frac{\pi}{1+n}\right)^{1-\alpha}$ will be negative or positive when n decreases or π increases. Therefore, it still remains that population aging on the steady state level of per capita income would be negative.

(ii) Pension Reform

We have confirmed that if the fiscal authority raises the rate of payroll tax, τ, then h^* will increase, leading to a decline in the labor force participation of the elderly. It is easy to see that such an impact leads to downward shifts in both the graphs of (28) and (29), leading to a decline in k. Additionally, (33) shows that a rise in τ unambiguously lowers the steady state level of per capita income. We obtain the same outcomes, if a reduction in the

μ by the government increases the share of the pension paid to the elderly in the labor force. Conversely, if the government lowers τ or raises η, then the steady state level of per capita capital increases. Similarly, in the case of population aging, the impact of a pension reform on the steady state level of real wage cannot be clarified without imposing further restrictions on the parameter values in the model. Consequently, a policy change favoring the retired elderly, that is, a rise in the payroll tax levied on the young generation, does not necessarily increase the steady state level of the per capita pension revenue, p. As sown by Proposition 1, while a higher τ increases h^*, it may yield a negative effect on w. Although a higher τ increases h^*, its total effect on p would be negative if the steady state level of the real wage is reduced by a rise in τ.

[Figure 4-a, b]

(iii) Income Distribution

In view of (32), the relative income between a fully retired old and a working old with $h > h^*$ is expressed as

$$\frac{(1-\mu)\,p + hw}{p} = 1 + \mu\left(\frac{h}{h^*} - 1\right).$$

Hence, given h and μ, all the factors that increase labor force participation of the elderly (a reduction in h^*) enhance income discrepancy between the fully retired elderly and the working elderly, except for a reduction in μ that produces opposite effects on the relative income. For example, population aging reduces h^*, increasing the income discrepancy between the retired and active old agents. Our findings, in this section, are summarized by the following proposition:

Proposition 3 *(i) Population aging increases the steady state level of capital stock per young agents, but its impact on the steady state level of income per total population is analytically ambiguous; (ii) a less generous pension reform, that is, a reduction in* τ*, or a rise in* μ*, increases steady state levels of capital and income per capita; (iii) population aging enhances income discrepancy between the retired and working elderly, while a more generous pension reform (a rise in* τ *or a reduction in* μ*) narrows the income difference between them.*

4.3 Numerical Examples

First, we focus on the government budget constraint, equation (19), which determines the cutoff level of labor efficiency, h^* this, in turn, determines the labor force participation rate of the elderly, $\frac{1-h^*}{1-\eta}$. We assume that one period spans across 40 years and $\pi = 0.5$; this ensures that the average time span of the old agents is 20 years. We also assume that the baseline population growth rate is 2% per year; this ensures that $1 + n = (1 + 0.02)^{40} = 2.$ 208. As a baseline case, we set:

$$\psi = 1.5, \ \eta = 0.1, \quad \mu = 0.5, \quad \tau = 0.3.$$

In this baseline setting, $h^* = 0.467,$ and thus the equilibrium rate of labor force participation of the elderly is

$$\frac{L_t^0}{N_{t-1}} = \frac{1-h^*}{1-0.1} = 0.354.$$

Before examining the effects of changes in policy parameters and the degree of population aging, we first inspect the relationships between the shape of the distribution function, $F(h)$, and the cutoff level, h^*, and the rate of the labor force participation of the elderly, L_t^o/N_{t-1}. The graphs in Figure 5 depict those relationships. As confirmed in the previous analytical discussion, a decrease in heterogeneity among the elderly, that is, an increase in the shape parameter, ψ, lowers the cutoff level of efficiency. This ensures that the labor force participation rate of the elderly increases. However, an increase in the minimum efficiency, η, increases h^*. Although these facts have been confirmed in the analytical discussion, the graphs capture the nonlinear profiles clearly. Particularly, Figure 5-b reveals that the old agent becomes more homogenous (and hence ψ takes a higher value)—the rate of labor force participation of the old agent converges to about 0.4. Additionally, it must be pointed out that, as shown by Figure 5-d, in our numerical example, a higher η increases the rate of labor force participation of the elderly. If a rise in η is considered an enhancement of the health status of the elderly, then the relationship between η and L_t^o/N_{t-1}, as depicted by Figure 5-d, would fit well to the positive link between the heath condition and the labor supply of the elderly confirmed by the empirical studies mentioned in Section 1.

[Figure 5]

Figure 6-a to 6-d describe the impacts of changes in the degree of population aging, that is, a change in n or π, on h^* and $L_t^0/\pi N_{t-1}$. The graphs are depicted under alternative levels of ψ and η. The graphs demonstrate that a change in ψ exerts a relatively small effect on the graph profiles. In contrast, a change in the minimum level of efficiency, η, has a substantial effect on the graph profile. Similarly, Figures 6-e, 6-f, 6-g, and 6-h depict the relationships between policy parameters, τ and μ and h^* and L_t^o/L_{t-1}. Again, the figures show that a change in η leads to a relatively large shift in the graph.

[Fifure 6]

Similarly, the panels in Figure 7 depicts the imacts of changes in policy parameters on h^* and L_t^0/N_{t-1}, which confirm our analytical findings mentioned before.

[Figure 7]

The graphs in Figure 8 show the relationship between the steady state level of capital per population of the young generation and the key parameter values. A notable fact in these graphs is that alternative levels of ψ lead to very small shifts in the graph, whereas different levels of μ lead to a relatively large shift in the graphs.

[Figure 8]

Finally, Figure 9 gives the relationship between the steady state level of real wage and policy parameters. While the analytical discussion given above cannot specify the relationships, our numerical examples demonstrate that these relationships are mostly monotonic, and that the profiles of linkages are essentially the same as the relationship between the steady state level of k_t and the policy parameters.

[Figure 9]

5 The Case of Endogenous Growth

Until now, we have used a neoclassical growth model in which the long-run level of per capita income stays constant in the steady state equilibrium. In this section, we modify the base model to allow for the continuous growth of the per capita income in the long-run equilibrium. Our main interest in this modified model is to explore the relationship between the balanced growth rate of the economy and the labor participation of the elderly.

5.1 A Modified Model

In what follows, we employ a simple model of endogenous growth in which the aggregate production technology has an AK property. The production function of the representative firm is now replaced by the following:

$$Y_t = AK_t^{\alpha}(\bar{K}L_t)^{1-\alpha}, \quad 0 < \alpha < 1.$$

In the above, $\bar{K}_t^{1-\alpha}$ denotes the external effects of the capital that represents the intangible knowledge associated with physical capital in the sense of Romer (1986). We assume that the mass of firms is normalized to one, so that, in the equilibrium, it holds that $\bar{K}_t = K_t$. As a result, the social production function is

$$Y_t = AL_t^{1-\alpha}K_t. \tag{34}$$

Additionally, the rate of return to private capital and the real wage rate, Respectively, given by

$$r_t = \frac{\partial Y_t}{\partial K_t} = \alpha AL_t^{1-\alpha}, \tag{35}$$

$$w_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha)\, AL_t^{-\alpha}K_t. \tag{36}$$

Since the model considered here is an AK growth model with a variable labor supply, the model is associated with scale effect. Hence, to establish a feasible balanced growth equilibrium, we assume that the mass of each generation, N_t, stays constant over time so that $N_t = N$ for all $t \geq 0$.

Setting $N_t = N$ and $n = 0$ in (16) , we see that the labor market equilibrium condition is given by

$$L_t = \left\{ 1 + \frac{\pi \psi \eta^{\psi - 1}}{(1 - \eta^{\psi})(\psi - 1)} \left[(h^*)^{1-\psi} - 1 \right] \right\} N.$$

As a result, the aggregate production function and factor prices are, Respectively, expressed as follows:

$$Y_t = \hat{A} K_t, \tag{37}$$

$$r_t = \alpha \hat{A}, \tag{38}$$

$$w_t = (1 - \alpha) \hat{A} K_t \tag{39}$$

where

$$\hat{A} = A \left[1 + \frac{\psi \pi \eta^{\psi - 1}}{(1 - \eta^{-\psi})(\psi - 1)} \left[\left(\frac{h^*}{\eta} \right)^{1-\psi} - 1 \right] N \right]^{1-\alpha}. \tag{40}$$

Equation (37) means that the aggregate production technology has an AK property in which $\hat{A}$ represents an augmented total factor productivity of social capital that includes external effects associated with the aggregate private capital. From (40) , we see that all the factors that increase the labor force participation of the elderly increase $\hat{A}$; this implies that population aging (a rise in π) and a less favorable pension plan (decreases in τ and μ) gives rise to a higher TFP of the aggregate capital. This outcome is essentially the same as that in the baseline, exogenous growth model. One notable difference between exogenous and endogenous growth settings is the effect of a change in population. In the exogenous growth model, a decline in the population growth rate changes population aging, positively influencing the augmented TFP through a rise in the labor percolation of the elderly. In contrast, in the endogenous growth environment, a fall in population, N, uniformly lowers the productivity through a negative scale effect. This is because a lower N means that the size of the young generation shrinks as well, and this leads to a uniform decline in the productivity of the social level of capital.

5.2 The Balanced Growth Rate

Capital accumulation follows $K_{t+1} = s_t N$ from (18) and (23); this condition is written as

$$\frac{K_{t+1}}{N} = \frac{\beta\pi}{1+\beta\pi}(1-\tau)w_t - \frac{1}{(1+\beta\pi)(1+r_{t+1})}\left[\frac{1}{\mu} - 1 + \mu F(h^*)\right] p_{t+1}$$
$$+\frac{\psi\eta^{\psi-1}}{(\eta^{\psi}-1)(\psi-1)}\left((h^*)^{1-\psi}-1\right)w_{t+1}\bigg].$$

In view of (38) and (39), we obtain the following:

$$\frac{w_{t+1}}{(1-\alpha)\hat{A}} = \frac{\beta\pi}{1+\beta\pi}(1-\tau)Nw_t$$
$$-\frac{w_{t+1}N}{(1+\beta\pi)\left(1+\alpha\hat{A}\right)}\left[\frac{1}{\mu} - 1 + \mu F(h^*) + \frac{\psi\eta^{\psi-1}}{(1-\eta^{\psi})(\psi-1)}(1-F(h^*))\left((h^*)^{1-\psi}-1\right)\right].$$

We denote the gross balanced growth rate: $g = \frac{w_{t+1}}{w_t}$. Then, the above equation leads to

$$g = \frac{\frac{\beta\pi}{1+\beta\pi}(1-\tau)}{\frac{1}{(1-\alpha)\hat{A}N} + \frac{1}{(1+\beta\pi)\left(1+\alpha\hat{A}\right)}\left[\frac{1}{\mu} - 1 + \mu F(h^*) + \frac{\psi\eta^{\psi-1}}{(1-\eta^{\psi})(\psi-1)}(1-F(h^*))\left[(h^*)^{1-\psi}-1\right]\right]}, \tag{41}$$

which gives the balanced growth rate of the economy. It is to be noted that since the production technology holds an AK property, the economy always stays on the balanced growth path, and it holds that

$$\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = \frac{c_{t+1}}{c_t} = \frac{x_{t+1}}{x_t} = \frac{p_{t+1}}{p_t} = \frac{w_{t+1}}{w_t} = g.$$

Consequently, we have found:

Proposition 4 *In a growing economy with an AK technology and endogenous retirement, the balanced growth equilibrium is uniquely given, and the economy always stays on the balanced growth path.*

Although the balanced growth rate is uniquely given, it depends on the parameter values in a complex manner. First, consider the effect of population aging. A rise in π increases the numerator in the right-hand side of (41), while its impact on the denominator is not determined without imposing further conditions. In fact, a rise in $\hat{A}$, generated by an increase in π, has a positive impact on g. However, an increase in π lowers h^*, which yield a negative effect on g. Therefore, the total impact of population aging on the long-term growth rate of per capita income depends on the tension between these opposing effects generated by a higher π.

5.3 Numerical Examples

To obtain quantitative outcomes, we examine some numerical examples. Using the same baseline parameter values in the case of the exogenous growth model, we depict the relationship between the balanced growth rate given by (41) and the values of key parameters. The panels in Figure 10 summarize our findings[15]. As mentioned above, changes in the key parameters may yield mixed effects on the balanced growth rate of the aggregate income; this ensures that we do not have analytically unambiguous results. However, given our calibration, we see that a change in the parameters that increases the labor force participation of the elderly generally yields a positive effect on the balanced growth rate of the aggregate income. In our examples, an enhancement in population aging (a rise in π) as well as a less generous pension scheme (a decrease in τ) accelerates long-term economic growth. An exception is the effect of a rise in μ. As panels (e) and (f) in Figure 10 demonstrate, the relation between μ and the balanced growth rate is inversely U-shaped. Namely, a rise in μ (a less favorable pension plan for the working elderly) first accelarates long-run growth, while it yields a negative impact on growth if μ exceeds a threshold level.

The graphs in Figure 10 also reveal that the relationship between the balanced growth rate and the key parameters are sensitive to the shape of the distribution function of h. For example, comparing Figures 10-a, 10-c, and 10-f, we see that the relationship between the balanced growth rate and the tax rate, τ, is insensitive to alternative specifications of the shape parameter, ψ, whereas the relationship between g and π or the relationship between g and μ are relatively sensitive to the level of ψ.

[Figure 10]

[15] The balanced growth rate of income in Figure 9 measures the annual growth rate.

6 Conclusion

In general, the heterogeneity among the elderly is wider than that among the young people. In particular, discrepancy in health status and motivation for labor force participation are more prominent among the elderly than among the young. This study relies on this simple fact to determine the aggregate labor force participation of the elderly. In our model, the heterogeneity among the elderly is represented by a distribution function of their labor efficiency. We have shown that the aggregate level of labor force participation of the elderly is determined by pension schemes, the level of population aging, and the profile of distribution of labor efficiency among the elderly. We investigate how changes in pension scheme and the level of population aging affects the long-run performance of the aggregate economy. Our model shows that population aging and a decline in the payroll tax levied on the young generation promote labor force participation of the old generation, which increases the aggregate labor supply. At the same time, a higher income for the elderly increases their consumption. While a higher labor supply enhances production and investment, a higher income of the elderly increases their consumption, which has a negative impact on investment. The long-run effect of a rise in labor supply of the elderly on economic growth hinges upon the relative strengths of these opposing effects generated by a change in the labor force participation of the elderly. To investigate the net effect of a change in the elderly's labor supply on the long-term growth, we examined some numerical examples. Our numerical experiment demonstrates that an increase in the elderly's labor force participation enhances capital accumulation and long-run growth.

In order to obtain clear analytical outcomes, we have set some restrictive assumptions. Particularly, we have assumed that payroll taxes levied on the young generation is proportional to their wage income and that all the tax revenue of the government to expenditure for pension. Due to these restrictions, the aggregate labor participation rate of the elderly stays constant during the transition process of the economy. If we assume an alternative pension scheme in which the per capital level of pension is fixed, then it can be shown that the threshold level of labor efficiency, h_t^*, does not stay constant over time, leading to more complex aggregate dynamics than that treated in this study. Similarly, if we assume that the tax revenue of the government is spent for other purposes, in addition to pension, then the cutoff level, h_t^*, will change over time. Such an extension, again, requires us to treat a more complex dynamic analysis. This means that we should heavily rely on numerical considerations rather than analytical arguments. Finally, when discussing a model with endogenous growth, we use a simple AK growth model in which production externality sustains continuous growth. In a careful discussion of long-run growth with labor supply of the elderly, we need to consider alternative engines of growth such as human capital investment and R&D activities of firms. These possible extensions of our base model would be promising.

Appendix

Let us express $\Gamma(w_{t+1})$ as

$$\begin{aligned}\Gamma(w_{t+1}) &= \frac{(1+n)(1+\beta\pi)}{(1-\tau)\beta\pi}\Phi[(1-\alpha)A]^{-\frac{1}{\alpha}} w_{t+1}^{\frac{1}{\alpha}} \\ &\quad + \frac{1+\beta\pi}{(1-\tau)\beta\pi} \times \frac{\left[\frac{1}{\mu}-1+F(h^*)\right] h^* w_{t+1} + \frac{\psi\eta^{\psi-1}}{(1-\eta^{\psi})(\psi-1)}\left[1-F(h^*)\right]\left[(h^*)^{1-\psi}-1\right] w_{t+1}}{(1+\beta\pi)\left(1+A\left[(1-\alpha)A\right]^{\frac{1}{\alpha}-1} w_{t+1}^{1-\frac{1}{\alpha}}\right)} \\ &= B_0 w_{t+1}^{\frac{1}{\alpha}} + \frac{B_1 w_{t+1}}{(1+\beta)\left(1+B_2 w_{t+1}^{1-\frac{1}{\alpha}}\right)},\end{aligned}$$

where

$$B_0 = \frac{1+\beta\pi}{(1-\tau)\alpha\beta\pi}(1+n)\Phi[(1-\alpha)A]^{\frac{1}{\alpha}} > 0,$$

$$B_1 = \left[\frac{1+\beta\pi}{(1-\tau)\beta\pi}\right]\left[\left(\frac{1}{\mu}-1+\mu F(h^*)\right)h^* + \frac{\psi\eta^{\psi-1}}{\mu(1-\eta^{\psi})(\psi-1)}\left[1-F(h^*)\right]\left((h^*)^{1-\psi}-1\right)\right] > 0,$$

$$B_2 = A\left[(1-\alpha)A\right]^{\frac{1}{\alpha}-1} > 0.$$

Then, we find the following :

$$\Gamma'(w_{t+1}) = \frac{1}{\alpha}B_0 w_{t+1}^{\frac{1}{\alpha}-1} + \frac{B_1}{(1+\beta)\left(1+B_2 w_{t+1}^{1-\frac{1}{\alpha}}\right)} + \left(\frac{1}{\alpha}-1\right)\frac{B_1 B_2 w_{t+1}^{1-\frac{1}{\alpha}}}{\left(1+B_2 w_{t+1}^{1-\frac{1}{\alpha}}\right)^2} > 0,$$

$$\begin{aligned}\Gamma''(w_{t+1}) &= \frac{1}{\alpha}\left(\frac{1}{\alpha}-1\right)B_0 w_{t+1}^{\frac{1}{\alpha}-2} + \left(\frac{1}{\alpha}-1\right)\frac{B_1 B_2 w_{t+1}^{-\frac{1}{\alpha}}}{\left(1+B_2 w_{t+1}^{1-\frac{1}{\alpha}}\right)^2} \\ &\quad - \left(\frac{1}{\alpha}-1\right)^2 \frac{B_1 B_2 w_{t+1}^{-\frac{1}{\alpha}}}{\left(1+B_2 w_{t+1}^{1-\frac{1}{\alpha}}\right)^2} + \left(\frac{1}{\alpha}-1\right)^2 \frac{2B_1 B_2^2 w_{t+1}^{1-\frac{2}{\alpha}}}{\left(1+B_2 w_{t+1}^{1-\frac{1}{\alpha}}\right)^3}\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{\alpha}\left(\frac{1}{\alpha}-1\right) B_0 w_{t+1}^{\frac{1}{\alpha}-2} + \left(\frac{1}{\alpha}-1\right) \frac{B_1 B_2 w_{t+1}^{-\frac{1}{\alpha}}}{\left(1+B_2 w_{t+1}^{1-\frac{1}{\alpha}}\right)^2} \\
&+ \left(\frac{1}{\alpha}-1\right)^2 \frac{B_1 B_2 w_{t+1}^{-\frac{1}{\alpha}}}{\left(1+B_2 w_{t+1}^{1-\frac{1}{\alpha}}\right)^2} \left[\frac{2B_2 w_{t+1}^{1-\frac{1}{\alpha}}}{1+B_2 w_{t+1}^{1-\frac{1}{\alpha}}} - 1\right]
\end{aligned}
$$

$$
= \frac{1}{\alpha}\left(\frac{1}{\alpha}-1\right) B_0 w_{t+1}^{\frac{1}{\alpha}-2} + \left(\frac{1}{\alpha}-1\right)^2 \frac{B_1 B_2 w_{t+1}^{-\frac{1}{\alpha}}}{\left(1+B_2 w_{t+1}^{1-\frac{1}{\alpha}}\right)^2} \left[\frac{1}{\frac{1}{\alpha}-1} - 1 + \frac{2B_2 w_{t+1}^{1-\frac{1}{\alpha}}}{1+B_2 w_{t+1}^{1-\frac{1}{\alpha}}}\right].
$$

It is easy to see that $\alpha \in (0,1)$ means that $\Gamma(0) = 0$ and $\Gamma(+\infty) = +\infty$. Moreover, we find that $\frac{1}{\frac{1}{\alpha}-1} - 1 > 0$ for $\alpha \in (0.5, 1)$.

Therefore, $\Gamma''(w_{t+1}) > 0$ for all $w_{t+1} > 0$ if $0.5 < \alpha < 1$.

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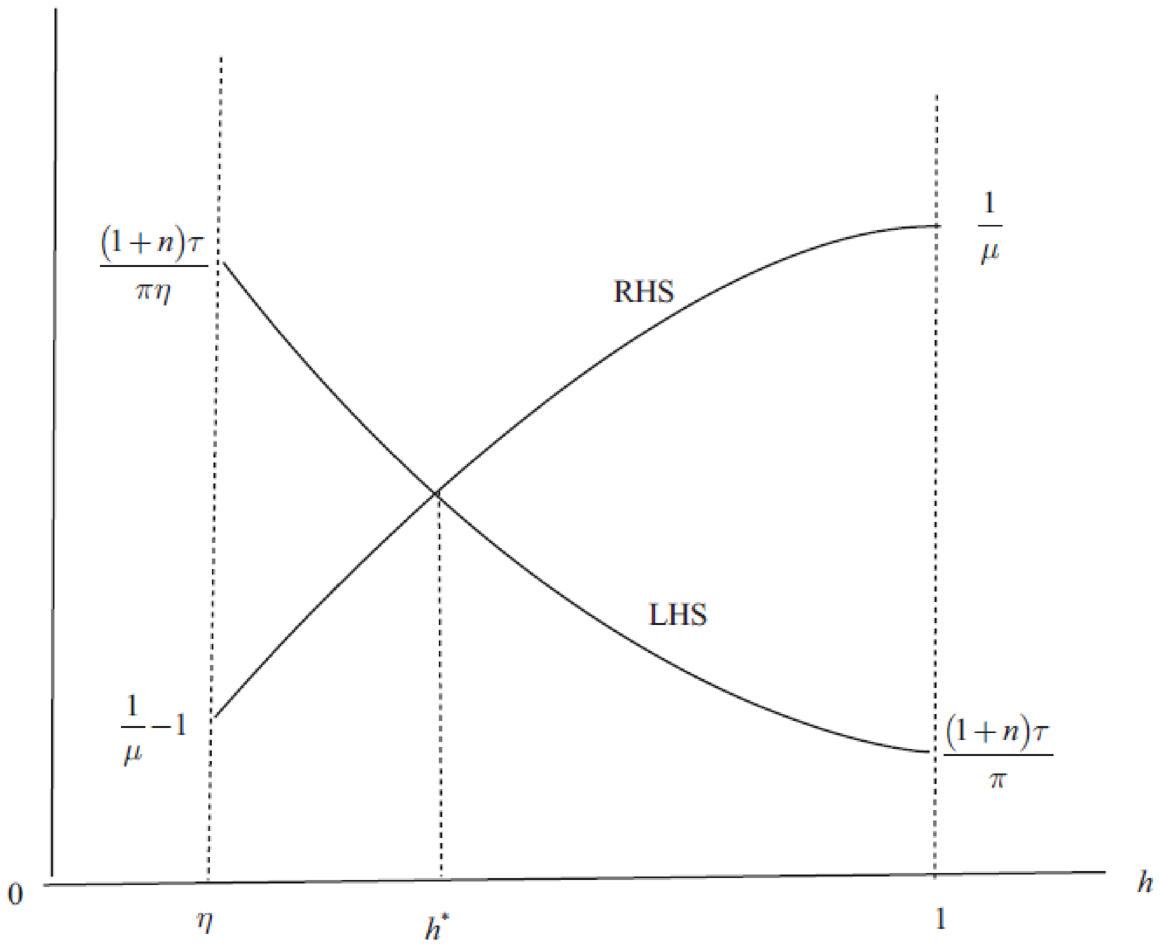


1

Figure 1: Existence of h^*

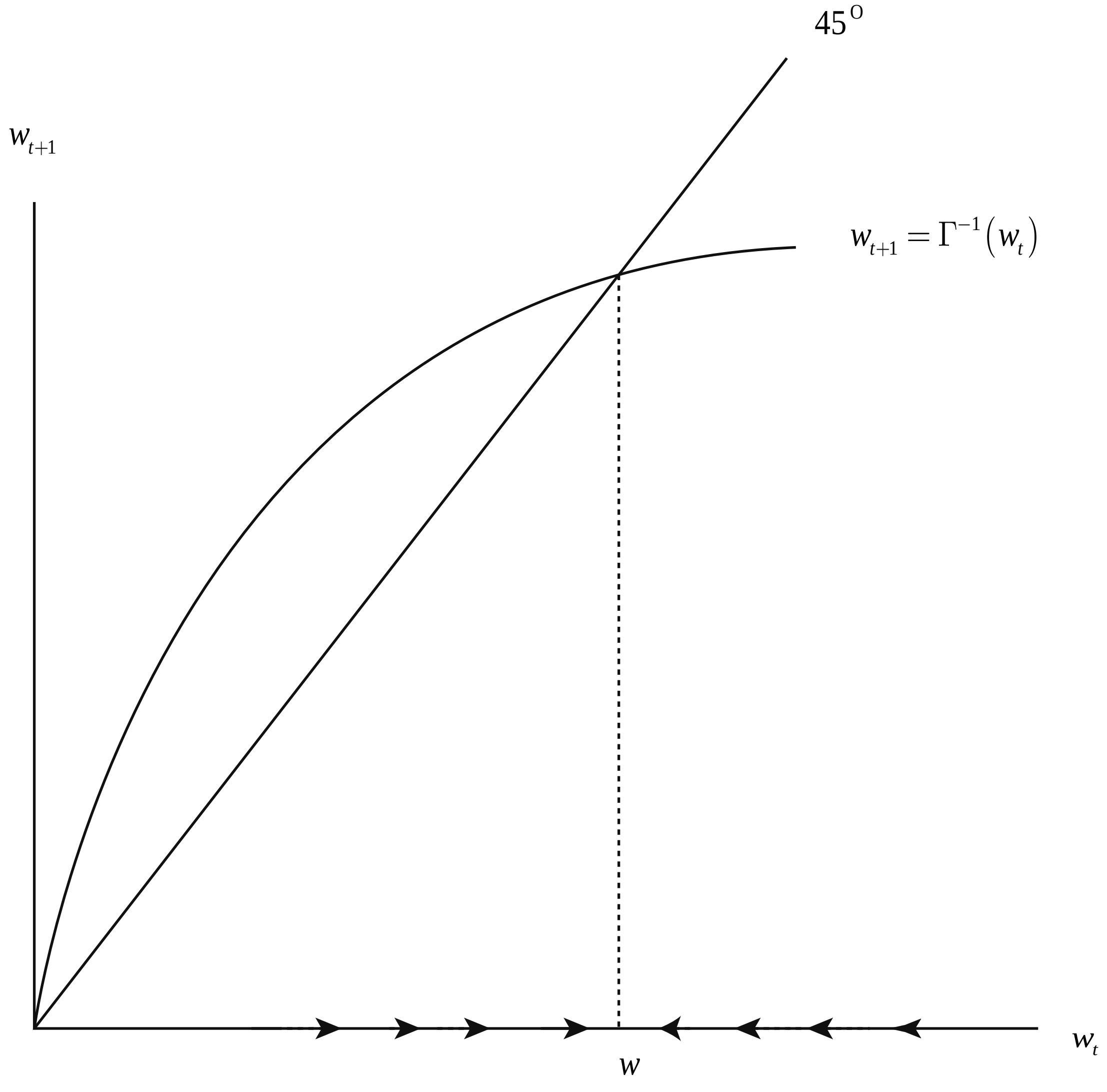
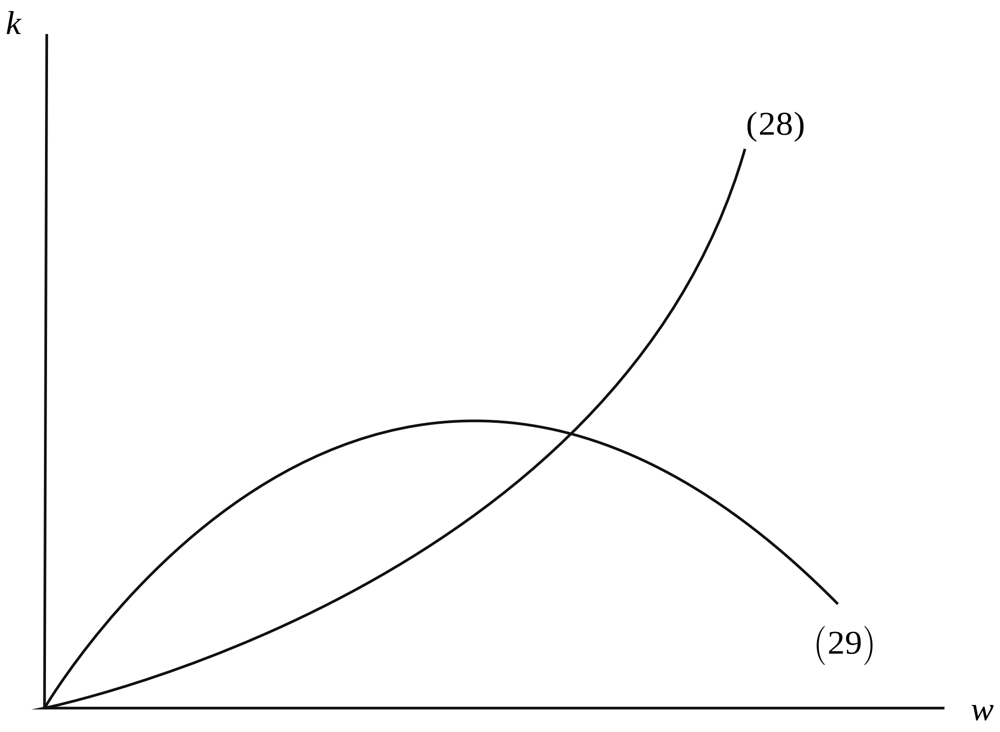


Figure 2



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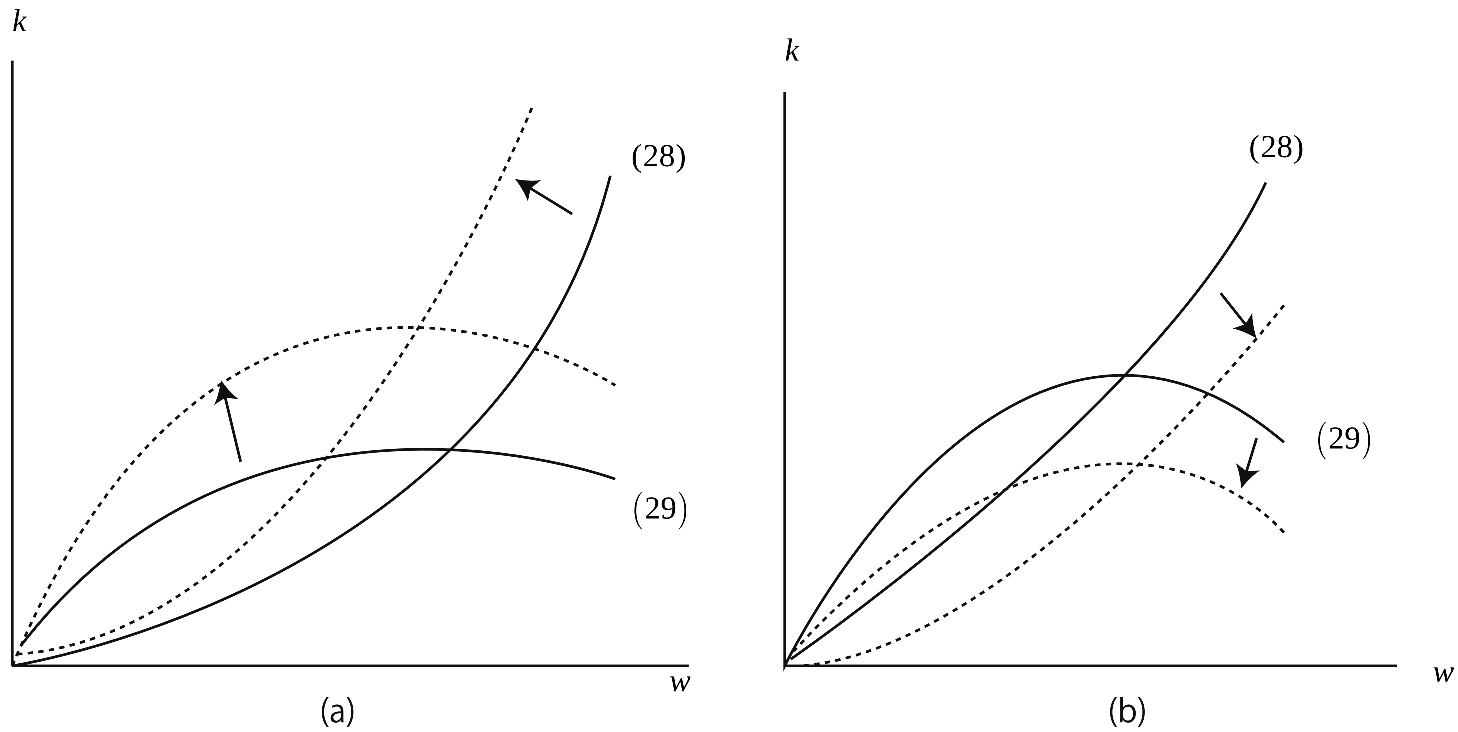

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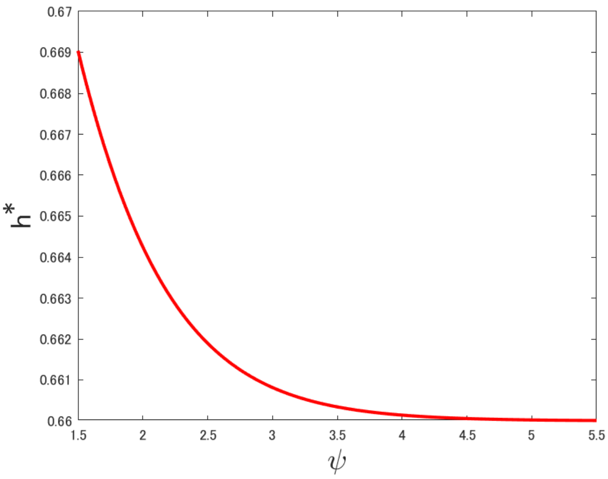

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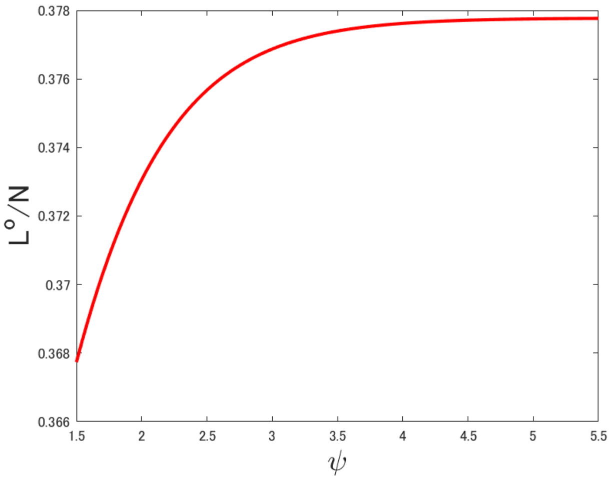

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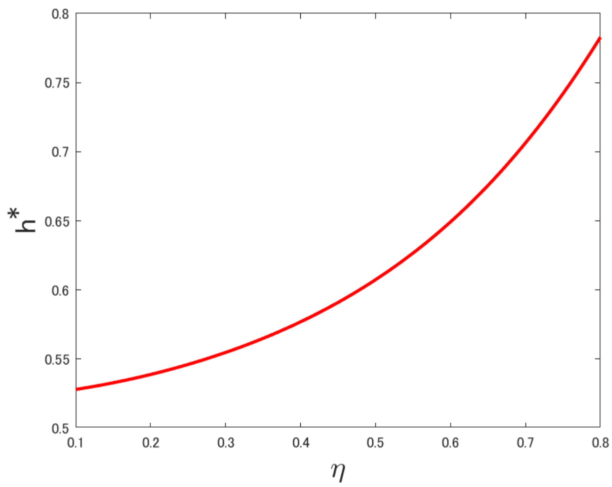

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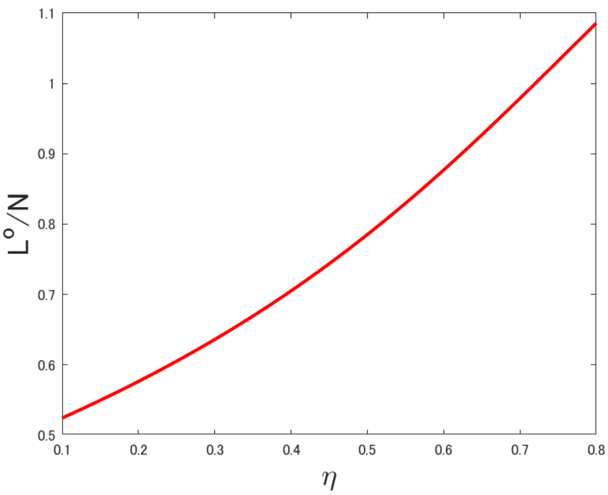
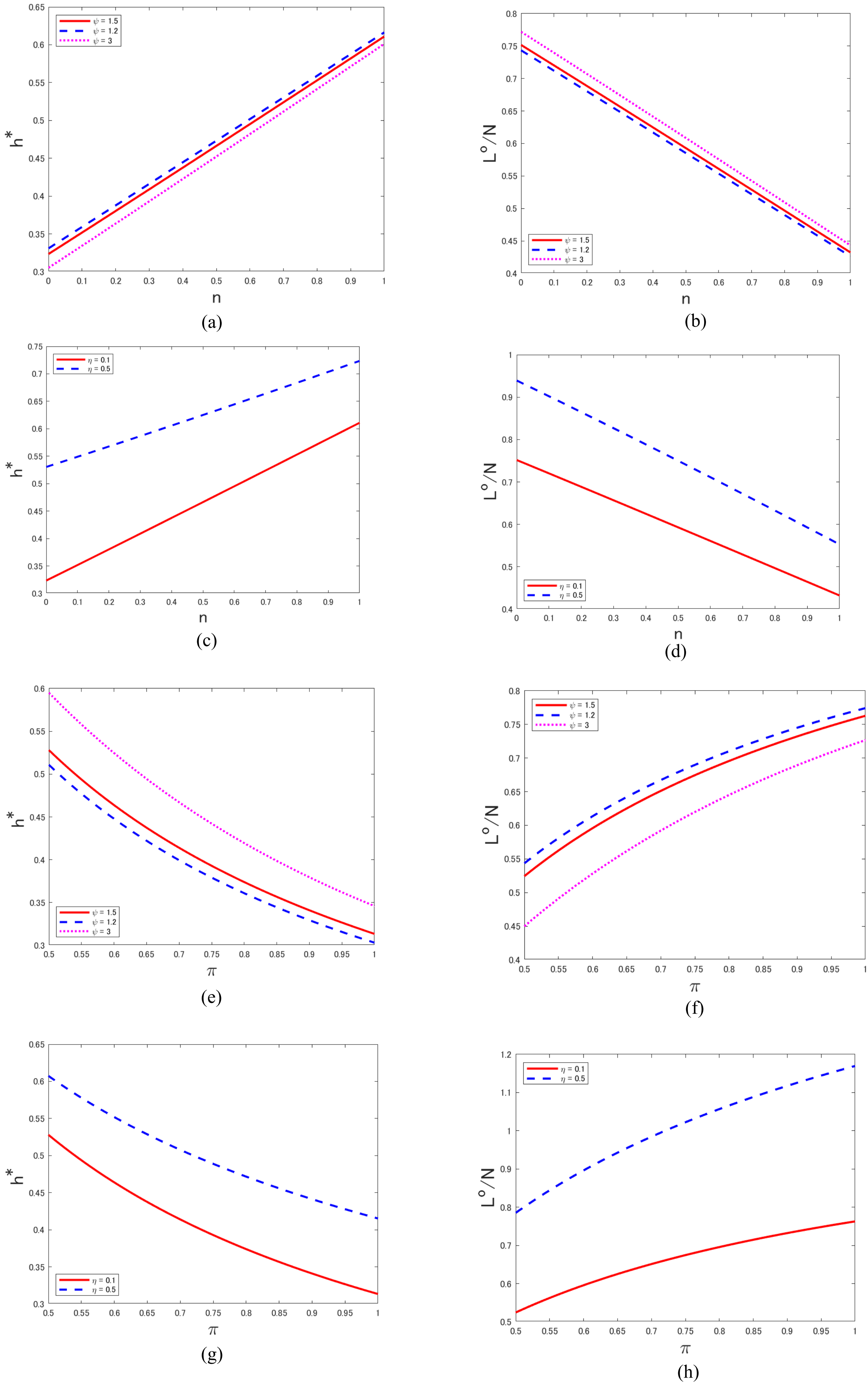


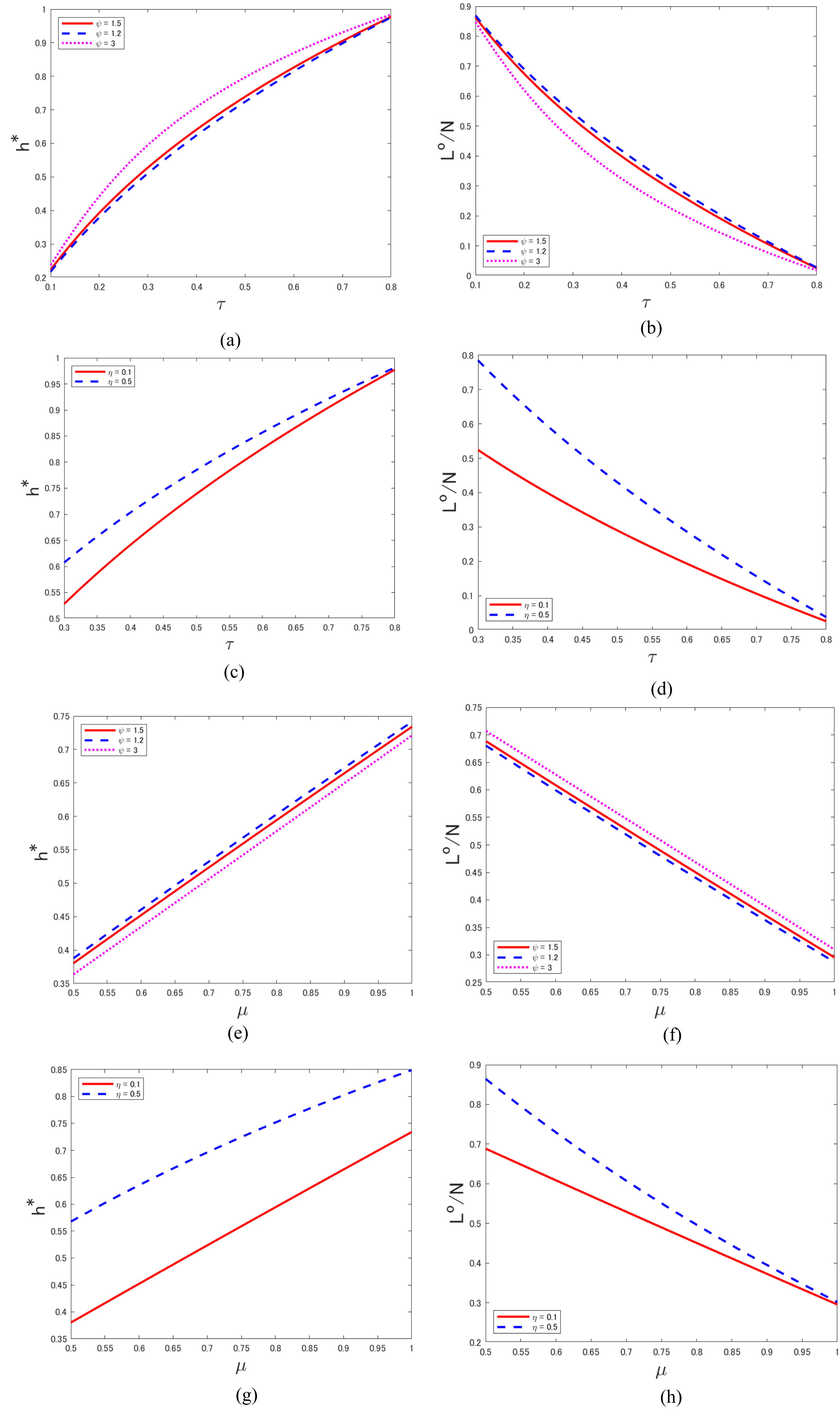
(d)

Figure 5



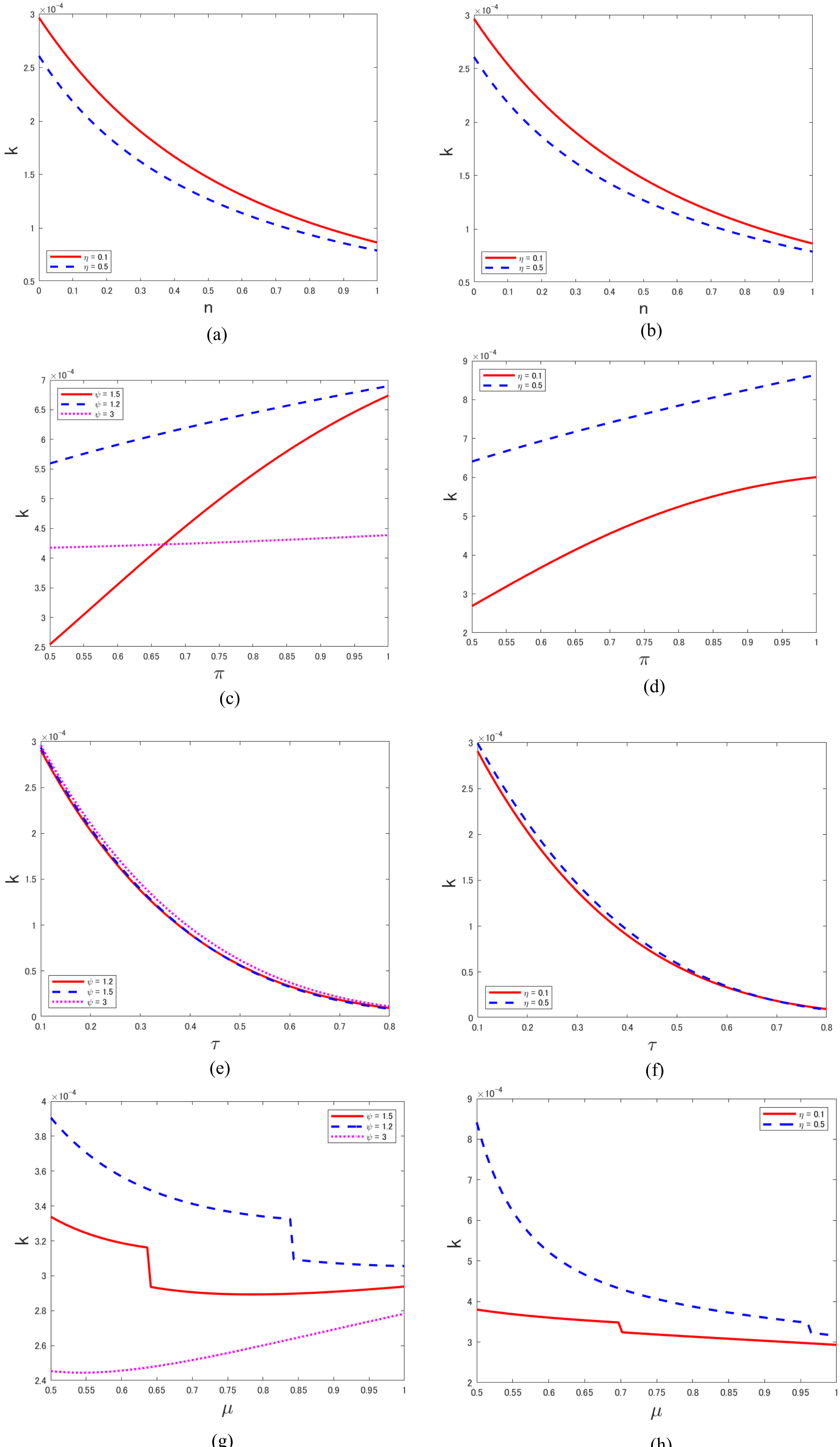
ψ = 1.5
ψ = 1.2
ψ = 3
h^*
n
(a)
ψ = 1.5
ψ = 1.2
ψ = 3
L^o/N
n
(b)
η = 0.1
η = 0.5
h^*
n
(c)
η = 0.1
η = 0.5
L^o/N
n
(d)
ψ = 1.5
ψ = 1.2
ψ = 3
h^*
π
(e)
ψ = 1.5
ψ = 1.2
ψ = 3
L^o/N
π
(f)
η = 0.1
η = 0.5
h^*
π
(g)
η = 0.1
η = 0.5
L^o/N
π
(h)

Figure 6



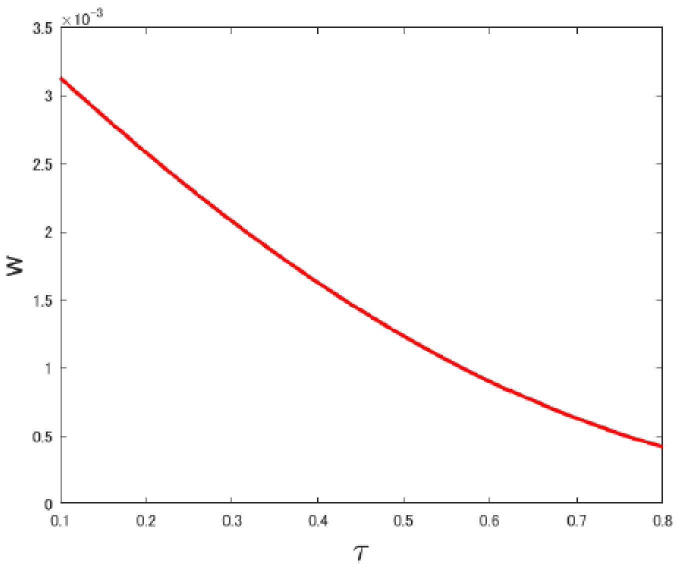
h*
τ
ψ = 1.5
ψ = 1.2
ψ = 3
(a)
L°/N
τ
ψ = 1.5
ψ = 1.2
ψ = 3
(b)
h*
τ
η = 0.1
η = 0.5
(c)
L°/N
τ
η = 0.1
η = 0.5
(d)
h*
μ
ψ = 1.5
ψ = 1.2
ψ = 3
(e)
L°/N
μ
ψ = 1.5
ψ = 1.2
ψ = 3
(f)
h*
μ
η = 0.1
η = 0.5
(g)
L°/N
μ
η = 0.1
η = 0.5
(h)

Figure 7



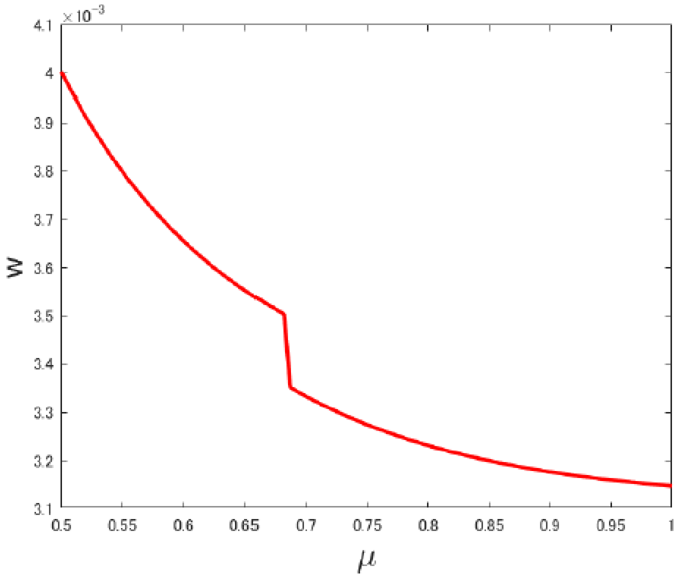
(a)
k
n
×10^-4
η = 0.1
η = 0.5
(b)
k
n
×10^-4
η = 0.1
η = 0.5
(c)
k
π
×10^-4
ψ = 1.5
ψ = 1.2
ψ = 3
(d)
k
π
×10^-4
η = 0.1
η = 0.5
(e)
k
τ
×10^-4
ψ = 1.2
ψ = 1.5
ψ = 3
(f)
k
τ
×10^-4
η = 0.1
η = 0.5
(g)
k
μ
×10^-4
ψ = 1.5
ψ = 1.2
ψ = 3
(h)
k
μ
×10^-4
η = 0.1
η = 0.5

Figure 8



$\times 10^{-3}$
3.5
3
2.5
2
1.5
1
0.5
0
W
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
τ

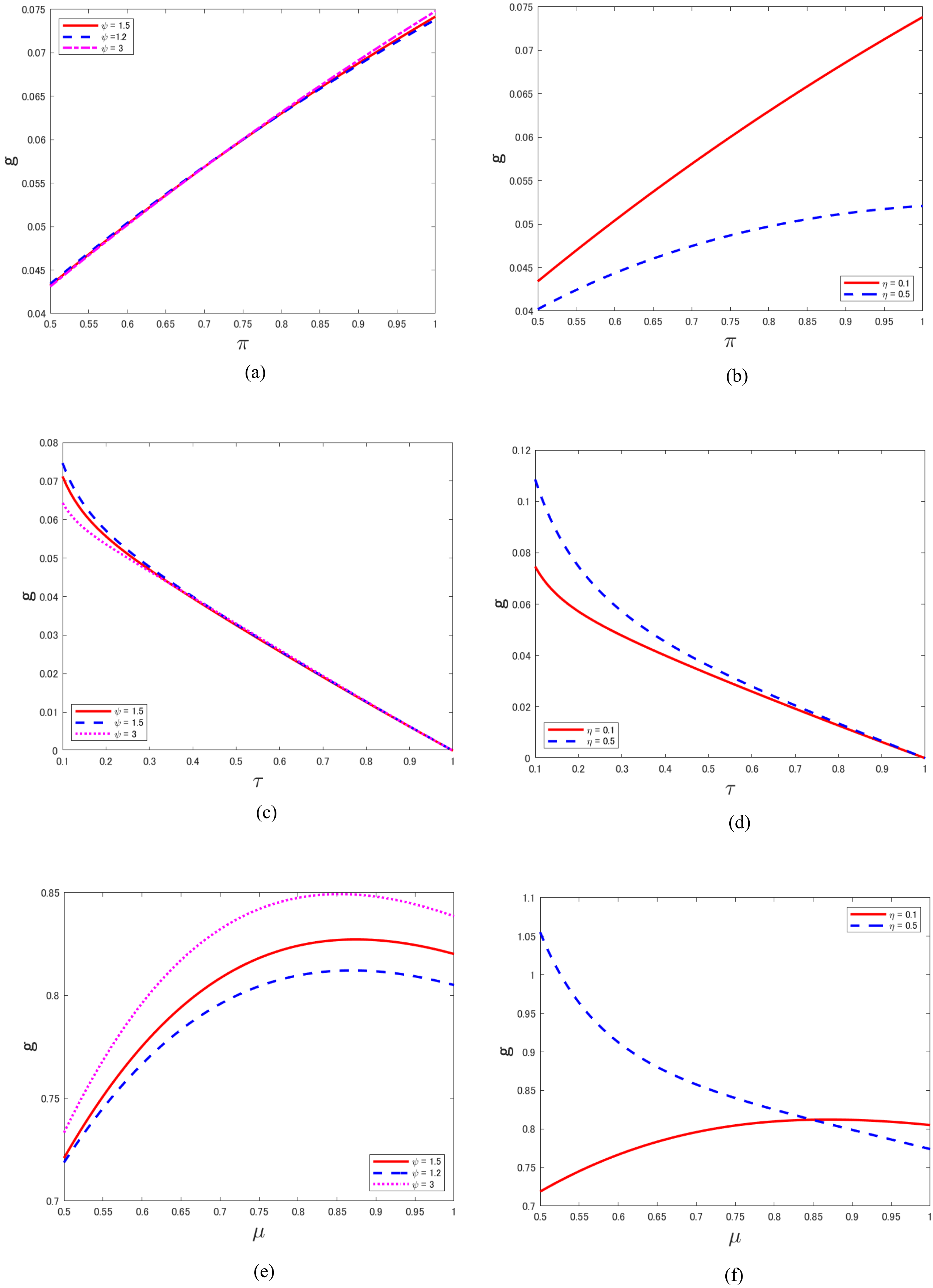
(a)



$\times 10^{-3}$
4.1
4
3.9
3.8
3.7
3.6
3.5
3.4
3.3
3.2
3.1
W
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
μ

(b)

Figure 9



g
π
ψ = 1.5
ψ =1.2
ψ = 3
(a)
g
π
η = 0.1
η = 0.5
(b)
g
τ
ψ = 1.5
ψ = 1.5
ψ = 3
(c)
g
τ
η = 0.1
η = 0.5
(d)
g
μ
ψ = 1.5
ψ = 1.2
ψ = 3
(e)
g
μ
η = 0.1
η = 0.5
(f)

Figure 10